IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)
An International Open Access, Peer-reviewed, Refereed Journal

Generation of prime labeled trees through corona product

J. Prabagaran[1] and K. Rajendran[2]

[1]Research Scholar
Department of Mathematics
Vels Institute of Science Technology and Advanced StudiesChennai, India

[2]Department of Mathematics
Vels Institute of Science Technology and Advanced StudiesChennai, India

Abstract

A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers $1, 2, 3, \ldots, |V|$ such that for each edge xy the labels assigned to x and y are relatively prime. In this paper, we prove that tree $T_n \odot K_1$, for any n admit prime labeling provided with the prime labeling of tree T_n. By the way, we prove that using corona product of tree with complete graph on one vertex, we can generate prime labeled trees. This procedure can be repeated so that we can generate prime labeled trees as long as the procedure is continued.

Mathematics Subject Classification: 05C78;05C05
Keywords: prime labeling; prime graphs; corona product of graphs;

1 Introduction

Graphs we considered in this paper are finite, undirected and simple graphs. For various graph theoretic teminologies, we refer the book [7]. Rosa [5] introduced various graph labeling and after that many researchers introduced various graph labeling for to de- compose graphs. One such graph labeling is prime labeling introduced by Entringer. A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers $1, 2, 3, \ldots, |V|$ such that for each edge xy the labels assigned to x and y are relatively prime. In the year 1980, Entringer conjectured that all trees have a prime
labeling. Seoud et.al., [6] proved the necessary and sufficient conditions for a graph to be prime.

They also gave a procedure to determine whether or not a graph is prime. Deretsky et al., [1] proved that cycles and disjoint union of cycles are prime. Lee et. al. [4] proved that complete graph does not have a prime labeling for $n \geq 4$ and wheel graphs W_n are prime if and only if n is even. For an exhaustive survey on Graceful Tree Conjecture, refer the excellent survey by Gallian [3]. In this paper, we prove that graphs $C_n \odot K_1$, for any $n \geq 3$ and $W_n \odot K_1$ are prime when n is even.

2 Corona Product of Graphs

Let G and H be two graphs and let n be the order of G. The corona product, or simply the corona, of graphs G and H is the graph $G \odot H$ obtained by taking one copy of G and n copies of H and then joining by an edge the ith vertex of G to every vertex in the ith copy of H. Given a vertex $g \in G$, the copy of H connected to g is denoted by H_g [2]. Complete graphs, stars and wheels are basic examples of corona product families.

Observation 1: When G is a tree T with m edges and $H \cong K_1$, the corona $T \odot K_1$ is also a tree with $2m + 1$ edges. Thus, the number of newly added vertices in the corona product of T and K_1 will be $m + 1$ and all of those vertices are of degree 1. An example of a tree T with 16 edges is shown in Figure 1 and its corona $T \odot K_1$ is shown in Figure 2.

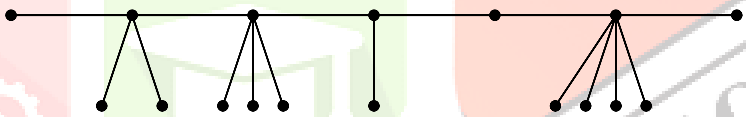

Figure 1: Tree T with 16 edges

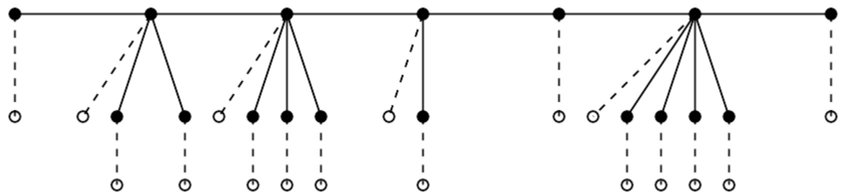

Figure 2: Corona Tree $T \odot K_1$ with 33 edges

Notation: For the sake of convenience, let $V(K_1) = \{w\}$ and if u be any vertex in the tree T, then the corresponding vertex added in the corona tree $T \odot K_1$ will be denotedas w_u.

3 Main Result

In this section, we prove our main result that given a prime labeled tree, an another prime labeled tree can be generated using the corona product of given prime labeled tree and a complete graph on one vertex K_1. Let f be a prime labeling of the tree T with m edges. Let $V(T) = \{u_0, u_1, \cdot\cdot\cdot, u_m\}$ and $V(K_1) = \{w\}$. Therefore, $V(T \odot K_1) = V_1 \cup V_2$, where $V_1 = \{u_0, u_1, \cdot\cdot\cdot, u_m\}$ and $V_2 = \{w_{u_0}, w_{u_1}, \cdot\cdot\cdot, w_{u_m}\}$. We will define the labeling function $\varphi : V(T \odot K_1) \to \{1, 2, \cdot\cdot\cdot, 2m + 2\}$ for the corona $T \odot K_1$.

First let us label the vertices in V_2 of $T \odot K_1$ as follows:

$$\varphi(w_{u_i}) = 2f(u_i), \text{ for } 0 \le i \le m \qquad (1)$$

Now, let us label the vertices in V_1 of $T \odot K_1$ as follows:

$$\varphi(u_i) = 2f(u_i) - 1, \text{ for } 0 \le i \le m \qquad (2)$$

Theorem 1. *Tree $T_n \odot K_1$ admits prime labeling.*

Proof. By the definition of labeling function φ, it is clear that the labels of vertices in V_1 are odd and the labels of vertices in V_2 are even. Since f is a prime labeling, the vertex labels of the corona $T \odot K_1$ are distinct and from the set $\{1, 2, 3, \ldots, 2m + 2\}$. Therefore, the labels of the adjacent vertices are always relatively prime being one end of any edge is an even number while the other end is an odd number. Thus, $T_n \odot K_1$ admits prime labeling. □

4 Example

In this section, we give example of a prime labeled tree T in Figure 3 and the primelabeling of corona $T \odot K_1$ in Figure 5.

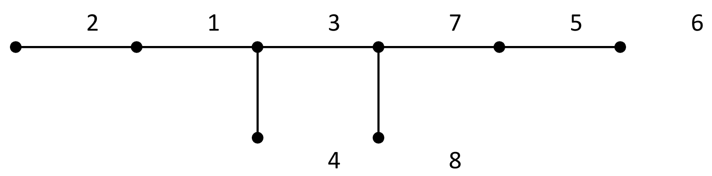

Figure 3: Prime labeling of tree T with 7 edges

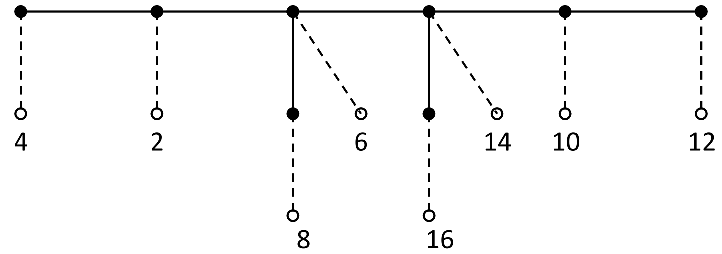

Figure 4: Corona Tree $T \odot K_1$ with all even labeled vertices as defined in Equation 1

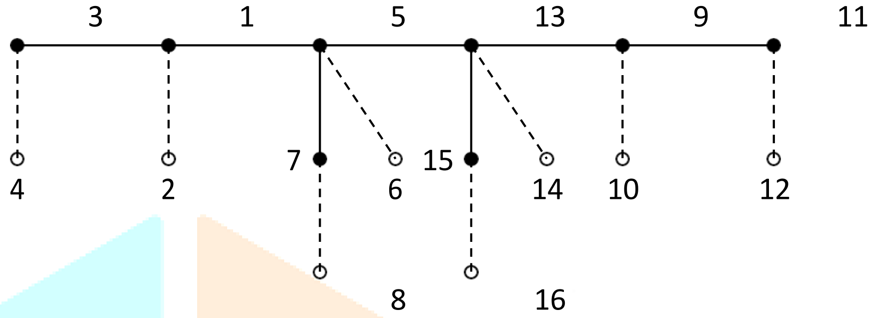

Figure 5: Prime labeling of corona tree $T \odot K_1$ with 15 edges

5 Conclusion

We proved that if T is a prime labeled tree, then the corona $T \odot K_1$ is also a prime labeled tree. The purpose of the note is to generate prime labeled trees through corona product. If we wish to generate prime graphs instead of prime tree, one can think of replacing the graph K_1 with other graphs. In this direction, we raise a question of how to generate prime labeled graphs using corona products.

References

[1] T. Deretsky, S. M. Lee, and J. Mitchem, *On vertex prime labelings of graphs*, in Graph Theory, Combinatorics and Applications Vol. 1, J. Alavi, G. Chartrand, O. Oellerman, and A. Schwenk, eds., Proceedings 6th International Conference Theory and Applications of Graphs (Wiley, New York, 1991) 359-369.

[2] Farrugia R. and Harary F, *On the corona of two graphs*, Aequationes Math, 4, (1970),322-325.

[3] Gallian J.A., *A Dynamic Survey of Graph Labeling*, The Electronic Journal of Com- binatorics, 18, (2015), #DS6.

[4] S. M. Lee, I. Wui and J. Yeh, *On the amalgamation of prime graphs*, Bull. Malaysian Math. Soc. (Second Series), 11 (1988) 59-67.

[5] Rosa A, *On certain valuations of the vertices of a graph*, Theory of graphs, (Interna- tional

Symposium, Rome, July 1966), Gordon and Breach, N.Y. and Dunod Paris, (1967), 349-355.

[6] M. A. Seoud, A. El Sonbaty, and A. E. A. Mahran, *On prime graphs*, Ars Com- bin.,104 (2012) 241-260.

[7] West D.B., *Introduction to Graph Theory*, Prentice Hall of India, 2nd Edition, 2001.