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Mathematical Analysis of two phase Coronary Blood flow in Arterioles with Silent Ischemia Disease.

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Abstract

Present paper envisions a model of two phased coronary blood flow in arterioles relevant to silent ischemia. Through arterioles the coronary circulation be made of the blood vessels that supply blood to the heart muscle to remove it. The blood enters coronary arterioles from coronary artery. We have implemented the Herschel Bulkley Non-Newtonian model in bio-fluid mechanical setup. We have accumulated pathological data in case of silent ischemia for the graphical study of blood pressure drop v/s hematocrit. Including everything the presentation is in tensorial form and solution techniques adopted is analytical as well as numerical.

Key words: Silent ischemia, coronary blood flow, hematocrit, Herschel Bulkley Non-Newtonian model.

Introduction

The specialized blood vessels known as arterioles may be small in stature, but they play a big role in heart health. As you might suspect, they are related to arteries, the blood vessels that carry oxygenated blood away from the heart and to the body's tissues. As you move down through the arterial network, these vessels get smaller and smaller like the branches of a tree. When arteries have decreased in size to less than 300 micro meters or 100 of an inch, they're referred to as arterioles. Arterioles share many of the properties of arteries. They are strong, have relatively thick walls, and contain a high percentage of smooth muscle, which means they are not under voluntary control. As the most highly regulated blood vessels in the body, arterioles also have the distinction of contributing the most to the rise and fall of blood pressure.

Arterioles are considered as the primary resistance vessels as they distribute blood flow into capillary beds. Arterioles provide approximately 80 % of the total resistance to blood flow through the body. Considering they are vital regulators of hemodynamics, contributive to the upstream pressure to the regional distribution of blood. They have significantly variable diameter depending on vascular bed and state of constriction or dilation. Therefore, size is not their main identifying feature, but the fact layers of smooth muscle in their wall-the wall of arterioles composed of three structurally distinct layers: intima, media and adventitia.

Blood flow in coronary circulation is not continuous as other parts of the body. Vascular resistance in the transmural portion of the coronary circulation is significantly higher during ventricular contraction resulting in decreased flew during systole. And regulation of coronary micro vascular resistance varies across different segments of the vasculature. Arterioles with less than 100 microns in diameter respond differently than larger arterioles at the level of auto regulation, myogenic control and control by metabolic factors. ^[9]



The word ischemia comes from a Latin term that means "Stopping blood". But if you have cardiac ischemia, blood flow to your heart doesn't actually stop. Instead, the supply of blood is temporarily less than the heart muscle needs. "People with heart disease may have 5 to 10 times as many episodes of silent ischemia as symptomatic ischemia," says Dr. Peter stone, a professor of medicine at Harvard medical school and director of the vascular profiling research group at Brigham and women's hospital. Silent ischemia is exactly the angina, except that you don't feel it. While some people have only angina and others have only silent episodes, most people with narrowed arteries have both types, in both conditions, the heart's nerves seem to react in the same way. But when silent ischemia occurs, the pain signal may be processed differently in the brain, says Dr. Stone.^[5]



Constitution of blood:

Blood is a fluid that moves through the vessels of a circulatory system. The percentage of volume covered by blood cells in whole blood is called hematocrit. The total volume concentration of leukocytes and thrombocytes is only about 1% which is negligible. Then we have considered only two phases of blood, which one phase is red blood cells and other phase is plasma. A hematocrit ranging from 42% to 52% in males and 35% to 47% in females is typically considered normal.^[15]



Components of Blood

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2. Real Model:

2.1. Choice of frame of reference:

We have to choose a frame of reference for mathematical modelling of the state of a moving blood keeping in mind the difficulty and generality of the problem of blood flow, we choose generalized three dimensional orthogonal curvilinear co-ordinate system summarized as E^3 , called as 3-dim Euclidean. We explain quantities related to blood flow in tensorial form. Let the co-ordinate axis be OX^i , where O is origin and i= 1,2,3. The mathematical description of the state is a moving blood is affected by means of functions which give the distribution blood velocity $v^k = v^k(x^i, t)$.

2.2. Choice of parameters:

Blood is Non-Newtonian fluids. The constitutive equations for fluids $\tau = \eta e^n$.

If n=1 then nature of fluid is Newtonian and if $n \neq 1$ then nature of fluid is Non-Newtonian. Where τ denoted by stress, e denoted by strain rate, η is denoted by viscosity and n is the parameter depends upon nature of fluid.

Now we have using two phase blood flow through arterioles whose constitutive equation is as follows-

$$T' = \eta_m e^n + T_P \quad (T' \ge T_P)$$

Where T_P is denoted by yield stress. When strain rate e = 0 ($T' < T_P$) a core region is formed which flow just like a plug.^[13]

2.3. Constitution of two phase blood volume:

The flow of blood is affected by the presence of blood cells, which is directly proportional to the volume occupied by blood cells.

Let the volume portion covered by blood cells in unit volume be $X, X = \frac{H}{100}$, where H is the Hematocrit. Then the volume portion covered by the plasma will be (1-X).



Fig 4. Unit volume

3. Mathematical formulation:

Blood is in the liquid form and it is Non-Newtonian through blood is not an ideal fluid, even to develop the equation of motion. The second important principle of fluid dynamics is the conservation of momentum. The equation of motion is motion is based on this principle. According to this principle, the total momentum of any fluid system is conserved in absence of external force.

 $\frac{dp}{dt} + P - F_v = 0 \qquad (\text{ External force})$

The blood can be considered as homogeneous mixtures of two phases. We derive the fundamental equation of continuity, which is a mathematical expression of principle of conservation of matter.

3.1. Equation of continuity:

If mass ratio of cells to plasma is r then

Where ρ_c and ρ_p are densities of blood cells and plasma respectively. Usually this mass ratio is not a constant. On the other hand mass ratio supposed to be constant in present context [Upadhyay V., 2000]^[13]. Mass ratio changes vessels to vessels. But in particular vessels (i.e. arterioles, arteries, capillary) mass ratio is constant. The both phase of blood, i.e. blood cells and plasma move with the common velocity. We consider the two phase of blood separately.

And

Where v^i = common velocity of two phase blood cells and plasma. $(X\rho_C v^i)_{,i}$ is covariant derivative of $(X\rho_C v^i)_{,i}$ with respect to X^i and $[(1 - X)\rho_P v^i]_{,i}$ is covariant derivative of $[(1 - X)\rho_P v^i]_{,i}$ with respect to X^i . If we define the uniform density of the blood ρ_m as follows

$$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_P} \tag{3.4}$$

Combined equation (3.2) and (3.3) we get

Where $\rho_m = X \rho_C + (1 - X) \rho_P$.

3.2. Equation of motion for blood flow:

The hydro dynamical pressure P between the two phases of blood can be supposed to be uniform because both the phases are always in equilibrium state in blood. Taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum. We get the equation of motion for two phase of blood cells as follows:

The equation of motion for plasma will be as follows-

$$(1-X)\rho_{c}\frac{\partial v^{i}}{\partial t} + ((1-X)\rho_{c}v^{j})v^{i}_{,j} = -(1-X)_{p,j}g^{ij} + (1-X)\eta_{c}(g^{jk}v^{i}_{,k})_{,j} \quad \dots \dots (3.7)$$

Now adding equation (3.6) and (3.7) and using relation (3.4), the equation of motion for blood flow with the both phases will be as follows:

Where $\eta_m = X\eta_c + (1 - X)\eta_P$ is the viscosity coefficient of blood flow decreases, the viscosity of blood increases. The velocity of blood decreases successively because of the fact that arterioles, veinules and veins these vessels is relatively low. Secondly these vessels is relatively narrow down more rapidly. In this situation, the blood cells line up on the axis to build up rouleaux. Hence a yield stress is produced. Though this yield stress is very small, even then viscosity of blood is increased.

Applying Herschel Bulkley law on the two phase blood flow through arterioles.^[7,8] The constitutive equation is as follows:

 $T' = \eta_m e^n + T_P$ $(T' \ge T_P)$ and e = 0 $(T' < T_P)$

Where T_P is yield stress. When strain rate e = 0 $(T' < T_P)$ A core region is formed which flow just like a plug. Let the radius of the plug be r_P , the stress acting on the surface of plug will be T_P . Equating the forces on the plug, we get

⇒

Or



Herschel <u>Bulkley</u> Blood Flow

Fig: 5

The constitutive equation for test part of blood vessel is

$$T' = \eta_m e^n + T_P$$
$$T' - T_P = \eta_m e^n = T_P$$

Where T_e = effective stress.

Whose generalized form will be as follows -

$$T^{ij}_{ij} = -Pg^{ij} + Te^{ij}$$

Where $Te^{ij} = \eta_m (e^{ij})^n$ while $e^{ij} = g^{jk} V_k^i$ Where the symbols have their usual meanings.

Now we describe the basic equations for Herschel Bulkley blood flow as follows-

Equation of continuity $-\frac{1}{\sqrt{g}}(\sqrt{g}v^i)_{,i} = 0$ Equation of motion $-\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^J)v^i_{,j} = -T^{\ iJ}_{e,J}$ (3.10)

Where all the symbols have their usual meaning.

4. Analysis:

The blood vessels supposed to be in this case are cylindrical; the above governing equations have to be transformed into cylindrical co-ordinates. As we know earlier:

$$X^1 = r , \qquad X^2 = \theta , \qquad X^3 = z.$$

Matrix of metric tensor in cylindrical co-ordinate system is as follows -

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

While matrix of conjugate metric tensor is as follows -

$$\begin{bmatrix} g^{ij} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Where the Christoffel's symbols of 2nd kind as follows –

Remaining others are zero.

Relation between contravariant and physical components of velocity of blood flow will be as follows –

Again the physical components of $P_{,J}g^{ij}$ are $\sqrt{g_{ij}}P_{,J}g^{ij}$

Now, equation (3.9) & (3.10) are transformed into cylindrical form so as to solve them as power law model to find

Equation of continuity – Equation of motion – $r - component: -\frac{\partial p}{\partial z} = 0$, $\theta - component: 0 = 0$

z - component: $0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r} \left[r \left(\frac{\partial v_z}{\partial r} \right)^n \right]$

Here, this fact has been taken in view that the blood flow is axially symmetric in arteries concerned, i.e. $v_{\theta} = 0$ and v_r , v_z and p = p(z) and

Since, pressure gradient –

$$r(\frac{dv}{dz})^n = -\frac{Pr^2}{2\eta_m} + A$$

Applying boundary condition at r = 0, $v = v_0$ then we get A = 0.

On integration above equation (4.2) under the no slip boundary condition v = 0 at r = R so we get:

Age – 60,

This is the formula for velocity of blood flow in arterioles. Putting $r = r_P$ to get the velocity v_P of plug flow as follows:

Sex – M

Where the value of r_P is taken from (3.7).

5. Bio-Physical Interpretation:

5.1. Patient case history:

Name – Brijbhan singh,

Diagnosis: Silent Ischemia (Coronary disease)

Date Hemoglobin Hematocrit Blood Arterioles Pressure drop (in pascal sec.) $\left(\frac{S+D}{2}\right) + D$ (3*HB) Pressure S + D(mmhg) 23/09/21 12.7 0.03595 110/70 36.66 12.4 0.03509 110/80 24/09/21 36.66 25/09/21 13.6 0.03849 130/80 43.33 26/09/21 12.9 0.03650 120/80 40.00 27/09/21 13.7 0.03877 120/70 36.66

 r_P

The two phase blood flow in arterioles is –

$$Q = \int_{0}^{r_{P}} 2\pi r V_{P} dr + \int_{r_{P}}^{r_{P}} 2\pi r V dr$$
$$= \int_{0}^{r_{P}} 2\pi r \frac{n}{n+1} (\frac{P}{2\eta_{m}})^{1/n} (R - r_{P})^{1/n+1} dr + \int_{r_{P}}^{R} 2\pi r \frac{n}{n+1} (\frac{P}{2\eta_{m}})^{1/n} \left[(R - r_{P})^{1/n+1} - (R - r_{P})^{1/n+1} \right] dr$$

R

Using equation (4.2) and (4.4) we get

$$Q = \frac{2\pi n}{n+1} \left(\frac{P}{2\eta_m}\right)^{1/n} \left(R - r_p\right)^{1/n+1} \left[\frac{r^2}{2}\right]_0^{r_p} + \frac{2\pi n}{(n+1)} \left(\frac{P}{2\eta_m}\right)^{1/n} \left[\frac{r^2}{2} \left(R - r_p\right)^{1/n+1} - \frac{r(r-r_p)^{1/n}}{1/n+1} + \frac{(r-r_p)^{1/n+3}}{(\frac{1}{n}+1)(\frac{1}{n}+3)}\right]_{r_p}^R$$

$$Q = \frac{2\pi n}{n+1} \left(\frac{P}{2\eta_m}\right)^{1/n} r_P^2 (R-r_P)^{1/n+1} + R^2 (R-r_P)^{1/n+1} - \frac{2R(R-r_P)^{1/n+2}}{1/n+2} + \frac{2R(R-r_P)^{1/n+3}}{\left(\frac{1}{n}+2\right)\left(\frac{1}{n}+3\right)} - r_P^2 (R-r_P)^{1/n+1}$$

$$Q = \frac{\pi n}{(n+1)} \left(\frac{p}{2\eta_m}\right)^{1/n} R^{1/n+3} \left[\frac{r_p^2}{R^2} \left(1 - \frac{r_p^2}{R}\right)^{1/n+1} + \left(1 + \frac{r_p}{R}\right)^{1/n+1} \left(1 - \frac{r_p}{R}\right)^{1/n+2} - \frac{2\left(1 - \frac{r_p}{R}\right)^{1/n+2}}{\left(\frac{1}{n} + 2\right)} + \frac{2\left(1 - \frac{r_p}{R}\right)^{1/n+3}}{\left(\frac{1}{n} + 2\right)\left(\frac{1}{n} + 3\right)} \qquad (5.1)$$

Now we have, $Q = 425 \frac{ml}{min} = 0.00708333 \ m^3/sec$
 $R = 1$, $r_p = \frac{1}{3}$ According to Gustafson, Daniel R.(1980)^[3]

 $\eta_p = 0.0015 \ pascal \ second$ (According to Glenn Elert (2010)^[2]

 $\eta_m = 0.035 \ pascal \ sec$

Arterioles pressure drop
$$(P_f - P_i)_{systolic} = 4887.58 \ pascal \ sec.$$

And H = 0.035095

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Coronary arterioles length = 0.05 m By using relation $\eta_m = \eta_c X + \eta_p (1 - X)$ Where $X = \frac{H}{100} = 0.00035095$ $0.035 = (0.00035095)\eta_c + 0.00149947$ $\eta_c = 95.4566930332$ Pascal sec Using this relation, we get $\eta_m = 0.95456693 \, H + 0.00149947$ Now putting the value of r_p and R in equation (5.1), we get $Q = \frac{n\pi}{n+1} \left[\frac{P}{2n_m} \cdot \frac{2}{3}\right]^{1/n} \left(\frac{2}{27}\right) \left[\frac{26n^2 + 33n + 9}{6n^2 + 5n + 1}\right]$ $\frac{27Q}{2\pi} = \left[\frac{P}{3\eta_m}\right]^{1/n} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1}\right]$ Or $A = \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1}\right]$ Let $\frac{P}{3n_m} = \left(\frac{27 \cdot Q}{2\pi A}\right)^n \quad \Rightarrow P = \left(\frac{27 \, Q}{2\pi A}\right)^n \cdot 3\eta_m$ $P = -\frac{dp}{dz} \implies -dp = P dz$ And limit of the pressure from z_f to z_i then $\int_{P_{f}}^{z_{i}} dP = -\int_{T}^{z_{i}} (\frac{27 Q}{2\pi A})^{n} . 3\eta_{m} dz$ Where $P_f - P_i$ = pressure drop and $z_f - z_i$ = coronary arterioles length $P_f - P_i = \left(\frac{27 Q}{2\pi A}\right)^n . 3\eta_m \left(z_f - z_i\right)$ $\frac{27 Q}{2\pi A} = \left[\frac{P_f - P_i}{(z_f - z_i)3\eta_m}\right]^{1/n}$ $\frac{27 Q}{2\pi A} = \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1}\right] \left[\frac{P_f - P_i}{(z_f - z_i)3n_m}\right]^{1/n}$ Substituting the value of Q, η_m , $(z_f - z_i)$ and $P_f - P_i$ and solve by numerical methods- $\frac{27 * 0.0070833}{6.28} = \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1}\right] \left[\frac{4887.58}{0.05 * 0.105}\right]^{1/n}$ 0.030438265 = $\left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1}\right] \left[\frac{4887.58}{0.00525}\right]^{1/n}$ 0.030438265 = $\left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1}\right] (930967.619)^{1/n}$ On solving, we get n = 2.635Now again using equation $P_f - P_i = \left(\frac{27 Q}{2\pi A}\right)^n \cdot 3\eta_m \left(z_f - z_i\right)$

$$\Delta P = \left(\frac{27 \ Q}{2\pi A}\right) \quad .3\eta_m \ \Delta z$$

= $\left(\frac{27 * 0.0070833}{6.28 * 5.606434993}\right)^{2.635} \quad .3\eta_m * 0.05$
= $(0.005429166)^{2.635} \quad .3\eta_m * 0.05$
= $(931061.2696) \quad .3\eta_m * 0.05$
 $\Delta P = 139659.19044 \ \eta_m$
139659.19044 [0.95456693 H + 0.00149947]
 $\Delta P = 133314.045 \ H + 209.414766289$

=

5.2. Table for Hematocrit v/s Blood pressure drop

1.2. Tuble for Hellautoern (75 blood pressure drop					
Hematocrit	0.03595	0.03509	0.03849	0.03650	0.03877
(H)					
Blood	5079474.529	4959491.88877	5439422.45077	5159462.95627	5479416.66427
pressure	27				
drop					

(a). Graphical presentation of Pathological data:



Graph (a): Relation between Hematocrit v/s Pathological Blood Pressure Drop

(b). Graphical Presentation of Mathematical Modulated Data:



Graph (b): Relation between Hematocrit v/s Mathematical Modulated Blood Pressure Drop.

6. Conclusion:

In the above we have taken blood characteristic as a homogeneous non-Newtonian fluid. An alternative approach is to describe it as a complex fluid that consists of blood plasma and blood cells. Graph b (Table 5.2) relations between mathematically modulated blood pressure drop v/s hematocrit. $\Delta P = 133314.045 H + 209.414766289$, Where Δp denoted by Relation between blood pressure drop v/s hematocrit (trained line). The linear model is found for the given data.

In the above graphs (a & b) we have observed "relation between pathologically clinically blood pressure drop and hematocrit" and "relation between mathematically modulated blood pressure drop v/s hematocrit" graph are shows different nature but there trend line are not different. Trend lines of above graphs shows that if trend is upward it means fluctuation of blood (pressure drop) increases sense with respect to the hematocrit. Graph Trend is downward that means fluctuation of blood pressure drop decreases with respect to the hematocrit. According to this study work concluded that designate the function of hematocrit inside the will power of blood pressure drop. For this reason the hematocrit is extended then the blood pressure drop is likewise multiplied.

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