# MANN-WHITNEY'S MINIMAL SPANNING TREE ALGORITHM OF CRITICAL PATH NETWORK ANALYSIS USING FERMATEAN PENTAGONAL FUZZY NUMBERS

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**Abstract:** In this article, a new algorithm for searching Mann-Whitney minimum spanning tree in a critical path network having fermeatean pentagonal fuzzy soft edge length is presented. The proposed algorithm is based on matrix approach to design un-directed fermeatean fuzzy weighted connected graph. Also, we provide numerical example to check the validity of the proposed algorithm using score function and accuracy function.

**Keywords:** fuzzy set,fuzzy number, validity, defuzzification, fermeatean fuzzy soft set, minimal spanning tree, pentagonal matrix, connected graph, algorithm, score function, accuracy function.

**1.Introduction:** A minimum spanningtree is a spanning tree that has the lowest possible weight. The MST is a spanning tree T of graph G which has the minimum cost among all the spanning tree of graph G. The MST problem is a fundamental and well known combinatorial optimization problem in the area of graph theory. It is widely applied in various fields of science and engineering, e.g., road network application, transportation, routing in communication channels and scheduling problems. In real life problems, edge costs are used to represent the parameter costs, capacities, demands, time, etc. Although in classical graph theory, real numbers are used in a MST problem to express the arc lengths of a connected weighted graph. we generally consider the classical MST problem as network optimization problem. In the classical MST problem, we assume the arc lengths (time, cost, distance, nature of connectivity between nodes, etc.) is certain. However, in real world problems, the information about any MST problems are generally not exact or precise due to several reasons such as incomplete data or insufficient data, less evidence, imperfect statistical. For e.g., in a weighted connected

graph, the arc length may describe the traversing time between two nodes. This traveling time depends on traffic jam, accident, weather etc., each of the criteria differ from day to day. Therefore, it becomes difficult for the DM to estimate exactly the edge cost. Several papers have been done on the uncertainty MST [1], [2], [3], [4], [6], [7], [12], [13]. The work by Itoh and Ishii [6] is first work on this uncertainty domain and worked a MST problem with fuzzy costs as a chance constrained programming based on the necessity measure. Following that, some methods based on the ranking index for arc weights of FMST were proposed by Chang and Lee [4]. Almeida et al. [3] described the fuzzy MST problem with uncertainty costs and introduced an algorithmic method to solve this FMST.Zadeh[14] introduce the idea of fuzzy set. They introduced a genetic algorithm founded on uncertainly logic and probability theory to compute the FMST with fuzzy weights. In [7], they described possibility theory to describe the minimum of edge of the fuzzy graph and to find a spanning tree where the fuzzy arc weights are in fuzzy intervals. Liu [8], [9] introduced the credibility theory including pessimistic value, credibility measure and expected value as fuzzy ranking methods. Senapati.T, R.R.Yager[11],introduced the concept of fermeatean fuzzy set in 2019(a). soft set theory is discussed by [10], based on the credibility theory, Gao and Lu [5] studied the fuzzy quadratic minimum spanning tree problem and formulated it as expected value model, chance-constrained programming and dependentchance programming according to different decision criteria. In this paper a new algorithm for searching defuzzification approach for validity of minimal spanning tree in a network having pentagonal fermeatean fuzzy soft edge length is presented. The proposed algorithm is based on a matrix approach to design undirected fermeatean fuzzy soft weighted connected graph.Also, we provide a numerical example to check the validity of the proposed algorithm.

## 2.Preliminaries

In this section, the concept of fermeatean fuzzy soft set and pentagonal fermeatean fuzzy soft numbers are presented to deal with uncertainty data, which can be defined as follows.

**2.1 Definition:** A uncertainty set in characterized by its membership function taking value from the domain space or universe of discourse mapped into the interval [0,1]. A uncertainty set D in the universal set X is defined us D={(x,$\alpha_D$(x)/x∈X}. Here $\alpha_D$:D→[0,1] is the grade of membership function and $\alpha_D$(x) is the grade value of x ∈ X in the fuzzy set D.

**2.2 Example:**Let D={$a, b, c, d$} be a set on X then $\alpha_D$(x) is defined as

| D | a | b | c | d |
|---|---|---|---|---|
| $\alpha_D$(x) | 0.4 | 0.1 | 0.3 | 0.7 |

Clearly 'D' is an uncertainty set on X.

**2.3 Definition:** Let X be an initial universe . P(X) be the power set of X, E be the set of all parameters and D ⊆ $E$. A soft set ($\alpha_D$(x), E) on the universe X is defined by the set of ordered pairs ($\alpha_D$(x),E)= {$e$, ($\alpha$D(e)) |$e$ ∈ E, $\alpha$D(e) ∈ p($\alpha$)}where $\alpha_D$:E——→P(X) such that $\alpha_D$(e)=$\phi$ if $e\epsilon$D. Here $\alpha_D$ is called an approximate of the soft set.

**2.4 Example:** Let X={$x_1,x_2,x_3,x_4$} be a set of four shirts and E={white($e_1$),red($e_2$),blue($e_3$)} be a set of parameters. If D={$e_1,e_2$}$\subseteq E$.

Let $\alpha_D(e_1)$ = { $x_1,x_2,x_3,x_4$} and $\alpha_D(e_2)$ = { $x_1,x_2,x_3$}. Then we write the soft

($\alpha_D$,E)= {($e_1, x_1,x_2,x_3,x_4$), ($e_2, x_1,x_2,x_3,x_4$)} over X which describe the "colour of the shirts" which Mr.'Z' is going to buy.

**2.5 Definition:** An uncertainty set 'D' is called normal if there exist an element $x \in$X whose membership value is 1. (i.e) $\alpha_D(x)$=1.

**2.6 Definition:** A fuzzy number 'D' is a subset of real line R, with membership function $\alpha_D$ satisfying the following properties

(i) $\alpha_D(x)$ is piecewise continuous in its domain.

(ii)'D' is normal (i.e) $\alpha_D(x)$=1 for all $x \in$X

(iii)'D' is convex (i.e) C($\lambda x_1$+(1-$\lambda$)$x_2$)=min{ $\alpha_D(x_1)$, $\alpha_D(x_2)$},for all $x_1,x_2\in$X.

Due to wide application of the fuzzy number, two types of fuzzy numbers, namely

  (i)Triangular fuzzy number.

  (ii)Trapezoidal fuzzy number are introduced in the field of fuzzy algebra.

Due to error in measuring techniques,instrumental faultiness etc., some data in our observation cannot impress or accurently defined.

   Let us consider that we measure whether temperatures and humidity. Simultaneously the temperature is approximately 35°c with normal humidity.

   The variation in temperature also affects the percentage of humidity. This is phenonmenous happens in general. This concept of variation leads to a new type of fuzzy number called the "pentagonal fuzzy number"(PFN)

Generally, a pentagonal is a S-tuple subset of a real number 'R' having five parameters.

  A pentagonal fuzzy number 'D' is denoted as D={$a_1,a_2,a_3,a_4,a_5$} where $a_3$ is the middle point and ($a_1,a_2$) and ($a_4,a_5$) are the left and rightside points of $a_3$ respectively.

 Now we define the mathematical construction of a pentagonal fuzzy number.

**2.7 Definition:** A fuzzy number D=( $a_1,a_2,a_3,a_4,a_5$) is called pentagonal fuzzy number (PFN) where the membership has the form

$$\alpha_D(x_1,w_1,w_2) = \begin{cases} W_1 \; x = \dfrac{x-a1}{a2-a1} \;, & a_1 \le x \le a_2 \\[2mm] 1-(1-w_1)\,\dfrac{x-a2}{a3-a2}\;, & a_2 \le x \le a_3 \\[1mm] 1 \;, & x=a_3 \\[2mm] 1-(1-w_2)\,\dfrac{x-a3}{a4-a3}, & a_3 \le x \le a_4 \\[2mm] w2\dfrac{x-a3}{a4-a3}\;, & a_4 \le x \le a_5 \\[2mm] 0 & ,\; x>a_5 \end{cases}$$
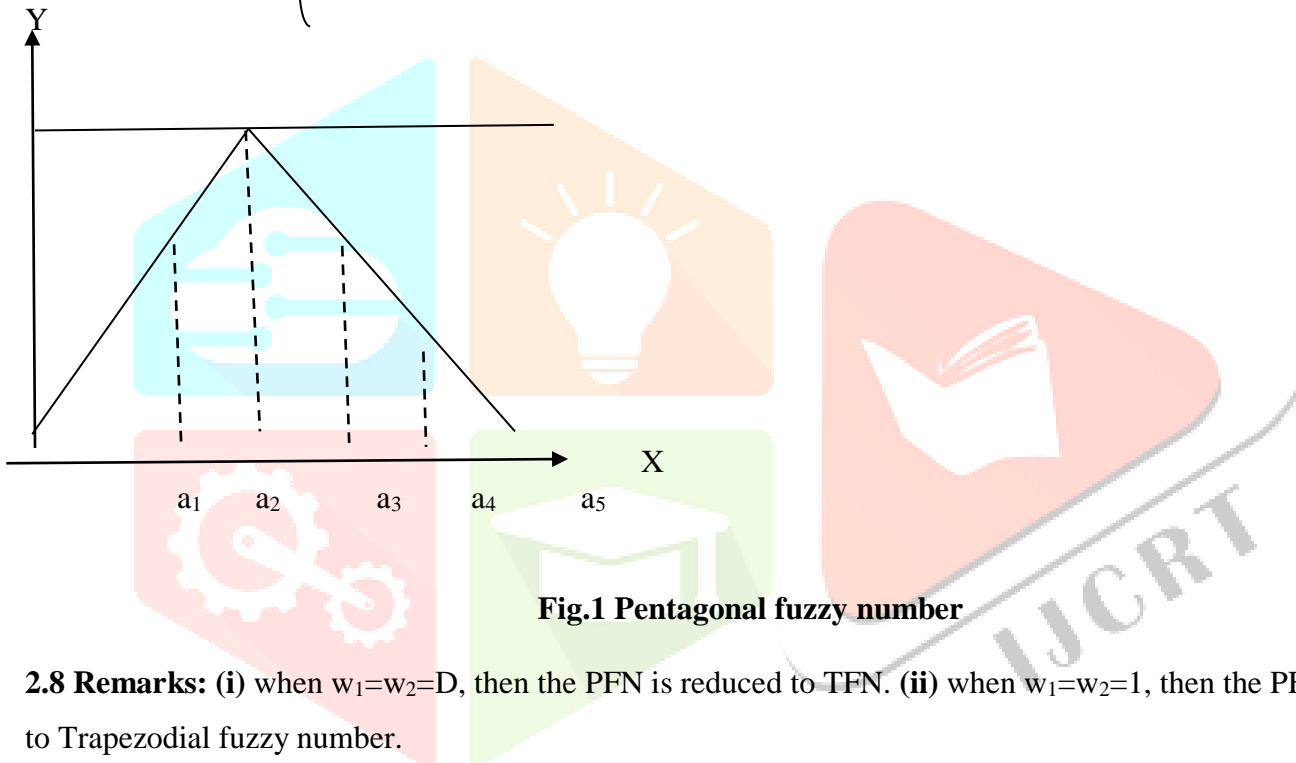


**Fig.1 Pentagonal fuzzy number**

**2.8 Remarks: (i)** when $w_1=w_2=D$, then the PFN is reduced to TFN. **(ii)** when $w_1=w_2=1$, then the PFN become a to Trapezodial fuzzy number.

**2.9 Definition:** Fermeatean fuzzy soft set: Let 'X' be a universe of discourse A. Fermeatean fuzzy soft set "F" in X is an object having the form $F = \{\langle x, m_F(x),\; n_F(x)\rangle / x \in X\}$,

where $m_F(x): X \to [0,1]$ and $n_F(x): X \to [0,1]$, including the condition

$0 \le \big(m_F(x)\big)^3 + \big(n_F(x)\big)^3 \le 1$, for all x ∈ X The numbers $m_F(x)$ signifies the level (degree) of membership and $n_F(x)$ indicate the non-membership of the element 'x' in the set F. All through this paper, we will indicate a fermeatean fuzzy soft set is FFSS.

For any FFSS 'F' and $x \in X$, $\pi_F(x) = \sqrt[3]{1-\big(m_F(x)\big)^2 - \big(n_F(x)\big)^2}$ is to find out as the degree of indeterminacy of 'x' to F. For convenience, senapathi and yager called $\big(m_F(x),\; n_F(x)\big)$ a fermatean fuzzy soft number (FFSN) denoted by $F = (m_F,\; n_F)$.
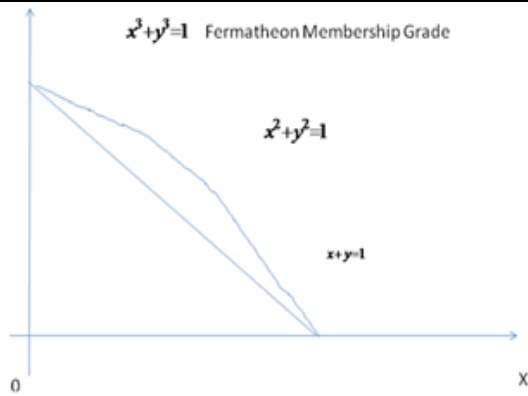
**Fig-1 Fermatean fuzzy membership grade**

We will explain the membership grades (MG's) related Fermeatean uncertainty collections as Fermeatean membership grades.

**2.10 Theorem:**[Senapati and Yager, 2019a] The collections of FMG's is higher than the set of pythagorean membership grades (PMG's) and bi –fuzzy membership grades (BMG's).

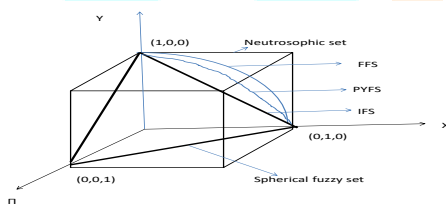**Proof:** This improvement can be evidently approved in the following figure.



Fig-2 Improvement of Fermatean fuzzy membership grade

**3.Operation on Pentagonal Fuzzy soft number:**

In this section, we study some arithmetic operations of pentagonal fermeatean fuzzy soft numbers.Let $A=\{a_1,a_2,a_3,a_4,a_5,\}$ and $B=\{b_1,b_2,b_3,b_4,b_5\}$ be two pentagonal fermeatean fuzzy soft set. Then

(i) Addition: $A+B = \{a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5\}$

(ii)Subtraction: $A-B = \{a_1-b_1, a_2-b_2, a_3-b_3, a_4-b_4, a_5-b_5\}$

(iii) Scalar multiplication: $\alpha A = \{\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4, \alpha a_5\}$

(iv) Multiplication: $AB = \{a_1b_1, a_2b_2, a_3b_3, a_4b_4, a_5b_5\}$

(v) Inverse: $A^{-1} = \{1/a_5, 1/a_4, 1/a_3, 1/a_2, 1/a_1\}$

(vi) Division: $A/B = \{a_1/b_5, a_2/b_4, a_3/b_3, a_4/b_2, a_5/b_1\}$

## 4.Mann-Whitney's Algorithm of Fermeatean Pentagonal Fuzzy Soft Numbers

**Input:** The weight matrix $D=[d_{ij}]_{mxn}$ for which is constructed for undirect weight fermeatean soft graph.

Step-1: Input pentagonal fermeatean fuzzy soft adjacency matrix

Step-2: Construct pentagonal fermeatean fuzzy soft number matrix inti a score matrix $[s_{ij}]_{mxn}$ by the using the score fun.

Step-3: Repeat step-4,step-5 up to time that all non-zero elements are marked 0 in alternative saying all (n-1) entries matrix of S are either marked or a set to zero.

Step-4: There are two ways to find out the weight matrix D that one is column-wise and another one is row-wise in order to determine the unmarked minimum entries $S_{ij}$ beside it determines the weight of the corresponding edge $e_{ij}$ in D.

Step-5:Set $S_{ij} = 0$ else mark $S_{ij}$ provided the corresponding edge $e_{ij}$ of selected $S_{ij}$, generate cycle with the proceeding marked entries of the score matrix S.

Step-6: Construct the graph T including the only marked entries from the score matrix S which shall be desired minimum cost spanning tree of G.

Step-7: Stop the iterations.

**4.1Defintion:** Let B be a pentagonal fermeatean fuzzy soft variable denoted as B = {$(a_1, a_2, a_3, a_4, a_5)$,$(b_1, b_2, b_3, b_4, b_5)$,$(c_1, c_2, c_3, c_4, c_5)$}.

Hence the score function and the accuracy function of pentagonal fermeatean fuzzy soft variables are denoted as below;

Score of (B) or S(B)= $1 / 15 [10 + (a_1 + a_2 + a_3 + a_4 + a_5) - (b_1 + b_2 + b_3 + b_4 + b_5) - (c_1 + c_2 + c_3 + c_4 + c_5)] \rightarrow$ (1)

accuracy of (B)or H(B)=$1/5 [(a_1 + a_2 + a_3 + a_4 + a_5) - (c_1 + c_2 + c_3 + c_4 + c_5)$ ------ (2)

**4.2Definition:**Let $B_1$ and $B_2$ be two pentagonal fermeatean fuzzy soft numbers variable defined on the set of real numbers. Hence the ranking method is defined as follows.

(i)$S(B_1) < S(B_2)$ , then $B_1 > B_2$, i.e $B_1$ is superior to $B_2$.

(ii)$S(B_1) = S(B_2)$ , then $H(B_1) > H(B_2)$, then $B_1$ is superior to $B_2$ .

**5.Numerical Example:** In this section, a numerical example of fermeatean pentagonal fuzzy soft numbers minimal spanning tree is used to demonstrate of the proposed algorithm.Steps involved in the construction of the minimum cost spanning tree are described as follows.
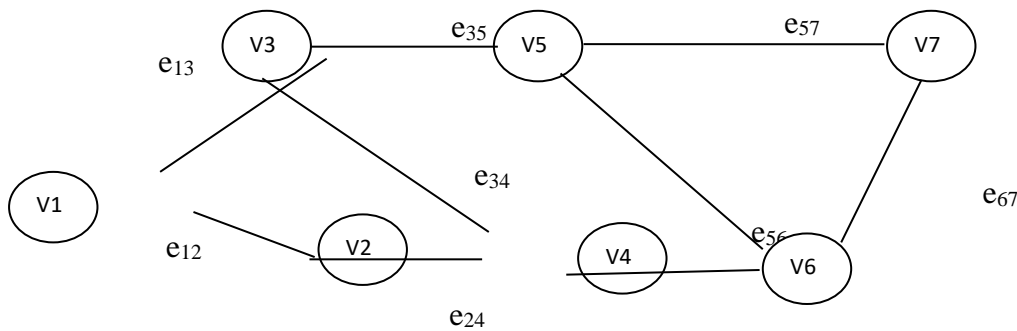


**Fig-4 A Fermeatean graph with pentagonal Fuzzy soft number edge weights**

step-1: The pentagonal fuzzy soft number adjacency matrix corresponding to the given graph fig-4 is

$$
A= 
\begin{array}{c}
\\ V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ V7
\end{array}
\begin{array}{c}
\begin{array}{ccccccc}
V1 & V2 & V3 & V4 & V5 & V6 & V7
\end{array} \\
\left(
\begin{array}{ccccccc}
0 & e12 & e13 & 0 & 0 & 0 & 0 \\
e12 & 0 & 0 & e24 & 0 & 0 & 0 \\
e31 & 0 & 0 & e34 & e35 & 0 & 0 \\
0 & e42 & e43 & 0 & 0 & e46 & 0 \\
0 & 0 & e53 & 0 & 0 & e56 & e57 \\
0 & 0 & 0 & e64 & e65 & 0 & e67 \\
0 & 0 & 0 & e75 & 0 & 0 & e76
\end{array}
\right)
\end{array}
$$

The values of edges weight are given by the following table

| e$ij$ | Edge weight | Edge weight | Edge weight |
|---|---|---|---|
| e$_{12}$ | (0.2, 0.3, 0.4, 0.6, 0.7) | (0.2, 0.3, 0.4, 0.6, 0.8) | (0.2, 0.3, 0.5, 0.6, 0.8) |
| e$_{13}$ | (0.3, 0.5, 0.6, 0.7, 0.8) | (0.1, 0.3, 0.4, 0.7, 0.8) | (0.3, 0.4, 0.6, 0.8, 0.8) |
| e$_{24}$ | (0.1, 0.3, 0.4, 0.6, 0.7 | (0.2, 0.4, 0.5, 0.6, 0.8) | (0.1, 0.3, 0.6, 0.7, 0.7) |
| e$_{34}$ | (0.2, 0.4, 0.6, 0.6, 0.8) | (0.2, 0.3, 0.3, 0.4, 0.6) | (0.3, 0.4, 0.4, 0.5, 0.7) |
| e$_{35}$ | (0.1, 0.3, 0.5, 0.7, 0.8) | (0.1, 0.3, 0.4, 0.5, 0.7) | (0.2, 0.3, 0.4, 0.6, 0.7) |
| e$_{46}$ | (0.2, 0.3, 0.4, 0.6, 0.8) | (0.2, 0.4, 0.4, 0.5, 0.7) | (0.1, 0.3, 0.3, 0.5, 0.7) |
| e$_{56}$ | (0.2, 0.2, 0.3, 0.4, 0.5) | (0.2, 0.3, 0.3, 0.5, 0.6) | (0.2, 0.3, 0.3, 0.5, 0.7) |
| e$_{57}$ | (0.1, 0.3, 0.4, 0.4, 0.5) | (0.1, 0.2, 0.4, 0.5, 0.7) | (0.1, 0.2, 0.4, 0.5, 0.6) |
| e$_{67}$ | (0.2, 0.5, 0.5, 0.6, 0.7) | (0.2, 0.3, 0.3, 0.5, 0.8) | (0.2, 0.3, 0.4, 0.5, 0.7) |

step-2: The score matrix $[S_{ij}]$ is obtained by using score function (By equation (1)

$$
A=
\begin{array}{c}
\phantom{V1}\\
V1\\
V2\\
V3\\
V4\\
V5\\
V6\\
V7
\end{array}
\begin{array}{ccccccc}
V1 & V2 & V3 & V4 & V5 & V6 & V7\\
0 & 0.826 & 0.513 & 0 & 0 & 0 & 0\\
0.826 & 0 & 0 & \boxed{0.480} & 0 & 0 & 0\\
0.513 & 0 & 0 & 0.567 \, 0.546 & 0 & 0\\
0 & 0.480 \, 0.567 & 0 & 0 & 0 & 0\\
0 & 0 & 0.546 & 0 & 0 & 0 & 0.533\\
0 & 0 & 0 & 0.546 \, 0.520 & 0 & 0.533\\
0 & 0 & 0 & 0 \, 0.520 \, 0 & 0.533
\end{array}
$$

from the above score matrix,we observe that the minimum value 0.480 is selected and the corresponding edge $e_{24}$ ($V_2 \rightarrow V_4$) is marked with box .The reduced undirected pentagonal fermeatean fuzzy soft graph is the next non-zero minimum entries in step-2 is 0.513 and the corresponding edges $e_{13}$ ($V_1 \rightarrow V_3$) the minimum value is
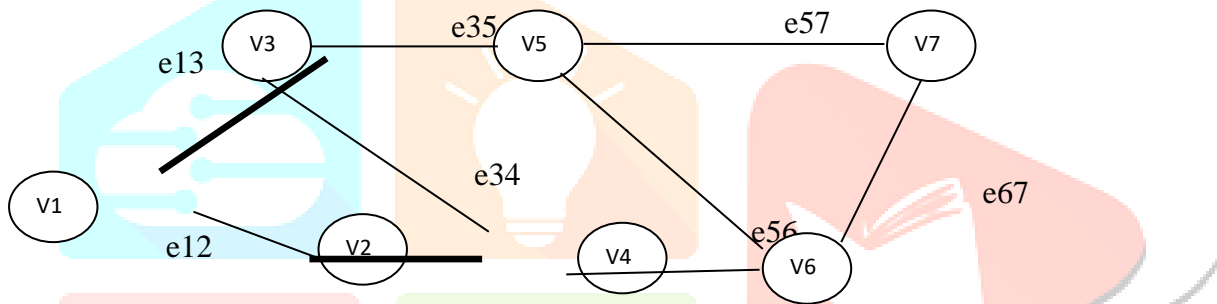


Fig-5 Marked pentagonal fermeatean fuzzy soft graph

$$
A=
\begin{array}{c}
\phantom{V1}\\
V1\\
V2\\
V3\\
V4\\
V5\\
V6\\
V7
\end{array}
\begin{array}{ccccccc}
V1 & V2 & V3 & V4 & V5 & V6 & V7\\
0 & 0.826 & \boxed{0.513} & 0 & 0 & 0 & 0\\
0.826 & 0 & 0 & \boxed{0.480} & 0 & 0 & 0\\
0.513 & 0 & 0 & 0.567 & 0.546 & 0 & 0\\
0 & 0.480 & 0.567 & 0 & 0 & 0 & 0\\
0 & 0 & 0.546 & 0 & 0 & 0 & 0.533\\
0 & 0 & 0 & 0.546 & 0.520 & 0 & 0.533\\
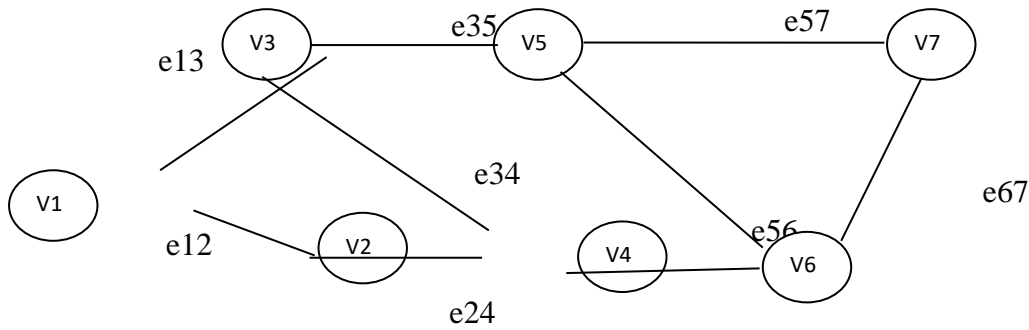0 & 0 & 0 & 0 & 0.520 & 0 & 0.533
\end{array}
$$

Fig-6 Marked pentagonal fermeatean fuzzy soft graph

Repeat the process until the iteration exists in the given table. The next non-zero minimum entries in the given score matrix (step-2) is 0.520 and the corresponding edge e35 (V3 → V5). Similarly

| Minimum Value | Edge | Node |
|---|---|---|
| 0.546 | $e_{56}$ | V5 →V6 |
| 0.553 | $e_{67}$ | V6 →V7 |

Hence the pentagonal fermeatean fuzzy soft graph for the corresponding minimum entries is an illustration of the marked edge.Based on the procedure of matrix approach applied to undirected pentagonal fuzzy soft graph.
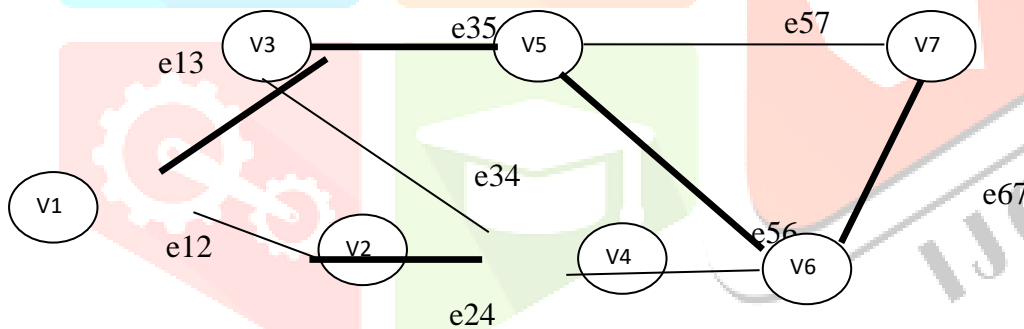


Fig-7 Marked minimum path pentagonal fermeatean fuzzy soft graph

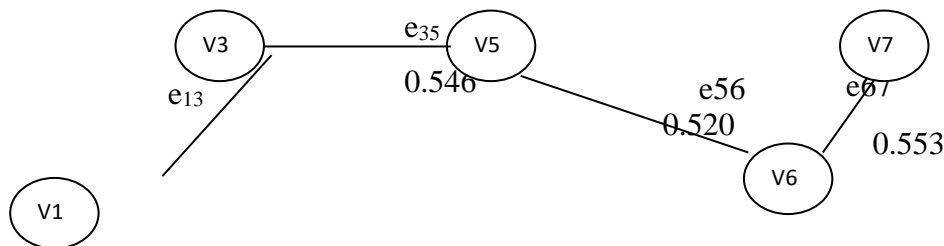The final path of minimum cost of spanning tree is



Fig-8 Final minimum path of cost of spanning tree G

Hence the length of the minimal spanning tree of pentagonal fermeatean fuzzy soft graph is $V_1 \to V_3 \to V_5 \to V_6 \to V_7$ and whose edge values represented as $0.513 \to 0.546 \to 0.520 \to 0.553$.

The accuracy function of the length of the minimal spanning tree is

| Edge | Accuracy function |
|------|-------------------|
| $e_{13}$ | 0 |
| $e_{34}$ | 0.02 |
| $e_{56}$ | -0.02 |
| **$e_{67}$** | **0.08** |

Therefore the validity of the minimal spanning tree occurs in $V6 \to V7$. Hence the maximum accuracy function occurs in edge is $e_{67}$ and value is $0.08$.

**Conclusion :** The main purpose of this paper is to propose a fermeatean fuzzy soft version of mann whitney's algorithm based in the matrix approach for reaching the cost minimum spanning tree in a network having pentagonal fermeatean fuzzy soft edge weighted validity of with accuracy function.

**Future work :** We can apply the proposed approach to the case of divided fermeatean fuzzy soft sets and other kinds of fermeatean fuzzy soft graphs including interval-valued rough graph and bipolar spherical fuzzy soft graphs.

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