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GENERALIZED SEMI-PRE CLOSED SETS IN WEAK STRUCTURES

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Abstract. In this paper, we introduce the concepts of generalized semi-pre w- closed sets and generalized semi-pre w-open sets. Further, we study some of their properties.

1. Introduction

Császár [4] introduced a new notion of structures called weak structures. Al-omari and Noiri [1] introduced generalized closed sets in weak structures. In this paper we introduce generalized semi-pre *w*-closed sets and generalized semi-pre *w*-open sets. The relation between semi-pre *w*-closed sets and other *w*-closed sets are given. And the relation between semi-pre *w*-open sets and other *w*-open sets are given. Also we study some of its properties.

2. Preliminaries

Throughout this paper, by a space X, we always mean a topological space (X, τ) with no separation properties assumed. Let H be a subset of X. We denote the interior, the closure and the complement of a subset H by int(H), cl(H) and $X \setminus H$ or H^c, respectively.

Definition 2.1. [7] *Let X be a space. A subset H of a space X is said to be semi-open if $H \subseteq cl(int(H))$. The family of all semi-open sets in X is denoted by SO(X).*

The complement of a semi-open set is called semi-closed.

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Definition 2.2. [3] *The semi-closure of the subset H of a space X is the intersectionof all semi-closed subsets of X containing H and it is denoted by scl(H).*

Definition 2.3. [2] *A subset H of a space X is called a semi-generalized closed set (briefly sg-closed) if scl(H)⊆U whenever H⊆U and U is semi-open in (X,τ).*

Theorem 2.4. [2] *Every semi-closed set is sg-closed but not conversely.*

Definition 2.5. [9] *A subset H of a space X is said to be preopen if H⊆int(cl(H)). The family of all preopen sets in X is denoted by PO(X).*
The complement of a preopen set is called preclosed.

Definition 2.6. [8] *The preclosure of the subset H of a space X is the intersection ofall preclosed subsets of X containing H and it is denoted by pcl(H).*

Definition 2.7. [8] *A subset H of a space X is called a pre-generalized closed set (briefly pg-closed) if pcl(H)⊆U whenever H⊆U and U is preopen in (X,τ).*

Definition 2.8. [4, 10] *Let X be a nonempty set and w⊆P(X) where P(X) is thepower set of X. Then w is called a weak structure (WS in short) on X if ∅ ∈ w.*
A non-empty set X with a weak structure w is called a weak structure space (WSS in short) and is denoted by (X, w). Each member of w is said to be w-open and the complement of a w-open set is called w-closed.

Definition 2.9. [10] *Let (X, w) be a WSS. Let H ⊆ X. Then the interior of H (briefly $i_w(H)$) is the union of all w-open sets contained in H and the closure of A (briefly $c_w(H)$) is the intersection of all w-closed sets containing H.*

Remark 2.10. [1] *If w is a WS on X, then $i_w(\emptyset)=\emptyset$ and $c_w(X)=X$.*

Theorem 2.11. [4] *If w is a WS on X and A,B ∈ w then*

(1) $i_w(A)\subseteq A\subseteq c_w(A)$,

(2) $A\subseteq B\Rightarrow i_w(A)\subseteq i_w(B)$ and $c_w(A)\subseteq c_w(B)$,

(3) $i_w(i_w(A))=i_w(A)$ and $c_w(c_w(A))=c_w(A)$,

(4) $i_w(X-A)=X-c_w(A)$ and $c_w(X-A)=X-i_w(A)$.

Lemma 2.12. [1] *If w is a WS on X, then*

(1) $x\in i_w(A)$ *if and only if there is a w-open set* $G\subseteq A$ *such that* $x\in G$,

(2) $x\in c_w(A)$ *if and only if* $G\cap A\neq\emptyset$ *whenever* $x\in G\in w$,

(3) If $A \in w$, then $A = i_w(A)$ and if A is w-closed then $A = c_w(A)$.

Definition 2.13. [1] *Let w be a WS on a space X. Then $H \subseteq X$ is called a generalized w-closed set (gw-closed in short) if $c_w(H) \subseteq U$ whenever $H \subseteq U \in \tau$.*

The complement of a gw-closed set is called gw-open.

Lemma 2.14. [1] *For a WS w on a space X, every w-closed set is a gw-closed setbut not conversely.*

Definition 2.15. [1] *A space X is called a w-$T_{\frac{1}{2}}$-space if for every gw-closed set H of X, $c_w(H) = H$.*

3. Properties of gsp-*w*-closed sets

In this section we introduce generalized semi-pre *w*-closed sets and study some oftheir properties.

Definition 3.1. *Let w be a WS on a topological space (X, τ). Then $A \subseteq X$ is called a generalized semi-pre w-closed set (gsp-w-closed set in short) if $spc_w(A) \subseteq U$ wheneverA $\subseteq$ U and U is open.*

Example 3.2. *Let X ={a, b, c}. Let w be a WS on a topological space (X, τ) where τ = {φ, {a}, {b}, {a, b}, X} and w = {φ, {a}, {a, b}}. Then the set A = {a, b} is gsp-w-closed set.*

Example 3.3. *Let X = {a, b, c}. Let w be a WS on a topological space (X, τ) whereτ = {φ, {a}, {b}, {a, b}, X} and w = {φ, {a}, {a, b}}. Then the set A = {a} is not gsp-w-closed set.*

Remark 3.4. *The union and intersection of two gsp-w-closed sets are not gsp-w- closed set in general.*

Example 3.5. *Let X = {a, b, c}. Let w be a WS on a topological space (X, τ) where τ = {φ, {a}, {b}, {a, b}, X} and w = {φ, {a, b}, {a, c}}. Then the set A = {a} and*

B = {b} are gsp-w-closed sets. But their union A ∪ B = {a, b} is not gsp-w-closed set.

Example 3.6. *Let X = {a, b, c}. Let w be a WS on a topological space (X, τ) whereτ = {φ, {a}, {b}, {a, b}, X} and w = {φ, {a}, {a, b}}. Then the set A = {a, b} and B = {a, c} are gsp-w-closed sets. But their intersection A ∩ B = {a} is not gsp-w-closed set.*

Theorem 3.7. *Let w be a WS on a topological space (X, τ). Then every w-closedset is gsp-w-closed set but not conversely.*

Proof. Let *w* be a WS on a topological space (X, τ). Let A be an *w*-closed set withA $\subseteq$ U and U is open. Since every *w*-closed set is sp-*w*-closed set, we have A is sp-*w*-closed set. Therefore $spc_w(A) = A$. Thus we have $spc_w(A) \subseteq U$ whenever A $\subseteq$U and U is open. Hence A is gsp-*w*-closed set.

Example 3.8. *gsp-w-closed set* a *w-closed set*

Let X = {a, b, c}. Let w be a WS on a topological space (X, τ) where τ = {φ, {a}, {a, b}, X} and w = {φ, {a}, {a, c}}. Then the set A = {c} is gsp-w-closed set but not w-closed set.

Theorem 3.9. *Let w be a WS on a topological space (X, τ). Then every α-w-closedset is gsp-w-closed set but not conversely.*

Proof. Let *w* be a WS on a topological space (X, τ). Let A be an *α-w*-closed set withA ⊆ U and U is open. Since every *α-w*-closed set is sp-*w*-closed set, we have A is sp-*w*-closed set. Therefore $spc_w(A)$ = A. Thus we have $spc_w(A)$ ⊆ U whenever A ⊆U and U is open. Hence A is gsp-*w*-closed set.

Example 3.10. *gsp-w-closed set* a *α-w-closed set*

Let X = {a, b, c}. Let w be a WS on a topological space (X, τ) where τ = {φ, {a}, X} and w = {φ, {a}, {c}}. Then the set A = {a, c} is gsp-w-closed set but not α-w-closed set.

Theorem 3.11. *Let w be a WS on a topological space (X, τ). Then every semi w-closed set is gsp-w-closed set but not conversely.*

Proof. Let *w* be a WS on a topological space (X, τ). Let A be a semi *w*-closed setwith A ⊆ U and U is open. Since every semi *w*-closed set is sp-*w*-closed set, we haveA is sp-*w*-closed set. Therefore $spc_w(A)$ = A. Thus we have $spc_w(A)$ ⊆ U whenever A ⊆ U and U is open. Hence A is gsp-*w*-closed set.

Example 3.12. *gsp-w-closed set* a *semi w-closed set*

Let X = {a, b, c}. Let w be a WS on a topological space (X, τ) where τ = {φ, {b}, {c}, {b, c}, X} and w = {φ, {c}, {a, c}, X}. Then the set A = {a, c} is gsp-w-closed set but not semi w-closed set.

Theorem 3.13. *Let w be a WS on a topological space (X, τ). Then every pre w-closed set is gsp-w-closed set but not conversely.*

Proof. Let *w* be a WS on a topological space (X, τ). Let A be a pre *w*-closed setwith A ⊆ U and U is open. Since every pre *w*-closed set is sp-*w*-closed set, we have A is sp-*w*-closed set. Therefore $spc_w(A)$ = A. Thus we have $spc_w(A)$ ⊆ U wheneverA ⊆ U and U is open. Hence A is gsp-*w*-closed set.

Example 3.14. *gsp-w-closed set* a *pre w-closed set*

Let X = {a, b, c}. Let w be a WS on a topological space (X, τ) where τ = {φ, {a}, {b, c}, X} and w = {φ, {b}, {a, b}}. Then the set A = {b} is gsp-w-closed set but not pre w-closed set.

Theorem 3.15. *Let w be a WS on a topological space (X, τ). Then every regular* *w-closed set is gsp-w-closed set but not conversely.*

Proof. Let w be a WS on a topological space (X, τ). Let A be a regular w-closed set with A ⊆ U and U is open. Since every regular w-closed set is sp-w-closed set, we have A is sp-w-closed set. Therefore spc_w(A) = A. Thus we have spc_w(A) ⊆ U whenever A ⊆ U and U is open. Hence A is gsp-w-closed set.

Example 3.16. *gsp-w-closed set* a *regular w-closed set*

Let X = {a, b, c}. Let w be a WS on a topological space (X, τ) where τ = {φ, {a}, {c}, {a, c}, X} and w = {φ, {b}, {c}, {a, b}}. Then the set A = {a} is gsp-w-closed set but not regular w-closed set.

Theorem 3.17. *Let w be a WS on a topological space (X, τ). Then every semi-pre losed set is gsp-w-closed set but not conversely.*

Proof. Let w be a WS on a topological space (X, τ). Let A be a semi-pre w-closedset with A ⊆ U and U is open. Since A is sp-w-closed set, we have spc_w(A) = A. Thus we have spc_w(A) ⊆ U whenever A ⊆ U and U is open. Hence A is gsp-w-closed set.

Example 3.18. *gsp-w-closed set* a *semi-pre w-closed set*

Let X = {a, b, c}. Let w be a WS on a topological space (X, τ) where τ = {φ, {b, c}, X} and w = {φ, {b}, {b, c}}. Then the set A = {b} is gsp-w-closed set but not semi-pre w-closed set.

Theorem 3.19. *Let w be a WS on a topological space (X, τ). Then every gw-closedset is gsp-w-closed set but not conversely.*

Proof. Let w be a WS on a topological space (X, τ). Let A be a gw-closed set. Then c_w(A) ⊆ U whenever A ⊆ U and U is open. Since spc_w(A) ⊆ cl_w(A), we have spc_w(A) ⊆ U whenever A ⊆ U and U is open. Hence A is gsp-w-closed set.

Example 3.20. *gsp-w-closed set* a *gw-closed set*

Let X = {a, b, c}. Let w be a WS on a topological space (X, τ) where τ = {φ, {a}, {c}, {a, c}, X} and w = {φ, {a, b}, {a, c}}. Then the set A = {a} is gsp-w-closedset but not gw-closed set.

Theorem 3.21. *Let w be a WS on a topological space (X, τ). If A is a gsp-w-closed, then spc_w(A)−A does not contain any non empty closed set.*

Proof. Let F be a closed subset of X such that F ⊆ spc_w(A)−A, where A is gsp-w-closed. Since

X−F is open, A ⊆ X−F and A is gsp-w-closed, $spc_w(A) \subseteq X-F$ andthus $F \subseteq X-spc_w(A)$. Thus $F \subseteq (X-spc_w(A)) \cap spc_w(A) = \emptyset$ and hence $F = \emptyset$.

Remark 3.22. *If $spc_w(A)-A$ does not contain any non empty closed subset of X, then A need not be gsp-w-closed.*

Example 3.23. *Let X = {a, b, c}. Let w be a WS on a topological space (X, τ) where τ = {φ, {b}, X} and w = {φ, {b}, {b, c}}. Let A = {b}. Then $spc_w(A)-A$ = {b, c}−{b} = {c}, does not contain any nonempty closed subset of X. But A is not gsp-w-closed set.*

Corollary 3.24. *Let w be a WS on a topological space (X, τ) and A ⊆ X be a gsp-w-closed set. Then $spc_w(A) = A$ if and only if $spc_w(A)-A$ is closed.*

Proof. Let A be a gsp-w-closed set. If $spc_w(A) = A$, then $spc_w(A)-A = \emptyset$, and $spc_w(A)-A$ is a closed set.

Conversely, let $spc_w(A)-A$ be a closed set, where A is gsp-w-closed. Then by Theo- rem 3.14, $spc_w(A)-A$ does not contain any non empty closed set. Since $spc_w(A)-A$is a closed subset of itself, $spc_w(A)-A = \emptyset$ and hence $spc_w(A) = A$.

Theorem 3.25. *A subset A of a topological space (X, τ) with a WS w on it is gsp-w-closed if and only if $cl(\{x\}) \cap A \;/\!= \emptyset$ for every $x \in spc_w(A)$.*

Proof. Let A be a gsp-w-closed set in X and suppose if possible that there exists $x \in spc_w(A)$ such that $cl(\{x\}) \cap A = \emptyset$. Therefore, $A \subseteq X-cl(\{x\})$, and so $spc_w(A) \subseteq X-cl(\{x\})$. Hence x /∈ $spc_w(A)$, which is a contradiction.

Conversely, suppose that the condition of the theorem holds and let U be any open set containing A. Let $x \in spc_w(A)$. Then by hypothesis $cl(\{x\}) \cap A \;/\!= \emptyset$, so there exists $z \in cl(\{x\}) \cap A$ and so $z \in A \subseteq U$. Thus $\{x\} \cap U \;/\!= \emptyset$. Hence $x \in U$, which implies that $spc_w(A) \subseteq U$. This shows that A is gsp-w-closed.

Theorem 3.26. *Let w be a WS on a topological space (X, τ) and A ⊆ B ⊆ $spc_w(A)$,where A is gsp-w-closed. Then B is gsp-w-closed.*

Proof. Let $B \subseteq U \in \tau$. Since A is gsp-w-closed and $A \subseteq U$, $spc_w(A) \subseteq U$. Now, $B \subseteq spc_w(A)$, $spc_w(B) \subseteq spc_w(A)$ and hence $spc_w(B) \subseteq U$.

Theorem 3.27. *Let w be a WS on a topological space (X, τ). Then the followingare equivalent:*

(1) For every open set U of X, $spc_w(U) \subseteq U$.

(2) Every subset of X is gsp-w-closed.

Proof. (1) ⇒ (2). Let A be any subset of X and $A \subseteq U \in \tau$. Then by (1) $spc_w(U) \subseteq U$ and hence $spc_w(A) \subseteq spc_w(U) \subseteq U$. Thus A is gsp-*w*-closed. (2) ⇒ (1). Let $U \in \tau$. Then by (2), U is gsp-*w*-closed and hence $spc_w(U) \subseteq U$.

Theorem 3.28. *Let w be a WS on a topological space (X, tau). If A is an open and gsp-w-closed subset of X, then $spc_w(A) = A$.*

Proof. Obvious.

Let us introduce *w*sp-$T_{\frac{1}{2}}$-space.

Definition 3.29. *A space (X, τ) is called a wsp-$T_{\frac{1}{2}}$-space if for every gsp-w-closed set A of X, $spc_w(A) = A$.*

Theorem 3.30. *Let w be a WS on a topological space (X, τ). Then the implication (1) ⇒ (2) holds. If $spi_w(\{x\}) \in w$ for every $x \in X$, then the following statements areequivalent:*

(1) X is a wsp-$T_{\frac{1}{2}}$-space.

(2) Every singleton is either closed or $\{x\} = spi_w(\{x\})$.

Proof. (1) ⇒ (2). Suppose {x} is not a closed subset for some $x \in X$. Then $X-\{x\}$ is not open and hence X is the only open set containing $X-\{x\}$. Therefore $X-\{x\}$ is gsp-*w*-closed. Since X is a *w*sp-$T_{\frac{1}{2}}$-space, $spc_w(X-\{x\}) = X-spi_w(\{x\}) = X-\{x\}$ and thus $\{x\} = spi_w(\{x\})$.

(2) ⇒ (1). Let A be a gsp-*w*-closed subset of X and $x \in spc_w(A)$. We show that $x \in A$. If {x} is closed and $x \notin A$, then $x \in (spc_w(A)-A)$. Then $\{x\} \subseteq X-A$ and hence $A \subseteq X-\{x\}$. Since A is gsp-*w*-closed set and $X-\{x\}$ is an open subset of X, $spc_w(A) \subseteq X-\{x\}$ and hence $\{x\} \subseteq X-spc_w(A)$. Therefore, $x \in A$.

If $\{x\} = spi_w(\{x\})$, since $x \in spc_w(A)$, then for every sp*w*-open set U containing x,we have $U \cap A \neq \emptyset$. But $\{x\} = i_w(\{x\})$ is sp*w*-open and $\{x\} \cap A \neq \emptyset$. Hence $x \in A$. Therefore, in both cases we have $x \in A$. Therefore, $spc_w(A) = A$ and hence X is a *w*sp-$T_{\frac{1}{2}}$-space.

4. Properties of gsp-*w*-open sets

In this section we introduce generalized semi-pre *w*-open sets and study some oftheir properties.

Definition 4.1. *Let w be a WS on a topological space (X, τ). Then A ⊆ X is calleda generalized semi-pre w-open set (gsp-w-open set in short) if the complement* A^c *is gsp-w-closed set.*

Example 4.2. *Let X = {a, b, c}. Let w be a WS on a topological space (X, τ) where τ = {φ, {a}, {b}, {a, b}, X} and w = {φ, {a}, {a, b}}. Then the set A = {c} is gsp-w-open set.*

Example 4.3. *Let X = {a, b, c}. Let w be a WS on a topological space (X, τ) whereτ = {φ, {a}, {b}, {a, b}, X} and w = {φ, {a}, {a, b}}. Then the set A = {b, c} isnot gsp-w-closed set.*

Theorem 4.4. *Let w be a WS on a topological space (X, τ). Then every w-open setis gsp-w-open set but not conversely.*

Proof. Let *w* be a WS on a topological space (X, τ). Let A be an *w*-open set. Then A^c is an *w*-closed set. By Theorem 3.7, A^c is gsp-*w*-closed set. Therefore A is gsp-*w*-openset.

Example 4.5. *gsp-w-open set* a *w-open set Let X = {a, b, c}. Let w be a WS on a topological space (X, τ) where τ = {φ, {a}, {b}, {a, b}, X} and w = {φ, {a}, {a, b}}. Then the set A = {a, c} is gsp-w-open set but not w-open set.*

Theorem 4.6. *Let w be a WS on a topological space (X, τ). Then every α-w-open set is gsp-w-open set but not conversely.*

Proof. Let *w* be a WS on a topological space (X, τ). Let A be an *α*-*w*-open set. Then A^c is an *α*-*w*-closed set. By Theorem 3.9, A^c is gsp-*w*-closed set. Therefore A is gsp-*w*-open set.

Example 4.7. *gsp-w-open set* a *α-w-open set*

Let X = {a, b, c}. Let w be a WS on a topological space (X, τ) where τ = {φ, {a}, *X*} *and w* = {*φ*, {*a*}, {*c*}}. *Then the set A* = {*b*} *is gsp-w-open set but not α-w-open set.*

Theorem 4.8. *Let w be a WS on a topological space (X, τ). Then every semi w-open set is gsp-w-open set but not conversely.*

Proof. Let *w* be a WS on a topological space (X, *τ*). Let A be a semi *w*-open set.Then A^c is a semi *w*-closed set. By Theorem 3.11, A^c is gsp-*w*-closed set. ThereforeA is gsp-*w*-open set.

Example 4.9. *gsp-w-open set* a *semi w-open set Let X* = {*a, b, c*}. *Let w be a WS on a topological space (X, τ) where τ* = {*φ*, {*a, b*}, *X*} *and w* = {*φ*, {*a*}, {*a, b*}}. *Then the set A* = {*b, c*} *is gsp-w-open set but not semi w-open set.*

Theorem 4.10. *Let w be a WS on a topological space (X, τ). Then every pre w-open set is gsp-w-open set but not conversely.*

Proof. Let *w* be a WS on a topological space (X, *τ*). Let A be a pre *w*-open set. Then A^c is a pre *w*-closed set. By Theorem 3.13, A^c is gsp-*w*-closed set. ThereforeA is gsp-*w*-open set.

Example 4.11. *gsp-w-open set* a *pre w-open set Let X* = {*a, b, c*}. *Let w be a WS on a topological space (X, τ) where τ* = {*φ*, {*a*}, {*c*}, {*a, c*}, *X*} *and w* = {*φ*, {*a, b*}}. *Then the set A* = {*a, c*} *is gsp-w-open set but not pre w-open set.*

Theorem 4.12. *Let w be a WS on a topological space (X, τ). Then every regular w-open set is gsp-w-open set but not conversely.*

Proof. Let *w* be a WS on a topological space (X, *τ*). Let A be a regular *w*-open set. Then A^c is a regular *w*-closed set. By Theorem 3.15, A^c is gsp-*w*-closed set. Therefore A is gsp-*w*-open set.

Example 4.13. *gsp-w-open set* a *regular w-open set Let X* = {*a, b, c*}. *Let w be a WS on a topological space (X, τ) where τ* = {*φ*, {*a*}, *X*} *and w* = {*φ*, {*a*}, {*c*}}. *Then the set A* = {*b, c*} *is gsp-w-open set but not regular w-open set.*

Theorem 4.14. *Let w be a WS on a topological space (X, τ). Then every semi-pre w-open set is gsp-w-open set but not conversely.*

Proof. Let *w* be a WS on a topological space (X, *τ*). Let A be a semi-pre *w*-openset. Then A^c is a semi-pre *w*-closed set. By Theorem 3.17, A^c is gsp-*w*-closed set. Therefore A is gsp-*w*-open set.

Example 4.15. *gsp-w-open set* a *semi-pre w-open set*

Let X = {*a, b, c*}. *Let w be a WS on a topological space (X, τ) where τ* = {*φ*, {*b, c*}, *X*} *and w* = {*φ*, {*b*}, {*b, c*}}. *Then the set A* = {*b, c*} *is gsp-w-open set but not semi-pre w-open set.*

Theorem 4.16. *Let w be a WS on a topological space (X, τ). Then every gw-open set is gsp-w-open set but not conversely.*

Proof. Let *w* be a WS on a topological space (X, τ). Let A be a g*w*-open set. ThenA^c is a g*w*-closed set. By Theorem 3.19, A^c is gsp-*w*-closed set. Therefore A is gsp-*w*-open set.

Example 4.17. *gsp-w-open set* a *gw-open set Let X = {a, b, c}. Let w be a WS on a topological space (X, τ) where τ = {φ, {a}, {c}, {a, c}, X} and w = {φ, {a, b}}. Then the set A = {a} is gsp-w-open set but not gw-open set.*

Theorem 4.18. *Let (X, τ) be a topological space and w be a WS on X. Then A is gsp-w-open if and only if F ⊆ spi_w(A) whenever F ⊆ A and F is closed.*

Proof. Let A be a gsp-*w*-open set and F ⊆ A, where F is closed. Then X−A is gsp-*w*-closed set contained in an open set X−F. Hence spc_w(X−A) ⊆ X−F, that is X−spi_w(A) ⊆ X−F. So F ⊆ spi_w(A). Conversely, suppose that F ⊆ spi_w(A) for any closed set F whenever F ⊆ A. Let X−A ⊆ U, where U ∈ τ. Then X−U ⊆ A and X−U is closed. By assumption, X−U ⊆ spi_w(A) and hence spc_w(X−A) = X−spi_w.(A) ⊆ U. Therefore X−A is gsp-*w*-closedand hence A is gsp-*w*-open.

Theorem 4.19. *Let w be a WS on a topological space (X, τ). If a subset A of X is gsp-w-open, then U = X whenever U is open and spi_w(A) ∪ (X−A) ⊆ U.*

Proof. Let U ∈ τ and spi_w(A) ∪ (X−A) ⊆ U for a gsp-*w*-open set A. Then X−U ⊆ (X−spi_w(A)) ∩ A. That is X−U ⊆ spc_w(X−A)−(X−A). Since X−A is gsp-*w*-closed, by Theorem 3.21, X−U = ∅ and hence X = U.

Theorem 4.20. *Let w be a WS on a topological space (X, τ). If a subset A of X is gsp-w-open and spi_w(A) ⊆ B ⊆ A, then B is gsp-w-open.*

Proof. We have X−A ⊆ X−B ⊆ X−spi_w(A) = spc_w(X−A). Since X−A is gsp-*w*- closed, it follows from Theorem 3.26 that X−B is gsp-*w*-closed and hence B is gsp- *w*-open.

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