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Application Of Bilinear Forms And Quadratic Forms

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Abstract:

In this paper I study bilinear forms on finite dimensional vector spaces. Then define a matrix of a bilinear form and verification of problems about the bilinear then we discussed to Quadratic form and their application to solve Reduction of a quadratic form to the diagonal form.

Keywords:

Bilinear, vector space, symmetric, Quadratic, diagonal.

Introduction:

Consider a finite dimensional, inner product space V over the field R of real numbers. The inner product is a function form $V \times V$ to R satisfying the following conditions.

(i) $<\alpha u_1 + \beta u_2, v> = \alpha <u_1, v> + \beta <u_2, v>$

(ii) $<u, \alpha v_1 + \beta v_2> = \alpha <u, v_1> + \beta <u, v_2>$

In other words, the inner product is a scalar valued function of the two variables u and v and is a linear function in each of the two variables.

This type of scalar valued functions is called bilinear forms. In this paper we introduce the concepts of the Bilinear form using finite dimensional vector space.

Preliminaries:

Definition:1

Let V be a vector space over a field F. A bilinear form on V is a function f: $V \times V \to F$. Such that

(i) $f(\alpha u_1 + \beta u_2, v) = \alpha f(u_1, v) + \beta f(u_2, v)$

(ii) $f(u, \alpha v_1 + \beta v_2) = \alpha f(u, v_1) + \beta f(u, v_2)$ Where $\alpha, \beta \in F$ and $u_1, u_2, v_1, v_2 \in V$.

In other words, f is linear as a function of any one of the two variables when the other is fixed.

Remark:

b (o, v) = b(v, o) = 0.

Examples:1

- b (x, y) =<x, y> in R^n is bilinear and symmetric for any scalar product.
- $b((x_1, y_1), (x_2, y_2)) = x_1x_2 + 2x_1y_2 + 3y_1x_2 + 4y_1y_2$ is bilinear, but not symmetric.

Definition:2

Let C = $V_1, \ldots, V_n$ basic of V and let b be a bilinear form on V. The matrix of b with respect to C is

$$[b]_c = \begin{bmatrix} b(v_1, v_1) & b(v_1, v_2)\ldots & b(v_1, v_n) \\ b(v_2, v_1) & b(v_2, v_2)\ldots & b(v_2, v_n) \\ b(v_n, v_1) & b(v_n, v_2)\ldots & b(v_n, v_n) \end{bmatrix}$$

Theorem:1

Let V be a vector space over a field F. Then L(V, V, F) is a vector space over F under addition and scalar multiplication defined by

$(f+g)(u, v)=f(u, v)+g(u, v)$ and $(\propto f)(u, v) = \propto f(u, v)$

Proof:

Let f, $g \in L(V, V, F)$ and $\alpha_1 \in F$.

We claim that f+g and $\alpha_1 f \in L(V, V, F)$.

$(f+g)(\alpha u_1 + \beta u_2, v) = f(\alpha u_1 + \beta u_2, v) + g(\alpha u_1 + \beta u_2, v)$

$$= \alpha f(u_1, v) + \beta f(u_2, v) + \alpha g(u_1, v) + \beta g(u_2, v)$$

$$= \alpha[f(u_1, v) + g(u_1, v)] + \beta[f(u_2 v) + g(u_2, v)]$$

$$=\alpha\left[(f+g)\left(u_1,v\right)\right]+\beta[(f+g)\left(u_2,v\right)]$$

Similarly, we can prove that

(f+g)(u,$\alpha v_1+\beta v_2) = \alpha\left[(f+g)\left(u,v_1\right)\right]+\beta[(f+g)\left(u,v_2\right)]$

Hence (f+g))$\in L(V,V,F)$.

Also $(\alpha_1 f)(\alpha u_1+\beta u_2,v)$

$$=\alpha_1 f\left(\alpha u_1+\beta u_2,v\right)$$

$$=\alpha_1\left[\alpha f\left(u_1,v\right)+\beta f\left(u_2,v\right)\right]$$

$$=\alpha_1\alpha f\left(u_1,v\right)+\alpha_1\beta f\left(u_2,v\right)$$

$$=\alpha\left[\left(\alpha_1 f\right)\left(u_1,v\right)\right]+\beta[\left(\alpha_1 f\right)\left(u_2,v\right)]$$

Similarly

$$\left(\alpha_1 f\right)\left(u,\alpha v_1+\beta v_2\right)=\alpha[\left(\alpha_1 f\left(u,v_1\right)\right]+\beta[\left(\alpha_1 f\right)\left(u\ ,v_2\right)]$$

$\therefore \alpha_1 f\in L(V,V,F)$.

The remaining axioms if a vector space can be easily verified.

Definition:3

A bilinear form f defined on a vector space V is called **symmetric bilinear form** if f(u, v)=f(u,v) for all u,v$\in V$.

Definition:4

Let f be a symmetric bilinear form defined by V. Then the **quadraticform** associated with f is the mapping q: v$\to F$ defined by q(v)=f(v, v). The matrix of the bilinear form f is called the matrix of the associated quadratic form q.

Theorem:2

Let g be a symmetric bilinear form defined on V. Let q be the associated quadratic form.

(i) f(u, v)=$\frac{1}{4}$ {q(u+v)-q(u-v)}

(ii) f(u, v) =$\frac{1}{2}${q(u+v)-q(u)-q(v)}

Proof:

(i) $\frac{1}{4}\{q(u+v)-q(u-v)\} = \frac{1}{4}\{f(u+v, u+v)-f(u-v, u-v)\}$

$=\frac{1}{4}\{f(u, u)+f(u, v)+f(v, u)+f(v, v)-f(u, u)+f(u, v)+f(v, u)\}$

$=\frac{1}{4}\{4f(u, v)\}$

$= f(u, v).$

(ii) $\frac{1}{2}\{q(u+v)-q(u)-q(v)\}$

$=\frac{1}{2}\{f(u+v, u+v)-f(u)-f(v)\}$

$=\frac{1}{2}\{f(u,u) + f(u,v) + f(v,u) + f(v,v) -f(u,u)- f(v,v)$

$=\frac{1}{2}\{2f(u, v)\}$

$= f(u, v).$

Problem:1

Find the matrix of the bilinear form

$F(x, y)=x_1y_2-x_2y_1$ with respect to the standard basis $v_2(R)$.

Proof:

$a_{11} = (e_1,e_1)=f((1,0), (1,0))=(1). (0) – (0). (1) =0$

$a_{12} = (e_1,e_2)=f((1,0), (0,1)) = (1). (1) - (0). (0) =1$

$a_{21} = (e_2,e_1)=f((0,1), (1,0)) = (0). (0) - (1). (1) =-1$

$a_{22} = (e_2,e_2)=f((0,1), (0,1)) = (0). (1)-(1). (0) =0$

The matrix of $f_i \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

Definition:5

$a_1x_1^2+a_2x_2^2+..........+a_nx_n^2$

Which is known as the **diagonal form**.

Definition:6

Let f be a bilinear form on V. Fix a basis $\{v_1,v_2,......v_n\}$for v.

Let u= $\alpha_1 v_1 + ... + \alpha_n v_n$ and v= $\beta_1 v_1 + ...\beta_n v_n$.

Then f (u, v)

$$= f\,(\alpha_1 v_1 + ... + \alpha_n v_n, \beta_1 v_1 + ... + \beta_n v_n)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \alpha \quad \beta$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} \alpha_i \beta_j \quad \text{where } f\,(v, v_j) = a_{ij}$$

$$=(\alpha_1, ...\alpha_n) \begin{bmatrix} a_{11}.......a_{1n} \\ .... \\ a_{n1}.......a_{nn} \end{bmatrix} \begin{bmatrix} \beta_1 \\ ..... \\ \beta_n \end{bmatrix}$$

∴ f(u, v)=XAY^T Where

X= $(\alpha_1, ...\alpha_n), A = \left(a_{ij}\right) and Y = \left(\beta_1, ...\beta_n\right)$.

The n×n matrix A is called the **matrix of the bilinear form** with respect to the chosen basis.

Conversely, given any n×n matrix A=(a_{ij}) the f: V×V→F defined by f(u, v)=XAY^T is a bilinear form on V and f(v_i,v_j)=a_{ij}. Also, if g is any other bilinear form on V such that g(v_i,v_j)=a_{ij}, then f = g.

Problem:2

find the matrix of quadratic form $x_1^2+4x_1x_2+3x_2^2$,v_2 in (R)

Solution

$Q(x_1,x_2) = x_1^2+4x_1x_2+3x_2^2$

$$x_1^2+4x_1x_2+3x_2^2 = (x_1,x_2)\begin{bmatrix} a & b \\ b & c \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= ax_1^2+2bx_1x_2+cx_2^2$$

a=1, b=4/2, c=3

$$A=\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

Conclusion:

In this paper I have discussed on Application of Bilinear forms and Quadratic forms and studies some of properties. Also, I provided a constructive characterization for all Application of Bilinear forms and Quadratic forms. This construction is a promising tool for proving further properties of Application of Bilinear forms and Quadratic forms.

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