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GENERALIZED PROPERTIES OF MATRIX NORMS AND ITS APPLICATION:-

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Abstract:- Actually norms can be regarded as generalization of the absolute value function of numbers.In this paper we discuss about the characterization of M-matrices and H-matrices,With positive diagonal elements.Also we explain about the different types of norms of matrices and relation between the matrix norms and spectral radius.The analysis of matrix based algorithms often requires use of matrix norms.These algorithms needs a way to quantify the size of matrix.It is our interesting fact to see how this study culverted the properties of matrix norms specialy H-matrix and M-matrix which gives a new concept on linear algebra.In this ellustrative article we demonstrate why one would want to study different type of matrix norms.

Keywords:- H-matrix,M-matrix,notation of norm,spectral radius,matrix norms computation,Conditioning number,Frobenius norm,Submultiplicative norms.

1.Introduction:-

The study of linear algebra has become highly demotic in the last few decades.People are charmed to this topics as of its fineness and connections with many other pure and applied areas.In theoretical improvement of this topics as well as many applications we necessity

to compute the size of matrix.For this purpose norm functions are considered on vector space.We discuss how the matrix norm on R^n can be obtained from eigenvalues of an associated matrix and computed norm of matrix.Now we starting about the applications of matrix norms.

2. Application of Matrix norms:-

Matrix norms use to measure the size of a matrix which is used in determining whether the solution,x of a linear system Ax=b can be trusted,and determining the convergence rate of a vector sequence,among other things.Also it allows to quantity the difference between matrices.Matrix Norm is most useful in the study of finite Markov chain in the field of probability.

Next we demonstrate about the M-matrix and H-matrix.

Definition-1.

M-matrix:-

A , Z-matrix X is called a M-matrix if it satisfies any one of the following equivalent conditions.

a. All principal minors of the matrix X are positive.

b. The leading principal minors of matrix X are positive.

c. X can be written in the form X=αI-Y where Y is a non-negative matrix whose spectral radius is strictly less than α.

d. All real eigen values of X are positive.

e. The real part of any eigen value of X is positive.

f. The matrix X is non-singular and the inverse of X is not negative.

g. X+D is non singular for every non negative diagonal matrix D.

h. X+αI is non-singular for all α≥0.

i. There exist a positive diagonal matrix D such that the matrix $DX+X^tD$ is positive definition.

j. The matrix X can be factorized as LU decomposition where L is lower triangular , U is upper triangular and the diagonal elements of both L and U are positive.

Now we define about H-matrix as follows.

Definition 1.1:-

H-matrix :-

The definition of H- matrix is related with M- matrix.Let $X=x_{ij}$ be a n× n complex matrix.Now we define the comparison matrix of X such as

$M(X)=a_{ij}$ by

a_{ij}= X

, if i=j

=- X
, if i≠j

If X= M(X) and eigenvalues of the matrix X have positive real parts.Then the matrix X is H-matrix if M(X) is a M-matrix. Next we will discuss about the norm of different matrix and their properties.

3. Matrix Norms:-

For simplicity of our demonstration about this article, we will consider the vector spaces $A_n(R)$ and $A_n(C)$ of square matrices.

Most results also hold for the spaces $A_{m\times n}(R)$ and $A_{m\times n}(C)$ of rectangular matrices .Since n×n matrices can be multiplieds, the idea behind matrix norms is that they should behave "well" with respect to matrix multiplication.

Definition 1.2:- A matrix norm $\|\|$ on the space of

square n × n matrices is $A_n(F)$, with F= R or F = C, is a norm on the vector space $A_n(F)$ with the additional property that

$\|XY\| \leq \|X\|\|Y\|, \forall X,Y \in A_n(F)$

Since $I^2=I$,from $\|I\|=\|I\|\ \| \leq \|I\|$ then we get $\|I\| \geq I$, for every matrix norm

Before giving examples of matrix norms, we necessary to recall the basic definitions about matrices. Given any matrix X= X ∈ $A_{m\times n}$ (C),the conjugate X of X is the matrix such that

$X_{ij} = X_{ij}$, $1\leq i \leq m$, $1 \leq j \leq n$.

The transpose of X is the n × m matrix X^t such that

X = x ,$1 \leq i \leq m$, $1 \leq j \leq n$.

If X is a real matrix { X ∈ $A_n(R)$}, we say that X is symmetric if

and if X∈$A_n(R)$, it is normal if $XX^t = X^tX$.

Where R and C indicate set of real and complex number and set of complex number respectively.

A matrix P∈ $A_n(C)$ is unitary if $PP_* = P_*P = I$.

A real matrix U ∈ $A_n(R)$ is orthogonal if

$UU^t = U^tU = I$,Where I indicate the identity matrix.

Given any matrix X= x ∈ $A_n(C)$ or X ∈ x ∈ $A_n(R)$,the trace tr(X) of X is the sum of its diagonal entries

$tr(X) = x_{11} + x \quad + ... + x_{nn}$

It is not hard to show that trace is a linear mapping, so that tr(αX) = αtr(X)

and tr(X + Y) = tr(X)+ tr(Y)

Furthermore, if X is a m×n and Y is a n×m matrix, it is easy to proved that

tr(XY) = tr(YX).

Also we recall about eigenvalues and eigenvectors. We captivate ourselves with definition including matrices.

Definition 1.3.

Given any square matrix X ∈ $A_n(C)$, a complex number α ∈ C is an eigenvalue of X if there is some nonzero vector I ∈ C^n, such that

XI = αI.

If α is an eigenvalue of X then the nonzero vectors $l \in C^n$ such that $Xl = \alpha l$ are called eigenvectors of X connected with α; along with the zero vector, these eigenvectors form a subspace of C denoted by $V_\alpha(X)$, and called the eigenspace connected with α.

Remark-1: Noted that Definition 1.3 need an eigen vector to be nonzero.

A somewhat unfortunate consequence of this necessity is that the set of eigenvectors is not a subspace, since the zero vector is not present.

On the other side, whenever eigenvectors are involved, there is no require to say that they are nonzero. If X is a square real matrix $X \in A_n(R)$,We restrict definition 1.3 to real eigenvalues $\alpha \in R$ and as well as real eigenvectors.

However, it should be described that every complex matrix always has at least some complex eigenvalue, a real matrix may not have any real eigenvalues. For example, the matrix

$$X = \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix}$$

has the complex eigenvalues i and -i but there are no real eigen- values. Hence for real matrices, we consider complex eigenvalues.

We Notice that $\alpha \in C$ is an eigenvalues of X

Iff $Xl = \alpha l$, for some non –zero vector $l \in C^n$

Iff $(\alpha l - X)l = 0$

Iff the matrix αl-X define a linear mapping which has a nonzero kernel, that is

Iff αl-X not invertible.

However αl-X is not invertible iff

$\det(\alpha l - X) = 0$

Now $\det(\alpha l - X)$ is a polynomial of degree n in the indeterminate α, in fact, of the form

Thus, we see that the eigenvalues of X are the zeros of the above polynomial. Since every complex polynomial of degree n has exactly n roots, counted with their multiplicity, we have the following definition.

Definition 1.4. Given any square n × n matrixX∈ $A_n(C)$, the polynomial $\det(\alpha l - X) = \alpha^n - tr(X)\alpha^{n-1} + \cdots + (-1)^n \det(X)$

is called the characteristic polynomial of X. Then the roots $\alpha_1, \alpha_2, \ldots, \alpha_n$ of the characteristic polynomial are all the eigenvalues of X and constitute the spectrum of X.

We let $\rho(X) = \max_{1 \le i \le n} |\alpha_i|$

be the largest modulus of the eigenvalues of X, called the spectral radius of X.

Proportion 1:-

For any matrix norm II II on $A_n(C)$ and for any square

n × n matrix X, we have

$\rho(X) \leq \|X\|$.

Remark 2:

Proposition 1 still holds for real matrices $X \in A_n(R)$, but a different proof is necessary since in the above proof the eigenvector I may be complex.

Now, it turns out that if X is a real n×n symmetric matrix, then the eigenvalues of X are all real and there is some orthogonal matrix U such that

$X = U^t \, diag(\alpha_1, \alpha_2, .., \alpha_n)U$,

where $diag(\alpha_1, \alpha_2, .., \alpha_n)$ indicate the matrix whose only nonzero elements are its diagonal element, which are the real eigenval then the eigenvalues of X.

In the same way if X is a complex n×n Hermitian matrix then the eigenvalues of X are all real and there is some unitary matrix P such that

$X = P_* \, diag(\alpha_1, \alpha_2, . .., \alpha_n)P$,

where $diag(\alpha_1, \alpha_2, .., \alpha_n)$ indicate the matrix whose only nonzero elements are its diagonal elements, which are the eigenvalues of X.

Now we again go to matrix norms. We starting with the so called Frobenius norm, which is just the norm, Which is just the norm $\| \; \|_2$ on C where the n×n matrix X is recall as the vector obtained by concatenating along with the rows or the columns of X.

It is easy to show that for any n×n complex matrix $X = (x_{ij})$, the following relation is hold good.

$\sum_{,} |x| = \{tr(X^*X)\} = \{tr(XX^*)\}$

Definition 1.5

The Frobenius norm $\| \; \|_k$ is define so that for every square n×n matrix $X \in A_n(C)$

$\|X\|_k = \sum_{,} \; x \quad = \{tr(XX^*)\} = \{tr(X^*X)\}$

The following proposition show that the Frobenius norm is a matrix norm satisfying other nice properties.

Proposition 1.1. The Frobenius norm $\| \; \|_k$ on $A_n(C)$ satisfies the following properties:

i> It is a matrix norm;that is

$\|XY\|_k \leq \|X\|_k \|Y\|_k$, $\forall$ X, Y $\in A_n(C)$.

ii> It is unitarily invariant, which means that for all unitary matrices P, Q , we have

$\|X\|_k = \|PX\|_k = \|XQ\|_k = \|PXQ\|_k$

iii> $\{\rho(X^*X)\} \leq \|X\|_k \leq n \{\rho(X^*X)\}$, $\forall$ X $\in A_n(C)$.

Remark 3 :-

The Frobenius norm is also known as the Hilbert-Schmidt norm or the Schur norm. So many populer names connected with such a simple thing.

We now give another method for obtaining matrix norms using subordinate norms.

First, we necessary a proposition that shows that in a finite- dimensional space, the linear mapping induced by a matrix is bounded, and thus continuous.

Proposition 1.2.

For every norm on C

constant D > 0, such that

(or R_n), for every matrix X ∈ A_n(C) or X∈ A_n(R), there is a real

‖X‖‖ ≤ D_x ‖‖‖‖

for every vector l∈ C

(or l ∈ R

if X is real).

Proposition 1.2 says that every linear mapping on a finite- dimensional space is bounded . This implies that every linear mapping on a finite-dimensional space is continuous.

Really, it is not easy to proved that a linear mapping on a normed vector space V is bounded if and only if it is continuous, regardless of the dimension of V.

Also this proposition indicate that for every matrix X ∈ A_n(C) or X ∈ A_n(R),

‖ ‖

∈ ‖ ‖ ≤ D_x

Now,since ‖αl‖ = |α| ‖l‖, it is easy to show that

∈ ‖ ‖ ‖ ‖ = ∈‖Xa‖ ‖ ‖

Similarly ∈ ‖ ‖ ‖ ‖ = ∈ ‖Xa‖ ‖ ‖

Definition 1.6.

If ‖ ‖ is any norm on C^n, we define the function ‖ ‖ on A_n(C) Such as ‖X‖= ∈ ‖ ‖ ‖ ‖

= ∈ **‖Xa‖.** ‖ ‖

The function X→‖X‖ is called the subordinate matrix norm or operator norm induced by the norm ‖‖.

It is easy to check that the function X→‖X‖ in indeed a norm, and by definition, it satisfies the property

llXall ≤ llXllllall. , ∀a ∈C^n

This implies that

llXYll ≤ llXllllYll,

For all X,Y∈ A (C),showing X→llXll is a matrix norm. Notice that the subordinate matrix norm is also defined by llXll=inf {α ∈ R: llXall ≤ αllXll, ∀a ∈ C }

The definition also implies that

llIll=1

The above explanation show that the Frobenius norm is not a subor- dinate matrix norm.

Remark - 4:

We can also use Definition 1.6 for any norm ll ll on R^n and define the function ll ll_R on A_n(R) by

llXll ∈ ll ll = ∈ ll ll llXall

The function X→llXll is a matrix norm on A (R),and

llXll ≤ llXll, ∀X ∈ A (R).

However, it is possible to formulate vector norms ll ll on C and matrices X such that

llXll < llXll.

In order to avoid this kind of difficulties, we define sub- ordinate matrix norms over A_n(C).

Luckily it turns out that ll ll_1,ll ll_2 and || ||

Proposition 1.3.

For every square matrix X= x ∈ A (C),We have

llXll = llXall = ∑ x
∈ ll ll

ǁXǁ=∊
ǁ ǁ

ǁXaǁ =

∑x
ǁXǁ =

∊

ǁXaǁ = ρ(X∗
X)= ρ(XX∗)ǁ ǁ

Likewise,ǁX*ǁ = ǁXǁ ,the norm ǁ ǁ$_2$, is unitarily invariant,which means that

ǁXǁ =ǁPXQǁ

for all unitary matrices P, Q , and if X is a normal matrix, then The norm ǁXǁ is often called the spectral norm.

We see that property (iii) of proposition 1.1 says that ǁXǁ ≤ ǁXǁ ≤ √nǁXǁ ,

which shows that the Frobenius norm is an upper bound on the spectral norm. The Frobenius norm is much easier to compute than the spectral norm.

The above proof still holds if the matrix X is real, confirming the fact that ǁXǁ = ǁXǁ for the vector norms ǁ ǁ $_1$,ǁ ǁ$_2$ and || || .

It is also easy to verify that the proof goes through for rectangular matrices, with the same method.

Similarly, the Frobenius norm is also a norm on rectan- gular matrix. For these norms, whenever XY makes sense, we have

ǁXYǁ ≤ ǁXǁ ǁYǁ.

The following proposition will be needed when we deal with the condition number of a matrix.

Proposition 1.4.

Let ||| be any matrix norm and let Y be a matrix such that ǁYǁ < 1.

(1) If ||| is a subordinate matrix norm, then the matrix I+Y is invertible and

1
ǁ(I+Y) ǁ ≤ 1−ǁYǁ

(2) If a matrix of the form I + Y is singular, then ǁYǁ ≥ 1 for every matrix norm (not necessarily subordinate).

Remark-5:

Another result that we will not prove here but that plays a role in the convergence of sequences of powers of matrices is the following. For every matrix X ∈ A (C) and every ∈> 0, there is some sub-ordinate matrix norm II II such that

$\|X\| \leq \rho(X)+\epsilon$

Note that equality is generally not possible; consider the matrix

X=0 0
1 0

For which (X) = 0 < IIXII,since X ≠ 0.

4. An Application to Conditioning of Linear system:-

Here we consider the linear systems By=a in which solutions are not stable under small perturbations of either a or B.

For example, consider the system

10787	a		32
7565	a	=	23
86109	a		33
75910	a		31

We observed that it has the solution a = (1, 1, 1, 1). If we perturb marginally the right-hand side, we obtaining the new system

10787
7565
86109
75910

a **+ ∆a**
a **+ ∆a**
a **+ ∆a**
a **+ ∆a**

32.2
= **22.8**
33.2
30.8

the new solutions turns out to be

a=(9.21, −12.57,4.45, −1.12)

In other words, a relative error of the order 1/200 in the data (here, a) produces a relative error of the order 10 in the solution, which represents an amplification of the relative error of the order 2000.

Now, let us perturb the matrix marginal, obtaining the new system

10	7	8.01	7.02	a	+ ∆a
7.08	5.04	6	5	a	+ ∆a
8.1	5.98	9.97	9	a	+ ∆a
6.99	4.98	9	9.98	a	+ ∆a

32

= $^{23}{}_{33}$

31

In this case, the solution is a = (−81, 137, −34, 22).

Again, if we doing a small change in the data alters the result rather acutely.

Yet, the original system is symmetric, has determinant 1, and has integer elements.

The problem is that the matrix of the system is astrocious , a concept that we will now explain.

An invertible matrix B, first assume that we perturb a to a + δa, and let us analyze the change between the two exact solutions y and y + δy of the two systems.

By = a

And B(y + δy) = a + δa.

We also assume that we have some norm ‖ ‖ and we use the subordinate matrix norm on matrices. From

By = a

And By + Bδy = a + δa,

we get

δy = B^{-1}δa,

and we conclude that

‖δy‖ ≤ ‖B ‖‖δa‖

‖a‖ ≤ ‖B ‖‖y‖

Consequently, the relative error in the result ‖δy‖/‖y‖is bounded in terms of the relative error ‖δa‖/ ‖a‖ in the data as follows:

$$\frac{\|\delta y\|}{\|y\|} \le (\|B\|.\ \|B\ \|)\ \frac{\|\delta a\|}{\|a\|}$$

Now let us assume that B is perturbed to B + δB, and let us explain the change between the accurate solutions of the two systems

By=a

And $(B + \Delta B)(y + \Delta y) = a$

After some calculations, we get

$$\frac{\|\Delta y\|}{\|y + \Delta y\|} \leq (\|B\|. \|B\ \ \|) \frac{\|\Delta B\|}{\|B\|}$$

We observe that the above reasoning is valid even if the matrix $B+\Delta B$ is singular, as long as $y + \Delta y$ is a solution of the second system.

Likewise if $\|\Delta B\|$ is small enough it is not unreasonable to expect that the ratio $\|\Delta y\|/\|y + \Delta y\|$ is close to $\|\Delta y\|/\|y\|$.

In summary, for each of the two perturbations, we see that the relative error in the result is bounded by the relative error in the data, multiplied the number $\|B\|\|B\ \|$.

In fact, this factor turns out to be optimal and this suggests the following definition:

Definition 7:-

For any subordinate matrix norm $\|\|$, for any invertible matrix B, the number

$K(B)=\|B\|\|B\ \|$, [K(B) is denote condition number] is called the condition number of B relative to $\|\|$.

The condition number K(B) measure the sensitivity of the linear system By=a to variations in the data a and B; a feature referred to as the condition of the system.

Thus, when we says that a linear system is atrocious , we mean that the condition number of its matrix is large.

Now we can sharpen the preceding analysis as follows:

Proposition 1.5.

Let B be an invertible matrix and let y and $y + \delta y$ be the solutions of the linear systems

$By = a$

And $B(y + \delta y) = a + \delta a$.

If $a \neq 0$, then the inequality

$$\frac{\|\delta y\|}{\|x\|} \leq K(B) \frac{\|\delta a\|}{\|a\|}$$

holds and is the best possible. This means that for a

given matrix B, there exist some vectors $a \neq 0$ and $\delta a \neq 0$ for which equality holds.

Proposition 1.6:-

Let B be an invertible matrix and let y and $y+\Delta y$ be the solution of the two systems

By=a And (B + ∆B)(y + ∆y) = a

If a≠ 0, the inequality

$$\frac{\|\Delta y\|}{\|y + \Delta y\|} \leq K(B) \frac{\|\Delta B\|}{\|B\|}$$

holds and is the best possible. This means that given a matrix B, there exist a vector a ≠ 0 and a matrix ∆B ≠ 0 for which equality holds.Likewise,if ‖∆B‖ is small enough we have

$$\frac{\|\Delta y\|}{\|y\|} \leq K(B) \frac{\|\Delta B\|}{\|B\|} \{1 + O(\|\Delta B\|)\}$$

In fact, we have

$$\frac{\|\Delta y\|}{\|y\|} \leq K(B) \frac{\|\Delta B\|}{\|B\|} \frac{1}{1-\|B^{-1}\|.\|\Delta B\|}$$

Remark-6:-

If B and a are perturbed simultaneously, so that we get the "perturbed" system

(B + ∆B)(y + ∆y) = a + δa,

It can be shown that if ‖∆B‖ < $\frac{1}{\|B^{-1}\|}$ (and a ≠ 0),

Then

$$\frac{\|\Delta y\|}{\|y\|} \leq \frac{K(B)}{1 - \|B^{-1}\|.\|\Delta B\|}$$

‖∆B‖ ‖δa‖ ‖B‖ + ‖a‖

We now list some properties of condition numbers and figure out what K(B) is in the case of the spectral norm.

First, we necessary to introduce a very important factorization of matrices, the singular value decomposition, for short form SVD.

It can be shown that given any n×n matrix B∈ A (C),there exist two unitary matrices P and Q, and a real diagonal matrix ∑=diag(ϑ , ϑ , … … . ϑ),with ϑ ≥ ϑ … … … ≥ ϑ ≥ 0, such that

B=Q∑P*

The non-negative numbers ϑ , ϑ , … … . ϑ are called the singular values of B.

If B is a real matrix, the matrices P and Q are orthogonal matrices.

The factorization B=Q∑P* implies that B*B= P∑ P* and BB* =Q∑ Q*,which shows that ϑ , ϑ … … … . ϑ are the eigenvalues of both B*B and BB*, that the columns of P are corresponding eigenvectors for B*B, and that the columns of Q are corresponding eigenvectors for BB*.

In the case of a normal matrix if α_1,α ,,.. .., α_n are the complex eigenvalues of B, then ϑ = |α |, 1 ≤ i ≤ n

Proposition 1.7:-

For every invertible matrix B ∈ A_n(C), the following properties hold:

(1) K(B) ≥ 1,

K(B)=K(B),

K(βB)=K(B),∀β ∈ C − {0}

(2) If K_2(B) denotes the condition number of B with respect to the spectral norm then K_2(B)=

Where ϑ ≥ ϑ … … . ≥ ϑ are the singular values of B.

(3) If matrix B is norm then

$K_2(B)$= | | | |

Where α , α , … … α are the eigenvalues of B so that |α | ≥ |α | ≥……….. ≥ |α |

(4) If B is a unitary or an orthogonal matrix then K_2(B)= 1

(5) The condition number K_2(B) is invariant under unitary transformations, which means that K_2(B) = K_2(PB) = K_2(BQ), for all unitary matrices P and Q .

Proposition 1.7 part (4) shows that unitary and orthogonal transformations are very flourishing, and part (5) shows that unitary transformations preserve the condition number.

In order to compute K_2(B), we need to compute the top and bottom singular values of B, which may be hard.

The inequality

‖B‖ ≤ ‖B‖ ≤ n ‖B‖

May be useful in getting an approximation of

K (B) = ǁBǁ ǁB ǁ if B can be determined.

Remark:-7

There is an interesting geometric characterization of K_2(B).

If Φ(B) indicate the least angle between the vectors Bu and Bv as u and v range over all pairs of orthogonal vectors, then it can be shown that

tan ∅( ) =
( )

Thus, if B is nearly singular, then there will be some orthonormal pair u, v such that Bu and Bv are nearly parallel; the angle ∅(B) will the small and cot ∅( ) will be large.

It should also be noted that in general (if B is not a normal matrix) a matrix could have a very large condition number even if all its eigenvalues are identical.

For example if we consider the square matrix whose order is n as follows

12 00.... 0 0
0120.... 0 0
0012.... 0 0
B= ....
....
0000....12
0000....01

It turns out that K (B) ≥ 2

Going back to our matrix

10787
B= 7565
86109
75910

which is a symmetric matrix, it can be shown that

$\alpha_1 \approx 30.2886 > \alpha_2 \approx 3.857 > \alpha_3 \approx 0.843 > \alpha_4 \approx 0.010$ so that

α
K (B) = α ≈ 2983.

It should proved that for the perturbation of the right-hand side a used earlier, the relative errors ǁ ǁ ǁ ǁ and ǁ ǁ ǁ ǁ satisfy the inequality

ǁ ǁ ≤ K(B) ǁ ǁ
ǁ ǁ ǁ ǁ

And comes close to equality .

5. FORMULA TO COMPUTE THE OPERATOR NORM ON N_n(R)

The principal idea to obtain the operator norm of X∈ N (R) is that the properties ǁXX ǁ = ǁX Xǁ = ǁXǁ tell us ǁXǁ is the square root of the operator norm of XX and X X

The following properties gives a formula to compute operator norms of matrices and that will lead to an operator norm formula on all matrices in N_n(R).

i> If X∈ N (R) satisfies X =X then all the eigenvalues of X are real and ‖α‖ = max eigenvalues |α|.

ii> for all X∈ N (R),the eigenvalues of XX and X X are all non-negative.

Let us consider the example to compute the operator norm of 2×2 matrices X= $\begin{matrix}1 & 2\\ 3 & 4\end{matrix}$

So X = 1 3
2 4

Therefore = $\begin{matrix}1 & 2\\ 3 & 4\end{matrix}\ \begin{matrix}1 & 3\\ 2 & 4\end{matrix} = \begin{matrix}5 & 11\\ 11 & 25\end{matrix}$

Therefore the characteristic polynomial of XX is α − 30α + 4 which has eigenvalues

15±√221 ≈ 13,29.86 .Therefore the operator norm of X is 15 + √221 and 15 + √221
≈ 5.46 is the smallest number with that property.

6.Conclusion:-

We have demonstrated about the importance of studying different norms on matrix and have shown how it is computed. In many situations ,this approach is efficient and allows one to use mathematical tools from other areas .Moreover using this knowledge and some general observations ,one can solve several other problems effectively. There are much potential for further development and application.

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