IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

AN M/G/1 RETRIAL QUEUE WITH MULTIPLE WORKING VACATIONS SETUP TIMES WITH N-POLICY AND LOCK DOWN TIMES

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ABSTRACT:

In this Paper An M/G/1 Retrial bulk queue with multiple working vacations, setup times with N-policy and Lockdown times considered in completion of a service, if the queue length is χ when $\chi < a$,then the server performs lockdown work. The server leaves for multiple working vacation of random length irrespective of queue length. When the server returns form a working vacation and if the queue length is still less than 'N' leaves for another working vacation and so on, until finds 'N'$(\mathrm{N} > b)$customers in the queue. That is, if the server finds at least 'N' customers waiting for service, then the requires a setup time 'R' to start the service. After the setup serves a batch of 'b' customers ,where $b \geq a$.Various characteristics of the queueing system and a cost model with the numerical solution for a particular case of the model are presented.

Key words:

Lockdown time, Supplementary variable method ,Multiple working vacations, Setup time and N-Policy

MSC: 60K25,60K30

1.Introduction

The paper concentrates on a working vacation system with lockdown time s and setup times with N-Policy, Server working vacation models are useful for the systems in which the server wants to utilize the idle time for different purposes. An Application of server working vacation models can be found in manufacturing systems, designing of local area networks and data communication systems, In practical situations, the lockdown time corresponds to the time taken for lockdown the service and setup time corresponds to the preparation time for starting the service . This objective of this paper is to analyse a situation that exists in a pipe manufacturing industry, A pipe manufacturing industry manufactures different types of pipes, which require shafts of various dimensions. The partially finished pipe shafts arrive at the copy turning center from the turning process only if the copy turning center form the turning centre. The operators starts the copy turning process only if required batch quantity of shafts is available shafts is less than the minimum batch quantity, then the operator will start doing other work such as making the templates for copy turning, checking the components, The operator always shuts down the machine and removes the templates before taking are more than the maximum batch quantity, the server resumes the copy turning process, for which some amount of time is required to setup the template in the machine, Otherwise the operator will continue with other until finds required number of shafts. The above process can be modeled as M/G/1 queueing system with multiple working vacation, setup times with N-policy and lockdown times.

A large number of researchers working in various fields have analyzed retrial queues. For a detailed review of literature on retrial queues one may refer Falin and Templeton (1979),Gomez-corral(1999), Renganathan et al.(2002),and kalyanaraman and Srinivasan(2003,2004). Recently, Recently, queueing system with vacations have been studied extensively, along with a comprehensive and excellent study on the vacation models ,including some applications such as production inventory system, communication systems, and computer systems As we known, there are mainly two vacation policies: classical vacation policy (also called ordinary vacation) and working vacation policy. The characteristic of a working vacation is that the server serves customers at a lower service rate during the vacation period, but in the case of classical vacation, the server stops the service completely during the vacation period.

In the literature of queueing systems with vacation has been discussed through a considerable amount of work in the recent past. Doshi (1990) has recorded prior work on vacation models and their applications in his survey paper. In recent year few authors who were concentrated on vacation queues are Madan and Gautam Choudhury (2005), Kalyanaraman and Pazhani Bala Murugan (2008) and Thangaraj and Vanitha(2010).

In this paper we study an Non-Markovian retrial queue with Multiple working vacation. The organization of the paper is as follows. In section 2 we describe the model. In section 3, we obtain the steady state probability generating function. Particular cases are discussed in section4. Some performance measures are obtained in section 5 and in section 6 numerical study is presented.

2.Model Description:

Let X be the group size random variable of the arrival,s_k be the probability that ‘k' customers arrive in a batch and X(z)be its probability generating function. Let S(.),V(.),R(.), and L(.) be the cumulative distributions of the service time, vacation time, setup time and lockdown time respectively. Further $s(x)$, $v(x)$, $r(x)$, and $l(x)$, are their respectively probability density functions. At an arbitrary time $s^0(t)$ denotes the remaining service time at time ‘t’ and $V^0(t)$, $L^0(t)$ and $R^0(t)$ denote the remaining vacation time, lockdown time and setup time at time ‘t’ respectively. Let us denote the Laplace transforms (LT) of $s(x)$, $v(x)$, $r(x)$, and $l(x)$ as S^*, V^*, R^*, L^*,respectively,

$N_s(t)$ = Number of customers in the service,

$N_q(t)$ = Number of customers in the queue,

We define the different states of the server at time ‘t’:

$$Y(t) = \begin{cases} 0 \text{ if the server is on busy with bulk service} \\ 1 \text{ if the server is doing lockdown job or setup job} \\ 2 \text{ if the server is on vacation} \end{cases}$$

and define N(t) = j , if the server is on j^{th} vacation starting from the idle period.

The Probabilities for the number of customers in the queue and service are defined as follows:

$$P_{i\,j}(x,t)dt = P\{ N_s(t) = i, N_q(t) = j, x \leq S^0(t) \leq x + dt, Y(t) = 0\}, a \leq i \leq b, j \geq 0$$

$$L_n(x,t)dt = P\{ N_q(t) = n, x \leq L^0(t) \leq x + dt, Y(t) = 1 \}, n \geq 0 \text{ and}$$

$$R_n(x,t)dt = P\{N_q(t) = n, x \leq R^0(t) \leq x + dt, Y(t) = 1\}, n \geq N$$

$$Q_{j\,n}(x,t)dt = P\{ N_q(t) = n, x \leq V^0(t) \leq x + dt, Y(t) = 2, N(t) = j \}, n \geq 0, j \geq 1$$

We define the Laplace Stieltjes Transform and the probability generating function as follows,

$$P^*_{i\ n}(\theta) = \int_0^\infty e^{-\theta x} P_{i\ n}(x)dx \ ;$$

$$Q^*_{j\ n}(\theta) = \int_0^\infty e^{-\theta x} Q_{j\ n}(x)dx \ ;$$

$$L^*_n(\theta) = \int_0^\infty e^{-\theta x} L_n(x)dx \ ;$$ and

$$R^*_n(\theta) = \int_0^\infty e^{-\theta x} R_n(x)dx$$

3.The Orbit Size Distribution for Multiple working vacation and Lock down:

$$-\frac{d}{dx}P_{i\,0}(x) = -\lambda P_{i\,o}(x) + \sum_{m=a}^{b} P_{mi}(0)s(x), \quad a \le i \le b \qquad (1)$$

$$-\frac{d}{dx}P_{i\,j}(x) = -\lambda P_{i\,j}(x) + \sum_{k=1}^{j} P_{i\,j-k}(x)\lambda s_k, \quad a \le i \le b-i, j \ge 1 \qquad (2)$$

$$-\frac{d}{dx}P_{b\,j}(x) = -\lambda P_{b\,j}(x) + \sum_{m=a}^{b} P_{m\,b+j}(0)s(x) + \sum_{k=1}^{\infty} P_{b\,j-k}(x)\lambda S_k \quad 1 \le j \le N-b-1 \qquad (3)$$

$$-\frac{d}{dx}P_{b\,j}(x) = -\lambda P_{b\,j}(x) + \sum_{m=a}^{b} P_{m\,b+j}(0)s(x) + \sum_{k=1}^{\infty} P_{b\,j-k}(x)\lambda S_k + R_{b+j}(0)s(x) \quad j \ge N-b \qquad (4)$$

$$-\frac{d}{dx}L_0(x) = -\lambda L_0(x) + \sum_{m=a}^{b} P_{m\,0}(x)L(x) \qquad (5)$$

$$-\frac{d}{dx}L_n(x) = -\lambda L_n(x) + \sum_{m=a}^{b} P_{m\,n}(0)L(x) + \sum_{k=1}^{\infty} L_{n-k}(x)\lambda S_k \quad 1 \le n \le a-1 \qquad (6)$$

$$-\frac{d}{dx}L_n(x) = -\lambda L_n(x) + \sum_{k=1}^{n} L_{n-k}(x)\lambda s_k \quad n \ge a \qquad (7)$$

$$-\frac{d}{dx}Q_{1\,0}(x) = -\lambda Q_{1\,o}(x) + L_0(0)V(x) \tag{8}$$

$$-\frac{d}{dx}Q_{1\,n}(x) = -\lambda Q_{1\,n}(x) + L_n(0)V(x) + \sum_{k=1}^{n} Q_{1\,n-k}(x)\lambda s_k, \quad n \geq 1 \tag{9}$$

$$-\frac{d}{dx}Q_{j\,0}(x) = -\lambda Q_{j\,o}(x) + Q_{j-1\,0}(0)V(x), \quad j \geq 2 \tag{10}$$

$$-\frac{d}{dx}Q_{j\,n}(x) = -\lambda Q_{j\,n}(x) + Q_{j-1\,n}(0)V(x) + \sum_{k=1}^{n} Q_{j\,n-k}(x)\lambda s_k$$

$$j \geq 2, 1 \leq n \leq N-1 \tag{11}$$

$$-\frac{d}{dx}Q_{j\,n}(x) = -\lambda Q_{j\,n}(x) + \sum_{k=1}^{n} Q_{j\,n-k}(x)\lambda s_k$$

$$j \geq 2, n \geq a \tag{12}$$

$$-\frac{d}{dx}R_n(x) = -\lambda R_n(x) + \sum_{1=1}^{\infty} Q_{1\,n}(0)r(x) + \sum_{k=1}^{n-N} R_{n-k}(x)\lambda s_{k,}$$

$$n \geq k \tag{13}$$

Now ,taking Laplace Stieltjes Transforms on both side of the equation(1) to (13) We get

$$\theta P_{i\,0}^{*}(\theta) - P_{i\,o}(0) = \lambda P_{i\,0}^{*}(\theta) - \sum_{m=a}^{b} P_{m\,i}(0)\,S^{*}(\theta)$$

$$a \leq i \leq b \tag{14}$$

$$\theta P_{i\,j}^{*}(\theta) - P_{i\,j}(0) = \lambda P_{i\,j}^{*}(\theta) - \lambda \sum_{k=1}^{j} P_{i\,j-k}^{*}(\theta) S_k \tag{15}$$

$$\theta P_{b\,j}^{*}(\theta) - P_{b\,j}(0) = \lambda P_{b\,j}^{*}(\theta) - \sum_{m=a}^{b} P_{m\,b+j}(0)\; S^{*}(\theta) - \lambda \sum_{K=1}^{j} P_{b\,j-k}^{*}(\theta) S_k$$

$$1 \leq j \leq N-b-1 \tag{16}$$

$$\theta P_{b\,j}^{*}(\theta) - P_{b\,j}(0) = \lambda P_{b\,j}^{*}(\theta) - \left[\sum_{m=a}^{b} P_{m\,b+j}(0) + R_{b+j}(0)\right] S^{*}(\theta) - \lambda \sum_{K=1}^{j} P_{b\,j+k}^{*}(\theta) S_k$$

$$j \geq N-b \tag{17}$$

$$\theta L_0^{*}(\theta) - L_0(0) = \lambda L_0^{*}(\theta) - \sum_{m=a}^{b} P_{m\,o}(0)\,L^{*}(\theta) \tag{18}$$

$$\theta L_n^{*}(\theta) - L_n(0) = \lambda L_n^{*}(\theta) - \sum_{m=a}^{b} P_{m\,n}(0)\,L^{*}(\theta) - \lambda \sum_{K=1}^{n} L_{n-k}^{*}(\theta)\,S_k$$

$$1 \leq n \leq a-1 \tag{19}$$

$$\theta L_n^*(\theta) - L_n(0) = \lambda L_n^*(\theta) - \lambda \sum_{K=1}^{n} L_{n-k}^*(\theta) S_k, \quad n \geq a \qquad (20)$$

$$\theta Q_{1\,0}^*(\theta) - Q_{1\,0}(0) = \lambda Q_{1\,0}^*(\theta) - L_0(0)V^*(\theta) \qquad (21)$$

$$\theta Q_{1\,n}^*(\theta) - Q_{1\,n}(0) = \lambda Q_{1\,n}^*(\theta) - L_n(0)V^*(\theta) - \lambda \sum_{k=1}^{n} Q_{1\,n-k}^*(\theta) s_k$$

$$n \geq 1 \qquad (22)$$

$$\theta Q_{j\,0}^*(\theta) - Q_{j\,o}(0) = \lambda Q_{j\,0}^*(\theta) - Q_{j-1\,0}(0)V^*(\theta)$$

$$j \geq 2 \qquad (23)$$

$$\theta Q_{j\,n}^*(\theta) - Q_{j\,n}(0) = \lambda Q_{j\,n}^*(\theta) - Q_{j-1\ n}(0)V^*(\theta) - \lambda \sum_{K=1}^{n} Q_{j\,n-k}^*(\theta) s_k$$

$$j \geq 2, 1 \leq n \leq N-1 \qquad (24)$$

$$\theta Q_{j\,n}^*(\theta) - Q_{j\,n}(0) = \lambda Q_{j\,n}^*(\theta) - \lambda \sum_{k=1}^{n} Q_{j\,n-k}^*(\theta) s_k$$

$$j \geq 2, n \geq N \qquad (25)$$

$$\theta R_n^*(\theta) - R_n(0) = \lambda R_n^*(\theta) - \sum_{l=1}^{\infty} Q_{1\ n}(0) R^*(\theta) - \lambda \sum_{k=1}^{n-N} R_{n-k}(\theta)\, s_k$$

$$n \geq N \qquad (26)$$

3.1 Queue size distribution

$$P_i^*(Z,\theta) = \sum_{n=0}^{\infty} P_{i\,n}^*(\theta) z^n \text{ and } P_i(z,0) = \sum_{n=0}^{\infty} P_{i\,n}(0) z^n; \quad a \leq i \leq b$$

$$Q_j^*(z,\theta) = \sum_{n=0}^{\infty} Q_{j\,n}^*(\theta) z^n \quad \text{and} \quad Q_j(z,0) = \sum_{n=0}^{\infty} Q_{j\,n}(0) z^n \quad j \geq 1$$

$$L^*(z,\theta) = \sum_{n=0}^{\infty} L_n^*(\theta) z^n \quad \text{and} \quad L(z,0) = \sum_{n=0}^{\infty} L_n(0) z^n$$

$$R^*(z,\theta) = \sum_{n=N}^{\infty} R_n^*(\theta) z^n \quad \text{and} \quad L(z,0) = \sum_{n=N}^{\infty} R_n(0) z^n \qquad (27)$$

Multiplying (21) by z^0 and equation (22) with z^n $(n \geq 1)$ summed over n from 0 to ∞ Using (27)

we get, $\big(\theta - \lambda + \lambda X(z)\big) Q_1^*(z,\theta) = Q_1(z,0) - L(z,0)V^*(\theta)$ (28)

Multiplying (23) by z^0 and equation (24) with z^n by $(1 \leq n \leq N-1)$ summed over

n from 0 to ∞ Using 27 we get,

$$\big(\theta-\lambda+\lambda X(z)\big)Q_j^*(z,\theta) = Q_j(z,0) - V^*(\theta)\sum_{n=0}^{N-1} Q_{j-1\,n}(0)z^n \qquad j \geq 2 \tag{29}$$

Multiplying (18) by z^0 and equation (19) by z^n $(1 \leq n \leq a-1)$ and (20) z^n $(n \geq a)$summed over j from 0 to ∞ Using 27 we get,

$$\big(\theta-\lambda+\lambda x(z)\big)L^*(z,\theta) = L(z,0) - L^*(\theta)\sum_{n=0}^{a-1}\sum_{m=a}^{b} P_{m\ n}(0)\, z^n \tag{30}$$

Multiplying (14) by z^0 and equation (15) with z^j $(j \geq 1)$ summed over j from 0 to ∞

Using 27 we get,

$$\big(\theta-\lambda+\lambda X(z)\big)P_i^*(z,\theta) = P_i(z,0) - S^*(\theta)\left[\sum_{m=a}^{b} P_{m\,i}(0)\right] \qquad a \leq i \leq b-1 \tag{31}$$

Multiplying (14) by z^0 and equation (16) with z^j $(1 \leq j \leq N-b-1)$ and (17) by

z^j $(j \geq N-b)$summed over j from 0 to ∞

Using 27 we get,

$$Z^b(\theta-\lambda)+\lambda X(z))P_b^*(z,\theta) = z^b P_b(z\,,0) - S^*(\theta)\left\{\left[\sum_{m=a}^{b}\left(P_m(z,0) - \sum_{j=0}^{b-1} P_{m\,j}(0)z^j\right)\right] - R(z\,,0)\right\} \tag{32}$$

Multiplying (26) by z^n, $(n \geq N)$ and summing up from n = N to ∞ Using 27 we get,

$$\big(\theta-\ \lambda\ +X(z)\big)\ R^*(z\,,\theta) = R(z\,,0) - R^*(\theta)\sum_{l=1}^{\infty}\left(Q_l(z\,,0) - \sum_{n=0}^{N-1} Q_{1\ n}(0)z^n\right) \tag{33}$$

By substituting $\theta = \lambda - \lambda X(z)$ in the equation 28 to 33 we get,

$$Q_1(Z\,,0) = V^*\big(\lambda-\lambda X(z)\big)L(Z\,,0) \tag{34}$$

$$Q_j(Z\,,0) = V^*\big(\lambda-\lambda X(z)\big)\sum_{n=0}^{N-1} Q_{j-1\,n}(0)\, z^n \qquad j \geq 2 \tag{35}$$

$$L(Z,0) = L^*(\lambda - \lambda X(z)) \sum_{n=0}^{a-1} \sum_{m=a}^{b} P_{m\ n}(0) z^n \tag{36}$$

$$R(z,0) = R^*(\lambda - \lambda X(z)) \sum_{l=1}^{\infty} \left(Q_1(z,0) - \sum_{n=0}^{N-1} Q_{1\ n}(0) z^n \right) \quad a \le i \le b-1 \tag{37}$$

$$P_i(z,0) = S^*(\lambda - \lambda X(z)) \sum_{m=a}^{b} P_{m\ i}(0) \tag{38}$$

And

$$Z^b P_b(Z,0) = S^*(\lambda - \lambda X(z)) \left\{ \left[\sum_{m=a}^{b-1} P_m(z,0) + P_b(z,0) - \sum_{j=0}^{b-1} \sum_{m=a}^{b} P_{m\,j}(0) Z^n \right] - R(Z,0) \right. \tag{39}$$

Now Solving for $P_b(Z,0)$ in 39 We have

$$\left(Z^b - S^*(\lambda - \lambda X(z))\right) P_b(Z,0) = S^*(\lambda - \lambda X(z)) \left\{ \left[\sum_{m=a}^{b-1} P_m(z,0) - \sum_{m=a}^{b} \sum_{j=0}^{b-1} P_{m\,j}(0) z^j \right] \right\} - R(Z,0) \tag{40}$$

From the equation 40 we get

$$P_b(Z,0) = \frac{S^*\left((\lambda - \lambda X(z)) f(z)\right)}{\left(Z^b - S^*\left((\lambda - \lambda X(z))\right)\right)} \tag{41}$$

Where

$$f(z) = \sum_{m=a}^{b-1} P_m(Z,0) - \sum_{j=0}^{b-1} \sum_{m=a}^{b} P_{m\,j}(0) Z^j + \sum_{1=1}^{\infty} \left(Q_1(Z,0) - \sum_{j=0}^{b-1} Q_{1,1}(0) Z^j \right)$$

Substituting the expression for $P_m(Z,0), (a \le m \le b-1)$from 32

$Q_1(Z,0)$ from 29 and $Q_j(Z,0), j \ge 2$ from 30 in f(Z) We get

$$f(Z) = S^*(\lambda - \lambda X(z)) \sum_{i=a}^{b-1} \sum_{m=a}^{b} P_{m\,i}(0) - \sum_{j=0}^{b-1} \sum_{m=a}^{b} P_{m\,j}(0)\, z^j + R^*(\lambda - \lambda X(Z)) \left\{ V^*(\lambda - \lambda X(Z)) \left[L^*(\lambda - \lambda X(z)) \sum_{n=0}^{a-1} \sum_{m=a}^{b} P_{m\,n}(0) Z^n + \sum_{n=0}^{a-1} \sum_{j=1}^{\infty} Q_{j\,n}(0) Z^n \right] - \sum_{j=1}^{\infty} \sum_{n=0}^{N-1} Q_{j\,n}(0)\, z^n \right\}$$

From the equation 28 and 34 we get,

$$Q_1^*(z\,,\theta) = \frac{[V^*(\lambda - \lambda X(Z)) - V^*(\theta)] L^*\left((\lambda - \lambda X(z))\right) \sum_{n=0}^{a-1} \sum_{m=a}^{b} P_{m\,n}(0) z^n}{(\theta - \lambda + \lambda X(z))} \qquad (42)$$

Similarly from the equations 29 and 35 we get,

$$Q_j^*(z\,,\theta) = \frac{\left[(V^*(\lambda - \lambda X(Z) -) V^*(\theta) \sum_{n=0}^{a-1} Q_{j-1\,n}(0)\, Z^n\right]}{(\theta - \lambda + \lambda X(z))}$$

$$j \geq 2 \qquad (43)$$

Similarly from the equations 30 and 36 we get

$$L^*(z\,,\theta) = \frac{\left[\left(L^*(\lambda - \lambda X(z)) - L^*(\theta)\right) \sum_{n=0}^{a-1} \sum_{m=a}^{b} P_{mn}\,(0) Z^n\right]}{(\theta - \lambda + \lambda X(z))} \qquad (44)$$

from the equations 33 and 37 we get

$$R^*(z\,,\theta) = \frac{[R^*(\lambda - \lambda X(Z)) - R^*(\theta)] \sum_{l=1}^{\infty} \left(Q_l(z\,,0) - \sum_{n=0}^{N-1} Q_{1\ n}\,(0) z^n\right)}{(\theta - \lambda + \lambda X(z))} \qquad (45)$$

from the equations 31 and 38 we get

$$P_i^*(z\,,\theta) = \frac{\left(S^*(\lambda - \lambda X(z))\right) - S^*(\theta) \sum_{m=a}^{b} P_{m\,i}(0)}{(\theta - \lambda + \lambda X(z))}$$

$$a \leq i \leq b - 1 \qquad (46)$$

From the equation 32 and 41

We get $$P_b^*(z,\theta) = \frac{[S^*(\lambda-\lambda X(z))-S^{*(\theta)}]f(z)}{(\theta-\lambda-\lambda X(z))\left(Z^b-S^*(\lambda-\lambda X(z))\right)} \qquad (47)$$

Let P(z)be the probability generating function of the queue size at an arbitrary time epoch, Then

$$P(z) = \sum_{m=a}^{b-1} P_m^*(z,0) + P_b^*(z,0) + +L^*(z,0) + R(z,0) \qquad (48)$$

Using the equation 42 to 47 in P(z) with $\theta = 0$

We get

$$P(z) = \frac{[S^*(\lambda-\lambda X(z))-1]\sum_{i=a}^{b-1}(\sum_{m=a}^{b} P_{m\ i}(0))}{(-\lambda+\lambda X(z))} + \frac{[S^*(\lambda-\lambda X(z))-1]f(z)}{(-\lambda+\lambda X(z))\left(z^b-S^*(-\lambda+\lambda X(z))\right)}$$
$$+\frac{[L^*((\lambda-\lambda X(z))-1]\sum_{n=0}^{a-1}\sum_{m=a}^{b} P_{mn}(0)z^n}{(-\lambda+\lambda X(z))}$$
$$+\frac{(V^*(\lambda-\lambda X(z)-1)L^*(\lambda-\lambda X(z)\sum_{n=0}^{a-1}\sum_{m=a}^{b} P_{m\ n}(0)z^n}{(-\lambda+\lambda X(z))}$$
$$+\frac{(V^*(\lambda-\lambda X(z)-1)\sum_{n=0}^{a-1}\sum_{j=1}^{\infty} Q_{j\ n}(0)z^n}{(-\lambda+\lambda X(z))} \qquad (49)$$

Let

$P_i = \sum_{m=a}^{b} P_{m\ i}(0)$, $q_i = \sum_{l=1}^{\infty} Q_{1\ i}(0)$, and $L_i = P_i + q_i$ (50)

Using 50 the equation 49 is simplified as

$$P(Z) = \frac{\left\{\begin{matrix} S^*(\lambda-\lambda X(z))\sum_{i=a}^{b-1}(z^b-z^i)P_i + (z^b-1)[R^*(\lambda-\lambda X(z))L^*(\lambda-\lambda X(z))V^*(\lambda-\lambda X(z))-1] \\ \sum_{i=0}^{a-1} P_i z^i + (Z^b-1)(V^*(\lambda-\lambda X(z))-1)R^*(\lambda-\lambda X(z))\sum_{i=0}^{N-1} q_n z^n \end{matrix}\right\}}{(-\lambda+\lambda X(z))\left(z^b-S^*(\lambda-\lambda X(z))\right)} \qquad (51)$$

Remark

Eq (51) has N+1 unknowns $p_0, p_1, \ldots p_{b-1}, q_0, q_1, q_2, \ldots q_{N-1}$. The following two theorem are prove to express q_i in term of p_i in such a way that the numerator has only b constants. The equation (51) gives the partial generating function of the number of customers involving only "b" unknowns, zeros inside and one on the unit circle |z|=1. Since P(z) is analytic within and on the unit circle, the numerator must vanish at these points, which gives b equations with b un Known .

Remark

The Probability generating function has to satisfy P(1) = 1 Applying L'Hospital's rulem in (51)

$$E(S)\sum_{i=a}^{b-1}(b-i)P_i + b\left(E(R)+E(C)+E(V)\right)\sum_{i=0}^{a-1} P_i + bE(V) + \sum_{i=0}^{N-1} q_i = \left(b-\lambda E(X)E(S)\right)$$

Since p_i and q_i are probabilities, it follows that left hand side of the above expression must be positive. Thus $P(1) = 1$ if and only if $b - \lambda E(X)E(S) > 0$. If $\rho = \lambda E(X)E(S)/b$,then$\rho < 1$ is the condition to be satisfied for the existence of steady state for the model under consideration.

Theorem 1:

$q_n = \sum_{i=0}^{n} K_i P_{n-1}$, $\quad n = 0,1,2,3,\ldots\ldots..a - 1$

Where

$K_n = \frac{h_n + \sum_{i=1}^{n} \alpha_i K_{n-1}}{1 - \alpha_0}$ $\quad n = 1,2,3,\ldots\ldots..a - 1$with $K_0 = \frac{\alpha_0\beta_0}{1-\alpha_0}$,$h_n = \sum_{i=0}^{n} \alpha_0 \beta_{n-1}$

Also α_i's and β_i's are probabilities of the 'i' customers arrive during Vacation time and closedown time respectively.

Proof :

Using 33 and 35 $\sum_{j=1}^{\infty} Q_j(z,0)$ simplifies to

$$\sum_{n=0}^{\infty} q_n z^n = V^*\big(\lambda - \lambda X(z)\big)\left[L^*\big(\lambda - \lambda X(z)\big)\sum_{n=0}^{a-1} P_n z^n + \sum_{n=0}^{N-1} q_n z^n\right]$$

$$= \left(\sum_{n=0}^{\infty} \alpha_n z^n\right)\left[\sum_{j=0}^{\infty} \beta_j z^j \sum_{n=0}^{a-1} P_n z^n + \sum_{n=0}^{N-1} q_n z^n\right]$$

$$= \sum_{n=0}^{a-1}\left(\sum_{i=0}^{n} \alpha_{n-i}\,(h_i + q_i)\right) z^n + \sum_{n=a}^{N-1}\left(\sum_{i=a}^{n} \alpha_{n-i}\, q_i\right) z^n + \sum_{n=N}^{\infty}\left(\sum_{n=a}^{N-1} \alpha_{n-i}\, q_i\right) z^n$$

With $h_n = \sum_{i=0}^{n} \alpha_0 \beta_{n-i}$, n= 0,1,2,…..a-1 (52)

Equating the coefficients of z^n on both sides of the above equation for n=0,1,2,…a-1,we have

$$q_n = \sum_{j=0}^{n}\left(\sum_{i=0}^{n-j} \alpha_i \beta_{n-i-j}\right) P_j + \sum_{i=0}^{n} \alpha_{n-i}\, q_i$$

on solving for q_n , we gt

$q_n = \frac{\left[\sum_{j=0}^{n}\left(\sum_{1=0}^{n-j} \alpha_i \beta_{n-i-j}\right)P_j + \sum_{i=0}^{n-1} \alpha_{n-i} q_i\right]}{(1-\alpha_0)}$ co-efficient of P_n in q_n is $K_0 = \frac{\alpha_0\beta_0}{1-\alpha_0}$, coefficient of P_n in q_{n-1} is $(h_i + \alpha_i$ coefficient of P_{n-1} in $q_n) / (1 - \alpha_0) = \frac{(h_i + \alpha_i k_0)}{(1-\alpha_0)} = K_1$

Hence the theorem

Theorem 2: If we let $\pi_n = \sum_{i=0}^{a-1}\left(\sum_{j=0}^{(a-1)^{-i}} \alpha_{n-i-j}\big(\beta_j + K_j\big)\right) P_i$

Then

$$q_n = \frac{\pi_n + \sum_{i=1}^{n-a} \alpha_i\, q_{n-i}}{(1-\alpha_0)}$$

when $n = a+1, a+2, \ldots, N-1$ (53)

Special case:

As a special case, it may noted that when lockdown times is zero then PGF obtained in (51) reduces to the following form

$$P(Z) = \frac{\left\{\begin{matrix} S^*(\lambda - \lambda X(z)) \sum_{i=a}^{b-1} (z^b - z^i) P_i + (z^b - 1)[R^*(\lambda - \lambda X(z)) - 1] \\ \sum_{i=0}^{a-1} P_i z^i + (Z^b - 1)(V^*(\lambda - \lambda X(z)) - 1) R^*(\lambda - \lambda X(z)) \sum_{i=0}^{N-1} q_n z^n \end{matrix}\right\}}{(-\lambda + \lambda X(z))\left(z^b - S^*(\lambda - \lambda X(z))\right)}$$

We have $q_n = \sum_{i=0}^{a-1} \alpha_{n-i}(h_i + q_i) + \sum_{i=a}^{n} \alpha_{n-i} q_i$

Where $h_n = \sum_{i=0}^{n} \alpha_i \beta_{n-1}$

$$\sum_{i=0}^{a-1} \alpha_{n-i} h_i + \sum_{i=0}^{a-1} \left[\sum_{j=0}^{a-1-i} K_i \, \alpha_{n-i-j} \right] h_i + \sum_{i=a}^{n} \alpha_{n-i} q_i$$

$= \pi_n + \sum_{i=a}^{n} \alpha_{n-i} q_i$ where $\pi_n = \sum_{i=0}^{a-1} \sum_{j=0}^{n-a} \alpha_{n-i-j} (\beta_j + K_j) P_i$

On solving for q_n, we get,

$$q_n = \frac{\pi_n}{1 - \alpha_0} \text{ and } q_n = \frac{\pi_n + \sum_{i=1}^{n-a} \alpha_i \, q_{n-i}}{1 - \alpha_0} \quad n = a+1, a+2, \dots N-1$$

Hence the theorem

Expected Length of Busy Period:

Let B be the busy Period random variable,

We define the another random variable M as

$$M = \begin{cases} 0 & \text{if the server finds} < 'a' \text{ customers after the first service} \\ 1 & \text{if the server finds } 'a' \text{ customers after the first service} \end{cases}$$

Now, expected length of busy period $E(B)$ is given by

$$E(B) = E\left(B/_M = 0\right) P(M = 0) + E\left(B/_M = 1\right) P(M = 1)$$

$$= E(S)P(M = 0) + [E(S) + E(B)]P(M = 1),$$

Where $E(S)$ is the expected service time.

Solving for

$$E(B) = \frac{E(S)}{\sum_{i=0}^{a-1} P_i} \quad (54)$$

Expected Length of Idle Period:

If I be idle period the random variable, then the expected length of idle period is given by,

$E(I) = E(L) + E(I_1) + E(R)$ where I_1 is the Idle period due to multiple vacation process', $E(L)$ is the expected lockdown time and $E(R)$ is the expected setup time.

Define the random variable U as

U = 0 , if the server finds at least N customers after first vacation

= 1 , if the server finds less than N customers after first vacation

Now $E(I_1) = E\left(\frac{I_1}{U} = 0\right) P(U = 0) + E\left(\frac{I_1}{U} = 1\right) P(U = 1)$

$= E(V)P(U = 0) + \big(E(V) + E(I_1)\big)P(U = 1)$

Solving for $E(I_1)$,We get,

$$E(I_1) = \frac{E(V)}{1 - P(U = 1)}$$

From (34) we get,

$$P(U = 1) = \sum_{n=0}^{a-1}\left(\sum_{i=0}^{n}\left(\sum_{j=0}^{n-1} \alpha_j\, \beta_{n-i-j}\right) P_i\right) + \sum_{n=a}^{N-1}\left(\sum_{i=0}^{a-1}\left(\sum_{j=0}^{n-i} \alpha_j\, \beta_{n-i-j}\right) P_i\right)$$

The (53) becomes

$$E(I_1) = \frac{E(V)}{1 - \left[\sum_{n=0}^{a-1}\left(\sum_{i=0}^{n}\left(\sum_{j=0}^{n-1} \alpha\beta_j\right)\right)\right]}$$

Expected Queue Length:

The expected queue length $E(Q)$ at arbitrary time epoch is obtained by differentiating $P(z)$ at z=1 and is given by

$$E(Q) = \frac{f_1 \sum_{i=a}^{b-1}\big(b(b-1)-i(i-1)\big)P_i + f_2 \sum_{i=a}^{b-1}(b-i)p_i + f_3 \sum_{i=0}^{a-1} p_i + f_4\left[\sum_{i=0}^{a-1} p_i + \sum_{i=0}^{N-1} q_i\right] + f_5 \sum_{i=0}^{a-1} p_i + f_6\left[\sum_{i=0}^{a-1} p_i + \sum_{i=0}^{N-1} q_i\right]}{2[\lambda E(X)(b-S\ 1)]^2}$$

where (56)

$f_1 = \lambda E(X)(b - S_1)S_1,$

$f_1 = \lambda E(X)(b - S2)S_2 - TS_1,$

$f_3 = \lambda E(X)(b - S_1)\Big[b(b - 1)\big(E(L) + E(R)\big) + b(R_2 + L_2) + 2b\big[E(L)\big(E(R) + E(V) - Tb - (L_1 + R_1)\big)\big]\Big]$

$f_4 = \lambda E(X)(b - S_1)[2bE(V)E(R) + b(b - 1)E(V) + bV_2] - TbE(V)$

$f_5 = \lambda E(X)(b - S_1)\big[b\big(E(L) + E(R)\big)\big]$

$f_6 = \lambda E(X)(b - S_1)bE(V)$ and

$S_1 = \lambda E(X)E(S)$, $V_1 = \lambda E(X)E(S)$, $R_1 = \lambda E(X)E(R)$, $L_1 = \lambda E(X)E(L)$,

$S_2 = 4\lambda E(S) + \lambda^2 E(X)^2 E(S^2)$, $V_2 = 4\lambda E(V) + \lambda^2 E(X)^2 E(V^2)$,

$R_2 = 4\lambda E(S) + \lambda^2 E(X)^2 E(R^2)$, $L_2 = 4\lambda E(L) + \lambda^2 E(X)^2 E(L^2)$

$X_2 = X''(1)$, $T = \lambda E(X)(b(b-1) - S_2) + \lambda X_2(b - S_1)$

Cost model

Proposed queueing model has a bulk service rule with a single server, The customers have to wait if sufficient batch quantity is not available, in such a case the server will be either on vacation of on lockdown work /setup period. By considering this situation of customers waiting and the effective utilization of the server and hence the Computer Numerical Control Machine, it is essential to have an optimal threshold value for a batch quantity. A cost is identified by which the total costs involved in the system can be minimized.

An Expression for finding the total average cost with the following assumptions is derived. Let C_s be the start up cost , C_h be the holding cost per customer per unit time, C_o be the operating cost per unit time, C_r be the reward per unit time due to vacation. C_u be the lockdown cost per unit time and C_v be the setup cost per unit time, The length of cycle is the idle period and busy period. Now, the expected length of cycle, $E(T_c)$ is obtained as

$$E(T_c) = E(I) + E(B) = E(L) + \frac{E(V)}{P(U=0)} + E(R) + \frac{E(S)}{\sum_{n=0}^{a-1} P_n}$$

Therefore , the total average cost per unit time is given by

Total Average Cost = **S**tart-up Cost per unit time + **H**olding Cost of number of Customer in the queue per unit time + **O**perating Cost per unit time ρ + **L**ock down cost per unit time cost + **S**etup cost per unit time – **R**eward due to vacation per unit time.

$$\text{Total Average Cost} = \left[C_s - C_r.\frac{E(V)}{P(U=0)} + C_u.E(L) + C_v E(R)\right].\frac{1}{E(T_c)} + C_h.E(Q) + L_0.\rho \qquad \textbf{(57)}$$

The Significance of the cost model is discussed belo

Numerical Example

A numerical example is presented in this section how to the management of pipe manufacturing industry can use the above results to take decision regarding the threshold vale of a batch that minimizes the total average cost with the following assumptions.

In the pipe manufacturing industry, the arrival of shafts in bulk from turning center to Computer Numerical Control copy turning center follows Poisson process with arrival rate λ.The operator takes sequence of vacation whenever finds that the number of available shafts is less than the minimum threshold value after lock down the machine. The operator utilizes this time for doing some other work .When the operator returns from other work and if the number of shafts available is greater than the threshold value 'N' requires a setup time to start the machine. In order to utilize the Computer Numerical Control machine as well as the operator efficiently, the management wishes to know the optimal threshold value (minimum batch quantity) that minimize the total average cost.

The above system can be modeled as M/G/1 queueing system. Parameters chosen ar3e being practical in nature, the following assumptions are made.

(i) Service time distribution is k – Erlang distribution with $k = 2, \ \mu = 7$

(ii) Batch arrival distribution is geometric

(iii) Vacation time, Lock down time setup time are exponential with parameters $\lambda = 10, \ \beta = 6 \ , \ \gamma = 7$

(iv) N = 14

Startup cost per unit time	Rs. 4.00
Holding cost per customer per unit time	Rs. 0.50
Operating cost per unit time	Rs. 5.00
Reward cost per unit time due to multiple vacations	Rs. 1.00
Lock down time cost per unit time	Rs. 0.25
Setup time cost per unit time	Rs. 0.50

The numerical results for various threshold values and performance measures with b = 10 fare presented the threshold value is vice versa total average cost is also shown From the table (a) and Figure (b) , one can observe that, f or a copy turning centre with the maximum batch size of 10 shafts at a time, the management has to fix the threshold value as 3 to minimize the total average cost. are presented in given table.

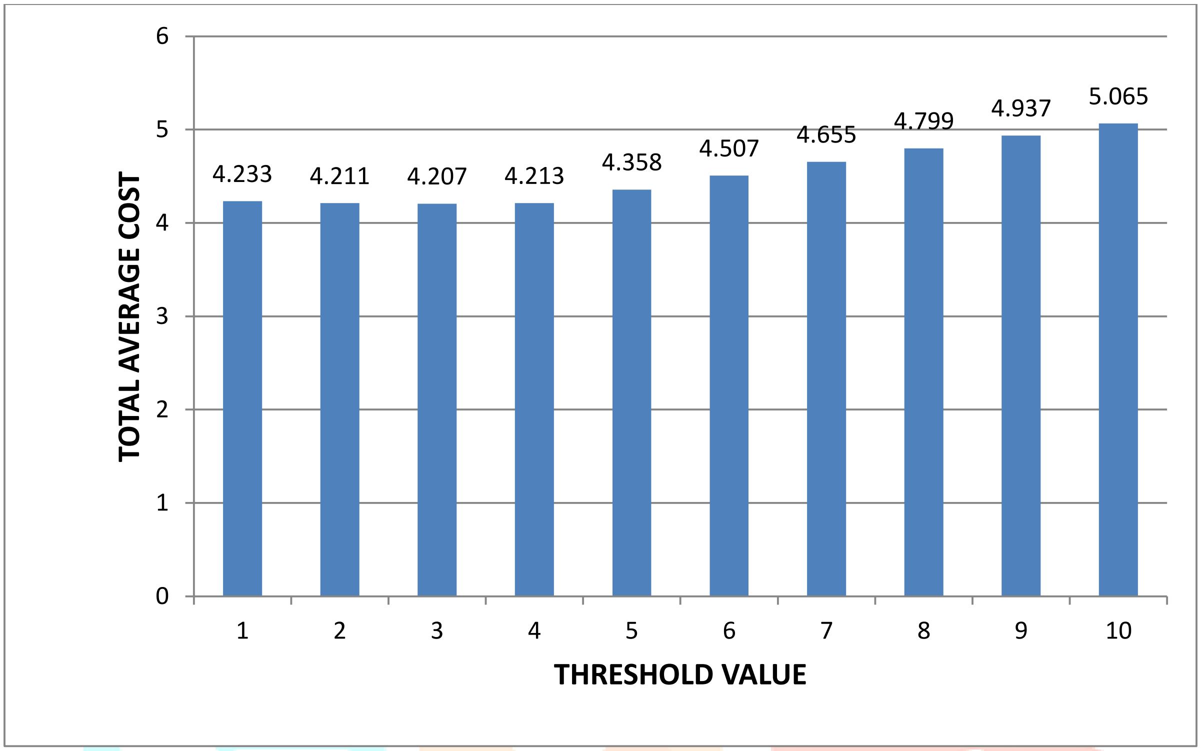
Table (a)

Threshold value and performance measures

a - Thereshold value, E(Q) – Expected queue length, E(B)- Expected length of busy period, E(I) - Expected length of Idle period,

a	Unknown probabilities								Total average cost
	P_1	P_3	P_5	P_7	P_9	E(Q)	E(B)	E(I)	
1	0.2536	0.0554	0.0647	0.0421	0.0242	2.4175	1.1266	0.4434	4.233
2	0.195	0.0447	0.0625	0.042	0.0246	2.4966	1.1704	0.4418	4.211
3	**0.1666**	**0.0302**	**0.0618**	**0.0424**	**0.025**	**2.5492**	**1.1921**	**0.441**	**4.207**
4	0.1505	0.028	0.0614	0.0428	0.0255	2.5881	1.2027	0.4406	4.213
5	0.1215	0.0242	0.0568	0.0458	0.0275	2.6698	1.1278	0.4433	4.358
6	0.0954	0.0209	0.0584	0.0492	0.0301	2.7597	1.0587	0.4462	4.507
7	0.0718	0.0182	0.0607	0.0446	0.0329	2.8549	0.9963	0.4493	4.655
8	0.0504	0.016	0.0635	0.0472	0.0357	2.9523	0.9405	0.4525	4.799
9	0.0313	0.0141	0.0647	0.051	0.0308	3.0483	0.8911	0.4557	4.937
10	0.0144	0.0125	0.07	0.0529	0.0327	3.1396	0.8478	0.4589	5.065

Figure (b)



Conclusion

An M/G/1 queue with multiple working vacations with Lock down times and set up times with N – Policy has been studied. The Partial Generating Function of queue size arbitrary time epoch is obtained. Some performance measures are also derived. The cost model proposed is studied numerically , An example is given to demonstrate the cost model is useful for management of manufacturing industry to take decision. A special case of the model is also presented.

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