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QUADRIPARTITIONED NEUTROSOPHIC CHAOTIC SET

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Abstract: The main objective of this paper is to propose a new type of set which is named as quadripartitioned neutrosophic chaotic set. We also intend to study and prove some of its basic properties. Using the new set we further introduce and develop the concept of “quadripartitioned neutrosophic chaotic topological spaces”. As the result of devising the new set and its respective topological space we have considerably discussed some of its properties supported by suitable examples.

***Index Terms* – neutrosophic chaotic set, quadripartitioned neutrosophic chaotic set, quadripartitioned neutrosophic chaotic topological spaces.**

I. INTRODUCTION

Fuzzy sets which allows the elements to have a degrees of membership in the set and it was introduced by Zadeh [30] in 1965. The degree of membership lies in the real unit interval [0, 1]. Intuitionstic fuzzy set (IFS) allows both membership and non membership to the elements andwas introduced by Atnassov [2] in 1983. By introducing one more components in intuitionstic fuzzy set Smarandache [26] introduced neutrosophic set in 1998. Neutrosophic set has three components truth membership function, indeterminacy membership function and falsity membership function respectively. This neutrosophic set helps to handle the indeterminate and inconsistent information effectively. Later Wang [29] (2010) introduced the concept of Single valued Neutrosophic set (SVNS) which is a generalization of classic set, fuzzy set, interval valued fuzzy set and intuitionstic fuzzy set. Neutrosophic set helps to solve many real life world problems [3-7] because of its uncertainty analysis in data sets. When indeterminacy component in neutrosophic set is divided into two parts namely 'Contradiction' (both true and false) 'Unknown' (neither true nor false) we get four components that is T,C,U,F which define a new set called 'Quadripartitioned Single valued neutrosophic set' (QSVNS) introduced by Rajashi Chatterjee., et al. [23] And this is completely based on Belnap's four valued logic and Smarandache's 'Four Numerical valued neutrosophic logic'. In the context of neutrosophic study however, the QSVNS looks quite logical. Also, in their study, Chatterjee[23] et al. has analyzed a real-life example for a better understanding of the QSVNS environment and showed that such situations occur very naturally. Suman Das, Rakhal Das, and Carlos Granados[26] introduced the concept of Topology on Quadripartitioned Neutrosophic Sets. The idea of neutrosophic topological space (NTS) was presented by Salama and Alblowi [25] in the year 2012. The neutrosophic semi-open mappings are studied by Arokiarani et. al. [1]. Following which, Iswaraya and Bageerathi [12] studied the concept of neutrosophic semi-open sets and neutrosophic semi-closed sets. The notion of neutrosophic b-open sets in NTSs was presented by Ebenanjar et al. [11]. Rao and Srinivasa [24] grounded the concept of pre open set and pre closed set via neutrosophic topological spaces. T. Madhumathi and F.Nirmala Irudayam [16] introduced the idea of neutrosophic pre-open sets in simple extended neutrosophic topology.

The concept of chaotic function in general metric space was introduced by R.L.Devaney[9]. It has many applications in many fields such traffic forecasting, animation, computer graphics, medical field, image processing, etc. T.Thrivikraman and P.B. Vinod Kumar[28] defined chaos and fractals in general topological spaces. The concept of the fuzzy chaotic set was introduced by R.Malathi and M.K. Uma[14] in 2018. In[15,17] we introduced the concept of neutrosophic orbit topological spaces and also neutrosophic chaotic continuous functions.

Motivated by the above said concepts in this article we introduce the idea of quadripartitioned in neutrosophic chaotic sets. The organization of the articles is as follows: Section 2 is dedicated to recalling some preliminary results; Section 3 introduces the concept of a quadripartitioned neutrosophic chaotic set. Section 4 deals with some basic set-theoretic operations over quadripartitioned neutrosophic chaotic sets. Section 5 extend the concept of quadripartitioned neutrosophic chaotic topological spaces. Section 6 concludes the paper stating future scope of research.

2. Preliminaries

2.1 Definition [26]
Let X be a universe. A neutrosophic set A on X can be defined as follows:

$$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$$

Where T_A , $I_A, F_A: U \to [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$
Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of inderminancy and $F_A(x)$ is the degree of non-membership.

2.2 Definition [23]
Let X be a universe. A quadripartitioned neutrosophic set A with independent neutrosophic components on X is an object of the form

$$A = \{< x, T_A(x), C_A(x), U_A(x), F_A(x) >: x \in X\}$$

and $0 \le T_A(x) + C_A(x) + U_A(x) + F_A(x) \le 4$
Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership and $F_A(x)$ is the false membership.

2.3 Definition [28]
Orbit of a point x in X under the mapping f is $O_f(x)=\{x, f(x), f^2(x),...\}$

2.4 Definition [28]
x in X is called a periodic point of f if $f^n(x) = x$, for some n ∈ Z+. Smallest of these n is called period of x.

2.5 Definition [28] f is sensitive if for each $\delta > 0$ there exist (a) $\varepsilon > 0$ (b) $y \in X$ and (c) $n \in Z_+$ such that $d(x,y) < \delta$ and $d(f^n(x), f^n(y)) > \varepsilon$.

2.6 Definition [28] f is chaotic on (X,d) if (i) Periodic points of f are dense in X (ii) Orbit of x is dense in X for some x in X and (iii) f is sensitive.

2.7 Definition [28] Let (X, τ) be a topological space and f : (X, τ) → (X, τ) be continuous map. Then f is sensitive at x ∈ X if given any open set U containing x there exists (i) y ∈ U (ii) $n \in Z^+$ and (iii) an open set V such that $f^n(x) \in V$, $f^n(y) \notin cl(V)$. We say that f is sensitive on a F if f|F is sensitive at every point of F.

2.8 Definition [28] Let (X, τ) be atopological space and F∈K(X). Let f : F → F be a continuous. Then f is chaotic on F if

(i) $cl(O_f(x)) = F$ for some x ∈ F.
(ii) periodic points of f are dense in F.
(iii) f ∈ S(F).

2.9 Notation [28] (i) C(F) = {f : F → F | f is chaotic on F} amd (ii) CH(X) = {F ∈NK(X) | C(F) ≠ φ}.

2.10 Definition [28] A topological space (X, τ) is called a chaos space if CH(X) ≠φ. The members of CH(X) are called chaotic sets.

2.11 Definition [17] Let X be a nonempty set and f : X → X be any mapping. Let α be any neutrosophic set in X. The neutrosophic orbit $O_f(\alpha)$ of α under the mapping f is defined as $O_{fT}(\alpha) = \{ \alpha, f^1(\alpha), f^2(\alpha),...f^n(\alpha)\}$, $O_{fI}(\alpha) = \{ \alpha, f^1(\alpha), f^2(\alpha),...f^n(\alpha)\}$, $O_{fF}(\alpha) = \{ \alpha, f^1(\alpha), f^2(\alpha),...f^n(\alpha)\}$ for $\alpha \in X$ and $n \in Z^+$.

2.12 Definition [17] Let X be a nonempty set and let f : X→ X be any mapping. The neutrosophic orbit set of α under the mapping f is defined as $NO_f(\alpha) = < \alpha, O_{fT}(\alpha), O_{fI}(\alpha), O_{fF}(\alpha)>$ for $\alpha \in X$, where $O_{fT}(\alpha) = \{ \alpha \wedge f^1(\alpha) \wedge f^2(\alpha) \wedge ... \wedge f^n(\alpha)\}$, $O_{fI}(\alpha) = \{ \alpha \vee f^1(\alpha) \vee f^2(\alpha) \vee ... \vee f^n(\alpha)\}$, $O_{fF}(\alpha) = \{ \alpha \vee f^1(\alpha) \vee f^2(\alpha) \vee ... \vee f^n(\alpha)\}$.

2.13 Definition [15] Let X be a nonempty set and let f : X → X be any mapping. Then a neutrosophic set of X is called neutrosophic periodic set with respect to f if $f_n(\Upsilon) = \Upsilon$, for some $n \in Z^+$. smallest of these n is called neutrosophic periodic of X.

2.14 Definition [15] Let (X, τ) be a neutrosophic topological space and $\lambda \in NF(X)$ (Where NF(X) is a collection of all nonempty neutrosophic compact subsets of X). Let f : X → X be any mapping. Then f is **neutrosophic chaotic** with respect to λ if

(i) cl $NO_f(\lambda) = 1$,
(ii) P is neutrosophic dense.

Example Let X = {a, b, c}. Define τ = {0, 1, μ_1, μ_2, μ_3, μ_4} where μ_1, μ_2, μ_3, μ_4 : X → [0,1] are defined as μ_1 (a) =<a,0.4,0.3,0.6>, μ_1 (b) =<b,0.8,0.7,0.2>, μ_1 (c) = <c,0.4,0.3,0.6>, μ_2 (a) = <a,0.4,0.3,0.6>, μ_2(b) =<b, 0.8,0.2,0.2>, μ_2 (c) = <c,0.5,0.2,0.5> μ_3 (a)=<a,0.8,0.2,0.2>, μ_3 (b)=<b,0.8,0.7,0.2>, μ_3 (c)=<c,0.6,0.2,0.4>, μ_4 (a)=<a,0.9,0.2,0.1>, μ_4 (b)=<a,0.8,0.7,0.2>, μ_4 (c)=<a,0.9,0.2,0.1>,
Let λ: X → I be defined as λ (a) =<a,0.3,0.2,0.7> (b) = <b,0.6,0.5,0.4> λ(c) =<c, 0.3,0.2,0.7>. Define f : X → X as f(a) = b, f(b) = c, f(c) = a. The neutrosophic orbit set of λ under the mapping f is defined as
NO_f (λ) = λ ∩f(λ) ∩ f2(λ) ∩…
NO_f (λ)(a) = <a,0.3,0.2,0.7>, NO_f (λ)(b) = <b,0.6,0.5,0.4>, NO_f (λ)(c) =<c, 0.3,0.2,0.7>. Therefore cl(NO_f (λ)) = 1. Here P(a) = <a,0.4,0.3,0.6>, P(b) =<b, 0.8,0.7,0.2> ,P(c) = <c,0.4,0.3,0.6> and cl (P) is neutrosophic dense. Hence f is neutrosophic chaotic with respect to λ.
2.15 Notation (i) NC (λ) = {f: X → X / f is neutrosophic chaotic with respect to λ}.
(ii) NCH(λ) = {λ∈ NF(X) / NC(λ) ≠ ϕ}.
2.16 Definition [15] A neutrosophic topological space (X, τ) is called a **neutrosophic chaos space** if NCH (λ) ≠ ϕ. If (X, τ) is neutrosophic chaos space then the element of the NCH(X) are called chaotic sets in X.

3. Quadripartitioned neutrosophic chaotic set

3.1 Definition

Let X be a non-empty set and $f: X \to X$ be any mapping. A quadripartitioned neutrosophic chaotic set A_{QNC} over X characterizes each element x in X by a truth membership function $\mathrm{T}_{A_{QNC}}$, contradiction membership function $\mathrm{C}_{A_{QNC}}$, ignorance membership function $\mathrm{U}_{A_{QNC}}$ and false membership function $\mathrm{F}_{A_{QNC}}$.

$$x \in X, \mathrm{T}_{A_{QNC}}, \mathrm{C}_{A_{QNC}}, \mathrm{U}_{A_{QNC}}, \mathrm{F}_{A_{QNC}} \in [0,1] \text{and } A_{QNC} = \left\{\left\langle x, \mathrm{T}_{A_{QNC}}, \mathrm{C}_{A_{QNC}}, \mathrm{U}_{A_{QNC}}, \mathrm{F}_{A_{QNC}} \right\rangle; x \in \mathrm{X}\right\}$$

$$0 \le \mathrm{T}_{A_{QNC}} + \mathrm{C}_{A_{QNC}} + \mathrm{U}_{A_{QNC}} + \mathrm{F}_{A_{QNC}} \le 4$$

3.2 Definition

Let A_{QNC} and B_{QNC} be two quadripartitioned neutrosophic chaotic sets of the universe X. If $\forall x_i \in X$, $\mathrm{T}_{A_{QNC}}(x_i) \le \mathrm{T}_{B_{QNC}}(x_i)$; $\mathrm{C}_{A_{QNC}(x_i)} \le \mathrm{C}_{B_{QNC}}(x_i), \mathrm{U}_{A_{QNC}}(x_i) \ge \mathrm{U}_{B_{QNC}}(x_i); \mathrm{F}_{A_{QNC}}(x_i) \ge \mathrm{F}_{\boldsymbol{B}_{QNC}}(x_i)$, then the quadripartitioned neutrosophic chaotic set A quadripartitioned neutrosophic chaotic included by B_{QNC}whore $1 \le i \le n$.

3.3 Definition

The complement of quadripartitioned neutrosophic chaotic set $A_{QNC}$is denotod by A^c_{QNC} and is defined by.

A^C=<x, $\mathrm{F}_{A_{QNC}}(x), \mathrm{U}_{A_{QNC}}(x), \mathrm{C}_{A_{QNC}}(x), \mathrm{T}_{A_{QNC}}(x)$>

i.e $\mathrm{T}_{A_{QNC}}(x) = \mathrm{F}_{A_{QNC}}(x)$,$\mathrm{C}_{A_{QNC}}(x) = \mathrm{U}_{A_{QNC}}(x), \mathrm{U}_{A_{QNC}}(x) = \mathrm{C}_{A_{QNC}}(x)$ and $\mathrm{F}_{A_{QNC}}(x) = \mathrm{T}_{A_{QNC}}(x)$

3.4 Definition

Let A_{QNC} be quadripartitioned neutrosophic chaotic set of the universe X where $\forall x_i \in X, \mathrm{T}_{A_{QNC}}(x) = 0; \mathrm{C}_{A_{QNC}}(x) = 0; \mathrm{U}_{A_{QNC}}(x) = 1; \mathrm{F}_{A_{QNC}}(x) = 1$.

Then, $A_{QNC}$is called null quadripartitioned neutrosophic chaotic set (0_{QNC} in Short). where $1 \leq i \leq n$.

3.5 Definition

Let A_{QNC} be quadripartitioned neutrosophic chaotic set of the universe X where $\forall\, x_i \in X, \mathrm{T}_{A_{QNC}}(x) = 1; \mathrm{C}_{A_{QNC}}(x) = 1; \mathrm{U}_{A_{QNC}}(x) = 0; \mathrm{F}_{A_{QNC}}(x) = 0$.

Then, A_{QNC} is called an absolute quadripartitioned neutrosophic chaotic set (1_{QNC}in short) whesene $1 \leq i \leq n$.

3.6 Definition

The union of two quadripartitioned neutrosophic chaotic sets A_{QNC} and B_{QNC} is denoted by $A_{QNC} \cup B_{QNC}$ is defined as

$$A_{QNC} \cup B_{QNC} =< \mathrm{x}, \max(\mathrm{T}_{A_{QNC}}(x), \mathrm{T}_{B_{QNC}}(x)), \max(\mathrm{C}_{A_{QNC}}(x), \mathrm{C}_{B_{QNC}}(x)), \min(\mathrm{U}_{A_{QNC}}(x), \mathrm{U}_{B_{QNC}}(x)),$$

$$\min(\mathrm{F}_{A_{QNC}}(x), \mathrm{F}_{B_{QNC}}(x)) >$$

3.7 Definition

The intersection of two quadripartitioned neutrosophic chaotic sets A_{QNC} and B_{QNC} is denoted by $A_{QNC} \cap B_{QNC}$ is defined as

$$A_{QNC} \cap B_{QNC} =< x, \min(\mathrm{T}_{A_{QNC}}(x), \mathrm{T}_{B_{QNC}}(x)), \min(\mathrm{C}_{A_{QNC}}(x), \mathrm{C}_{B_{QNC}}(x)), \max(\mathrm{U}_{A_{QNC}}(x), \mathrm{U}_{B_{QNC}}(x)),$$

$$\max(\mathrm{F}_{A_{QNC}}(x), \mathrm{F}_{B_{QNC}}(x)) >$$

3.8 Example

Consider two quadripartitioned neutrosophic chaotic sets over X, gives as

$$A_{QNC} = \langle x, 0.1,0.6,0.3,0.5, \rangle$$

$$B_{QNC} = \langle x, 0.2,0.5,0.6,0.8\rangle$$

$$A_{QNC} \cup B_{QNC} = \langle x, 0.2,0.6,0.3,0.5\rangle$$

$$A_{QNC} \cap B_{QNC} = \langle x, 0.1,0.5,0.6,0.8\rangle$$

4. Basic Properties

4.1 Theorem

Quadripartitioned neutrosophic chaotic set satisfy the following properties under the above mentioned set theoretic operations:

(i) Commutative law

$$a)\ M_{QNC} \cup N_{QNC} = N_{QNC} \cup M_{QNC}$$
$$\text{b)}\ M_{QNC} \cap N_{QNC} = N_{QNC} \cap M_{QNC}$$

(ii) Associate law

$$\text{c)}\ M_{QNC} \cup (N_{QNC} \cup O_{QNC}) = (M_{QNC} \cup N_{QNC}) \cup O_{QNC}$$
$$\text{e)}\ M_{QNC} \cap (N_{QNC} \cap O_{QNC}) = (M_{QNC} \cap N_{QNC}) \cap O_{QNC}$$

(iii) Distributive law

$$\text{e)}\ (M_{QNC} \cup N_{QNC}) \cap (M_{QNC} \cup O_{QNC}) = M_{QNC} \cup (N_{QNC} \cap O_{QNC})$$
$$\text{f)}\ (M_{QNC} \cap N_{QNC}) \cup (M_{QNC} \cap O_{QNC}) = M_{QNC} \cap (N_{QNC} \cup O_{QNC})$$

(iv) Absorption law

$$\text{g)}\ M_{QNC} = M_{QNC} \cup (M_{QNC} \cap O_{QNC})$$
$$h) M_{QNC} = M_{QNC} \cap (M_{QNC} \cup O_{QNC})$$

(v) Involution law

$$\text{i)}\ M_{QNC} = ({M^C}_{QNC})^C$$

(vi) Law of contradiction

$$j)\ M_{QNC} \cap {M_{QNC}}^{\mathrm{C}} = \theta$$

(vii) De Morgan's law

$$k)\ {M_{QNC}}^{\mathrm{C}} \cup {N_{QNC}}^{\mathrm{C}} = (M_{QNC} \cap N_{QNC})^C$$

$$l)\ {M_{QNC}}^{\mathrm{C}} \cap {N_{QNC}}^{\mathrm{C}} = (M_{QNC} \cup N_{QNC})^C$$

Proof

a) Let $M_{QNC} =< x, \mathrm{T}_{M_{QNC}}(x), \mathrm{C}_{M_{QNC}}(x), \mathrm{U}_{M_{QNC}}(x), \mathrm{F}_{M_{QNC}}(x) : x \in X >$ and

$$N_{QNC} =< x, \mathrm{T}_{N_{QNC}}(x), \mathrm{C}_{N_{QNC}}(x), \mathrm{U}_{N_{QNC}}(x), \mathrm{F}_{N_{QNC}}(x) : x \in X >$$

$M_{QNC} \cup N_{QNC} = N_{QNC} \cup M_{QNC}$

We know that,

$M_{QNC} \cup N_{QNC} =< x, \max(\mathrm{T}_{M_{QNC}}(x), \mathrm{T}_{N_{QNC}}(x)), \max(\mathrm{C}_{M_{QNC}}(x), \mathrm{C}_{N_{QNC}}(x)), \min(\mathrm{U}_{M_{QNC}}(x), \mathrm{U}_{N_{QNC}}(x)),$

$$\min(\mathrm{F}_{M_{QNC}}(x), \mathrm{F}_{N_{QNC}}(x))/x \in X. >$$

Throughout this paper, we denote $\mathrm{T}_{M_{QNC}}(x), \mathrm{C}_{M_{QNC}}(x), \mathrm{U}_{M_{QNC}}(x)$ and $\mathrm{F}_{M_{QNC}}(x)$ by $\mathrm{T}_{M_{QNC}}, \mathrm{C}_{M_{QNC}}, \mathrm{U}_{M_{QNC}}$ and $\mathrm{F}_{M_{QNC}}$

$$M_{QNC} \cup N_{QNC} =< x, \max(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{N_{QNC}}), \max(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{N_{QNC}}), \min(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{N_{QNC}}),$$

$$\min(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{N_{QNC}})/x \in X >$$

Let $x_i \in M_{QNC} \cup N_{QNC}$

$$\Rightarrow x_i \in< x, \max(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{N_{QNC}}), \max(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{N_{QNC}}), \min(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{N_{QNC}}),$$

$$\min(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{N_{QNC}})/x \in X >$$

$$\Rightarrow x_i \in < x, \max(\mathrm{T}_{N_{QNC}}, \mathrm{T}_{M_{QNC}}), \max(\mathrm{C}_{N_{QNC}}, \mathrm{C}_{M_{QNC}}), \min(\mathrm{U}_{N_{QNC}}, \mathrm{U}_{M_{QNC}}),$$

$$\min(\mathrm{F}_{N_{QNC}}, \mathrm{F}_{M_{QNC}})/x \in X >$$

$\Rightarrow x_i \in N_{QNC} \cup M_{QNC}$

$\Rightarrow M_{QNC} \cup N_{QNC} \subseteq N_{QNC} \cup M_{QNC}$ ------------- (1)

Let $y_i \in N_{QNC} \cup M_{QNC}$

$$\Rightarrow y_i \in< x, \max(\mathrm{T}_{N_{QNC}}, \mathrm{T}_{M_{QNC}}), \max(\mathrm{C}_{N_{QNC}}, \mathrm{C}_{M_{QNC}}), \min(\mathrm{U}_{N_{QNC}}, \mathrm{U}_{M_{QNC}}),$$

$$\min(\mathrm{F}_{N_{QNC}}, \mathrm{F}_{M_{QNC}})/x \in X >$$

$$\Rightarrow y_i \in< x, \max(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{N_{QNC}}), \max(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{N_{QNC}}), \min(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{N_{QNC}}),$$

$$\min(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{N_{QNC}})/x \in X >$$

$\Rightarrow y_i \in M_{QNC} \cup N_{QNC}$

$\Rightarrow N_{QNC} \cup M_{QCV} \subseteq M_{QNC} \cup N_{QNC}$ ------------- (2)

Therefore, from (1) and (2) we obtain

$$M_{QNC} \cup N_{QNC} = N_{QNC} \cup M_{QNC}$$

b) Similarly, we can prove $M_{QNC} \cap N_{QNC} = N_{QNC} \cap M_{QNC}$

c) Let $x_i \in M_{QNC} \cup (N_{QNC} \cup O_{QNC})$

$$\Longrightarrow x_i \in M_{QNC} \cup < x, \max(\mathrm{T}_{N_{QNC}}, \mathrm{T}_{O_{QNC}}), \max(\mathrm{C}_{N_{QNC}}, \mathrm{C}_{O_{QNC}}), \min(\mathrm{U}_{N_{QNC}}, \mathrm{U}_{O_{QNC}}),$$

$$\min(\mathrm{F}_{N_{QNC}}, \mathrm{F}_{O_{QNC}})/x \in X >$$

$$\Longrightarrow x_i \in< x, \max(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{N_{QNC}}, \mathrm{T}_{O_{QNC}}), \max(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{N_{QNC}}, \mathrm{C}_{O_{QNC}}), \min(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{N_{QNC}}, \mathrm{U}_{O_{QNC}}),$$

$$\min(\mathrm{F}_{M_{QNC}},\mathrm{F}_{N_{QNC}},\mathrm{F}_{O_{QNC}})/x \in X >$$

$$\Rightarrow x_i \in < x, \max(\mathrm{T}_{M_{QNC}},\mathrm{T}_{N_{QNC}}), \max(\mathrm{C}_{M_{QNC}},\mathrm{C}_{N_{QNC}}), \min(\mathrm{U}_{M_{QNC}},\mathrm{U}_{N_{QNC}}),$$

$$\min(\mathrm{F}_{M_{QNC}},\mathrm{F}_{N_{QNC}})/x \in X > \cup\, O_{QNC}$$

$\Rightarrow M_{QNC} \cup (N_{QNC} \cup O_{QNC}) \subseteq (M_{QNC} \cup N_{QNC}) \cup O_{QNC}$------------ (3)

Let $y_i \in (M_{QNC} \cup N_{QNC}) \cup O_{QNC}$

$$\Rightarrow y_i \in < x, \max(\mathrm{T}_{M_{QNC}},\mathrm{T}_{N_{QNC}}), \max(\mathrm{C}_{M_{QNC}},\mathrm{C}_{N_{QNC}}), \min(\mathrm{U}_{M_{QNC}},\mathrm{U}_{N_{QNC}}),$$

$$\min(\mathrm{F}_{M_{QNC}},\mathrm{F}_{N_{QNC}})/x \in X > \cup\, O_{QNC}$$

$$\Rightarrow y_i \in < x, \max(\mathrm{T}_{M_{QNC}},\mathrm{T}_{N_{QNC}},\mathrm{T}_{O_{QNC}}), \max(\mathrm{C}_{M_{QNC}},\mathrm{C}_{N_{QNC}},\mathrm{C}_{O_{QNC}}), \min(\mathrm{U}_{M_{QNC}},\mathrm{U}_{N_{QNC}},\mathrm{U}_{O_{QNC}}),$$

$$\min(\mathrm{F}_{M_{QNC}},\mathrm{F}_{N_{QNC}},\mathrm{F}_{O_{QNC}})/x \in X >$$

$$\Rightarrow y_i \in M_{QNV} \cup < x, \max(\mathrm{T}_{N_{QNC}},\mathrm{T}_{O_{QNC}}), \max(\mathrm{C}_{N_{QNC}},\mathrm{C}_{O_{QNC}}), \min(\mathrm{U}_{N_{QNC}},\mathrm{U}_{O_{QNC}}),$$

$$\min(\mathrm{F}_{N_{QNC}},\mathrm{F}_{O_{QNC}})/x \in X >$$

$\Rightarrow y_i \in M_{QNC} \cup (N_{QNC} \cup O_{QNC})$

$\Rightarrow (M_{QNC} \cup N_{QNC}) \cup O_{QNC} \subseteq M_{QNC} \cup (N_{QNC} \cup O_{QNC})$ ------------ (4)

Therefore, from (3) and (4) we obtain

$$M_{QNC} \cup (N_{QNC} \cup O_{QNC}) = (M_{QNC} \cup N_{QNC}) \cup O_{QNC}$$

d) Similarly, we can prove $M_{QNC} \cap (N_{QNC} \cap O_{QNC}) = (M_{QNC} \cap N_{QNC}) \cap O_{QNC}$

e) Let $x_i \in M_{QNC} \cup (N_{QNC} \cap O_{QNC})$

$$\Rightarrow x_i \in M_{QNV} \cup < x, \min(\mathrm{T}_{N_{QNC}},\mathrm{T}_{O_{QNC}}), \min(\mathrm{C}_{N_{QNC}},\mathrm{C}_{O_{QNC}}), \max(\mathrm{U}_{N_{QNC}},\mathrm{U}_{O_{QNC}}),$$

$$\max(\mathrm{F}_{N_{QNC}},\mathrm{F}_{O_{QNC}})/x \in X >$$

$$\Rightarrow x_i \in < x, \max\{\mathrm{T}_{M_{QNC}}, \min(\mathrm{T}_{N_{QNC}},\mathrm{T}_{O_{QNC}})\}, \max\{\mathrm{C}_{M_{QNC}}, \min(\mathrm{C}_{N_{QNC}},\mathrm{C}_{O_{QNC}})\},$$

$$\min\{\mathrm{U}_{M_{QNC}}, \max(\mathrm{U}_{N_{QNC}},\mathrm{U}_{O_{QNC}})\}, \min\{\mathrm{F}_{M_{QNC}}, \max(\mathrm{F}_{N_{QNC}},\mathrm{F}_{O_{QNC}})\}, : x \in X >$$

$$\Rightarrow x_i \in < x, \max(\mathrm{T}_{M_{QNC}},\mathrm{T}_{N_{QNC}}), \max(\mathrm{C}_{M_{QNC}},\mathrm{C}_{N_{QNC}}), \min(\mathrm{U}_{M_{QNC}},\mathrm{U}_{N_{QNC}}),$$

$$\min(\mathrm{F}_{M_{QNC}},\mathrm{F}_{N_{QNC}})/x \in X > \cap < x, \max(\mathrm{T}_{M_{QNC}},\mathrm{T}_{O_{QNC}}), \max(\mathrm{C}_{M_{QNC}},\mathrm{C}_{O_{QNC}}), \min(\mathrm{U}_{M_{QNC}},\mathrm{U}_{O_{QNC}}),$$

$$\min(\mathrm{F}_{M_{QNC}},\mathrm{F}_{O_{QNC}})/x \in X >$$

$\Longrightarrow x_i \in (M_{QNC} \cup N_{QNC}) \cap (M_{QNC} \cup O_{QNC})$

$M_{QNC} \cup (N_{QNC} \cap O_{QNC}) \subseteq (M_{QNC} \cup N_{QNC}) \cap (M_{QNC} \cup O_{QNC})$---------- (5)

Assume that $y_i \in (M_{QNC} \cup N_{QNC}) \cap (M_{QNC} \cup O_{QNC})$

$$\Longrightarrow y_i \in < x, \max(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{N_{QNC}}), \max(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{N_{QNC}}), \min(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{N_{QNC}}),$$

$$\min(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{N_{QNC}})/x \in X > \cap < x, \max(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{O_{QNC}}), \max(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{O_{QNC}}), \min(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{O_{QNC}}),$$

$$\min(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{O_{QNC}})/x \in X >$$

$$\Longrightarrow y_i \in < x, \max\{\mathrm{T}_{M_{QNC}}, \min(\mathrm{T}_{N_{QNC}}, \mathrm{T}_{O_{QNC}})\}, \max\{\mathrm{C}_{M_{QNC}}, \min(\mathrm{C}_{N_{QNC}}, \mathrm{C}_{O_{QNC}})\},$$

$$\min\{\mathrm{U}_{M_{QNC}}, \max(\mathrm{U}_{N_{QNC}}, \mathrm{U}_{O_{QNC}})\}, \min\{\mathrm{F}_{M_{QNC}}, \max(\mathrm{F}_{N_{QNC}}, \mathrm{F}_{O_{QNC}})\}, : x \in X >$$

$$\Longrightarrow y_i \in M_{QNC} \cup < x, \min(\mathrm{T}_{N_{QNC}}, \mathrm{T}_{O_{QNC}}), \min(\mathrm{C}_{N_{QNC}}, \mathrm{C}_{O_{QNC}}), \max(\mathrm{U}_{N_{QNC}}, \mathrm{U}_{O_{QNC}}),$$

$$\max(\mathrm{F}_{N_{QNC}}, \mathrm{F}_{O_{QNC}})/x \in X >$$

$\Longrightarrow y_i \in M_{QNC} \cup (N_{QNC} \cap O_{QNC})$

$(M_{QNC} \cup N_{QNC}) \cap (M_{QNC} \cup O_{QNC}) \subseteq M_{QNC} \cup (N_{QNC} \cap O_{QNC})$---------- (6)

From (5) and (6) we conclude that

$(M_{QNC} \cup N_{QNC}) \cap (M_{QNC} \cup O_{QNC}) = M_{QNC} \cup (N_{QNC} \cap O_{QNC})$

f) Similarly, we can prove

$$(M_{QNC} \cap N_{QNC}) \cup (M_{QNC} \cap O_{QNC}) = M_{QNC} \cap (N_{QNC} \cup O_{QNC})$$

g) Let $x_i \in M_{QNC} \cup (M_{QNC} \cap O_{QNC})$

$$\Longrightarrow x_i \in M_{QNC} \cup < x, \min(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{O_{QNC}}), \min(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{O_{QNC}}), \max(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{O_{QNC}}),$$

$$\max(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{O_{QNC}})/x \in X >$$

$$\Longrightarrow x_i \in < x, max\{\mathrm{T}_{M_{QNC}}, \min(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{O_{QNC}})\}, \max\{\mathrm{C}_{M_{QNC}}, \min(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{O_{QNC}})\},$$

$$\min\{\mathrm{U}_{M_{QNC}}, \max(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{O_{QNC}})\}, \min\{\mathrm{F}_{M_{QNC}}, \max(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{O_{QNC}})/x \in X >$$

$$\Longrightarrow x_i \in < x, \mathrm{T}_{M_{QNC}}(x), \mathrm{C}_{M_{QNC}}(x), \mathrm{U}_{M_{QNC}}(x), \mathrm{F}_{M_{QNC}}(x) >: x \in X >$$

$\Longrightarrow x_i \in M_{QNC}$

$\Longrightarrow M_{QNC} \cup (M_{QNC} \cap O_{QNC}) \subseteq M_{QNC}$ --------------------- (7)

Let $y_i \in M_{QNC}$

$$\Rightarrow y_i \in < x, \mathrm{T}_{M_{QNC}}(x), \mathrm{C}_{M_{QNC}}(x), \mathrm{U}_{M_{QNC}}(x), \mathrm{F}_{M_{QNC}}(x) >: x \in X >$$

$$< x, max\{\mathrm{T}_{M_{QNC}}, \min(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{O_{QNC}})\}, \max\{\mathrm{C}_{M_{QNC}}, \min(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{O_{QNC}})\},$$

$$\min\{\mathrm{U}_{M_{QNC}}, \max(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{O_{QNC}})\}, \min\{\mathrm{F}_{M_{QNC}}, \max(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{O_{QNC}})/x \in X >$$

$$\Rightarrow y_i \in M_{QNC} \cup < x, max\{\mathrm{T}_{M_{QNC}}, \min(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{O_{QNC}})\}, \max\{\mathrm{C}_{M_{QNC}}, \min(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{O_{QNC}})\},$$

$$\min\{\mathrm{U}_{M_{QNC}}, \max(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{O_{QNC}})\}, \min\{\mathrm{F}_{M_{QNC}}, \max(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{O_{QNC}})/x \in X >$$

$\Rightarrow y_i \in M_{QNC} \cup (M_{QNC} \cap O_{QNC})$

$\Rightarrow M_{QNC} \subseteq M_{QNC} \cup (M_{QNC} \cap O_{QNC})$ -------------- (8)

From (7) and (8) we conclude that

$$M_{QNC} = M_{QNC} \cup (M_{QNC} \cap O_{QNC})$$

h) Similarly, we prove that

$$M_{QNC} = M_{QNC} \cap (M_{QNC} \cup O_{QNC})$$

$M_{QNC} =< x, \mathrm{T}_{M_{QNC}}(x), \mathrm{C}_{M_{QNC}}(x), \mathrm{U}_{M_{QNC}}(x), \mathrm{F}_{M_{QNC}}(x): x \in X >$

${M^C}_{QNC} =< x, \mathrm{F}_{M_{QNC}}(x), \mathrm{U}_{M_{QNC}}(x), \mathrm{C}_{M_{QNC}}(x), \mathrm{T}_{M_{QNC}}(x): x \in X >$

Let $x_i \in ({M^C}_{QNC})^C$

$$\Rightarrow x_i \in (< x, \mathrm{F}_{M_{QNC}}(x), \mathrm{U}_{M_{QNC}}(x), \mathrm{C}_{M_{QNC}}(x), \mathrm{T}_{M_{QNC}}(x): x \in X >)^C$$

$\Rightarrow x_i \in < x, \mathrm{T}_{M_{QNC}}(x), \mathrm{C}_{M_{QNC}}(x), \mathrm{U}_{M_{QNC}}(x), \mathrm{F}_{M_{QNC}}(x): x \in X >$

$\Rightarrow x_i \in M_{QNC}$

$$\Rightarrow ({M^C}_{QNC})^C \subseteq M_{QNC} \text{ -------------- (9)}$$

Assume that $y_i \in M_{QNC}$

$$\Rightarrow y_i \in < x, \mathrm{T}_{M_{QNC}}(x), \mathrm{C}_{M_{QNC}}(x), \mathrm{U}_{M_{QNC}}(x), \mathrm{F}_{M_{QNC}}(x): x \in X >$$

$$\Rightarrow y_i \in (< x, \mathrm{F}_{M_{QNC}}(x), \mathrm{U}_{M_{QNC}}(x), \mathrm{C}_{M_{QNC}}(x), \mathrm{T}_{M_{QNC}}(x): x \in X >)^C$$

$$\Rightarrow M_{QNC} \subseteq ({M^C}_{QNC})^C \text{ -------------- (10)}$$

From (9) and (10), we get

$$M_{QNC} = ({M^C}_{QNC})^C$$

i) Let $x_i \in M_{QNC} \cap {M_{QNC}}^{\mathrm{C}}$

$$\Rightarrow x_i \in < x, \mathrm{T}_{M_{QNC}}(x), \mathrm{C}_{M_{QNC}}(x), \mathrm{U}_{M_{QNC}}(x), \mathrm{F}_{M_{QNC}}(x) : x \in X >$$

$$\cap < x, \mathrm{F}_{M_{QNC}}(x), \mathrm{U}_{M_{QNC}}(x), \mathrm{C}_{M_{QNC}}(x), \mathrm{T}_{M_{QNC}}(x) : x \in X >$$

$$\Rightarrow x_i \in < x, \min(\mathrm{T}_{M_{QNC}}(x), \mathrm{F}_{M_{QNC}}(x)), \min(\mathrm{C}_{M_{QNC}}(x), \mathrm{U}_{M_{QNC}}(x)), \max(\mathrm{U}_{M_{QNC}}(x), \mathrm{C}_{M_{QNC}}(x)),$$

$$\max(\mathrm{F}_{M_{QNC}}(x), \mathrm{T}_{M_{QNC}}(x)) : x \in X >$$

$$\Rightarrow x_i \in \theta$$

$$\Rightarrow M_{QNC} \cap {M_{QNC}}^{\mathrm{C}} \subseteq \theta \text{-------------------- (11)}$$

Assume that $x_i \in \theta$

$$\Rightarrow y_i \in < x, \min(\mathrm{T}_{M_{QNC}}(x), \mathrm{F}_{M_{QNC}}(x)), \min(\mathrm{C}_{M_{QNC}}(x), \mathrm{U}_{M_{QNC}}(x)), \max(\mathrm{U}_{M_{QNC}}(x), \mathrm{C}_{M_{QNC}}(x)),$$

$$\max(\mathrm{F}_{M_{QNC}}(x), \mathrm{T}_{M_{QNC}}(x)) : x \in X >$$

$$\Rightarrow y_i \in < x, \mathrm{T}_{M_{QNC}}(x), \mathrm{C}_{M_{QNC}}(x), \mathrm{U}_{M_{QNC}}(x), \mathrm{F}_{M_{QNC}}(x) : x \in X >$$

$$\cap < x, \mathrm{F}_{M_{QNC}}(x), \mathrm{U}_{M_{QNC}}(x), \mathrm{C}_{M_{QNC}}(x), \mathrm{T}_{M_{QNC}}(x) : x \in X >$$

$$\Rightarrow y_i \in M_{QNC} \cap {M_{QNC}}^{\mathrm{C}}$$

$$\Rightarrow \theta \subseteq M_{QNC} \cap {M_{QNC}}^{\mathrm{C}} \text{ --------------- (12)}$$

From (11) and (12), we get

$$M_{QNC} \cap {M_{QNC}}^{\mathrm{C}} = \theta$$

j) Let $x_i \in (M_{QNC} \cup N_{QNC})^C$

$$\Rightarrow x_i \in$$

$$(< x, \max(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{N_{QNC}}), \max(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{N_{QNC}}), \min(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{N_{QNC}}),$$

$$\min(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{N_{QNC}}) / x \in X >)^C$$

$$\Rightarrow x_i \in$$

$$< x, \min(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{N_{QNC}}), \min(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{N_{QNC}}), \max(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{N_{QNC}}), \max(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{N_{QNC}})$$

$$: x \in X >$$

$$\Rightarrow x_i \in$$

$$< x, \mathrm{F}_{M_{QNC}}, \mathrm{U}_{M_{QNC}}, \mathrm{C}_{M_{QNC}}, \mathrm{T}_{M_{QNC}} : x \in X > \cap < x, \mathrm{F}_{N_{QNC}}, \mathrm{U}_{N_{QNC}}, \mathrm{C}_{N_{QNC}}, \mathrm{T}_{N_{QNC}} : x \in X >$$

$\Rightarrow x_i \in (< x, \mathrm{T}_{M_{QNC}}, \mathrm{C}_{M_{QNC}}, \mathrm{U}_{M_{QNC}}, \mathrm{F}_{M_{QNC}} : x \in X >)^C \cap (< x, \mathrm{T}_{N_{QNC}}, \mathrm{C}_{N_{QNC}}, \mathrm{U}_{N_{QNC}}, \mathrm{F}_{N_{QNC}} : x \in X >)^C$

$\Rightarrow x_i \in \ {M_{QNC}}^{\mathrm{C}} \cap {N_{QNC}}^{\mathrm{C}}$

$\Rightarrow (M_{QNC} \cup N_{QNC})^C \subseteq {M_{QNC}}^{\mathrm{C}} \cap {N_{QNC}}^{\mathrm{C}}$ ------------- (13)

Again, Assume that $y_i \in \ {M_{QNC}}^{\mathrm{C}} \cap {N_{QNC}}^{\mathrm{C}}$

$\Rightarrow y_i \in (< x, \mathrm{T}_{M_{QNC}}, \mathrm{C}_{M_{QNC}}, \mathrm{U}_{M_{QNC}}, \mathrm{F}_{M_{QNC}} : x \in X >)^C \cap (< x, \mathrm{T}_{N_{QNC}}, \mathrm{C}_{N_{QNC}}, \mathrm{U}_{N_{QNC}}, \mathrm{F}_{N_{QNC}} : x \in X >)^C$

$\Rightarrow y_i \in < x, \mathrm{F}_{M_{QNC}}, \mathrm{U}_{M_{QNC}}, \mathrm{C}_{M_{QNC}}, \mathrm{F}_{M_{QNC}} : x \in X > \cap < x, \mathrm{F}_{N_{QNC}}, \mathrm{U}_{N_{QNC}}, \mathrm{C}_{N_{QNC}}, \mathrm{F}_{N_{QNC}} : x \in X >$

$$\Rightarrow y_i \in < x, \min(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{N_{QNC}}), \min(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{N_{QNC}}), \max(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{N_{QNC}}), \max(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{N_{QNC}}) : x \in X >$$

$$\Rightarrow y_i \in (< x, \max(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{N_{QNC}}), \max(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{N_{QNC}}), \min(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{N_{QNC}}), \min(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{N_{QNC}}) / x \in X >)^C$$

$\Rightarrow y_i \in (M_{QNC} \cup N_{QNC})^C$

$\Rightarrow {M_{QNC}}^{\mathrm{C}} \cap {N_{QNC}}^{\mathrm{C}} \subseteq (M_{QNC} \cup N_{QNC})^C$ --------------- (14)

From (13) and (14), we conclude that

${M_{QNC}}^{\mathrm{C}} \cap {N_{QNC}}^{\mathrm{C}} = (M_{QNC} \cup N_{QNC})^C$

k) Let $x_i \in (M_{QNC} \cap N_{QNC})^C$

$$\Rightarrow x_i \in (< x, \min(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{N_{QNC}}), \min(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{N_{QNC}}), \max(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{N_{QNC}}), \max(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{N_{QNC}}) : x \in X >)^C$$

$$\Rightarrow x_i \in < x, \max(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{N_{QNC}}), \max(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{N_{QNC}}), \min(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{N_{QNC}}), \min(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{N_{QNC}}) : x \in X >$$

$$\Rightarrow x_i \in$$

$$< x, \mathrm{F}_{M_{QNC}}, \mathrm{U}_{M_{QNC}}, \mathrm{C}_{M_{QNC}}, \mathrm{F}_{M_{QNC}} : x \in X > \cup < x, \mathrm{F}_{N_{QNC}}, \mathrm{U}_{N_{QNC}}, \mathrm{C}_{N_{QNC}}, \mathrm{F}_{N_{QNC}} : x \in X >$$

$$\Rightarrow x_i \in (< x, \mathrm{T}_{M_{QNC}}, \mathrm{C}_{M_{QNC}}, \mathrm{U}_{M_{QNC}}, \mathrm{F}_{M_{QNC}} : x \in X >)^C \cup (< x, \mathrm{T}_{N_{QNC}}, \mathrm{C}_{N_{QNC}}, \mathrm{U}_{N_{QNC}}, \mathrm{F}_{N_{QNC}} : x \in X >)^C$$

$$\Rightarrow x_i \in {M_{QNC}}^{\mathrm{C}} \cup {N_{QNC}}^{\mathrm{C}}$$

$$\Rightarrow (M_{QNC} \cap N_{QNC})^C \subseteq {M_{QNC}}^{\mathrm{C}} \cup {N_{QNC}}^{\mathrm{C}} \text{ ------------- } (15)$$

Again, Assume that $y_i \in {M_{QNC}}^{\mathrm{C}} \cup {N_{QNC}}^{\mathrm{C}}$

$$\Rightarrow y_i \in (< x, \mathrm{T}_{M_{QNC}}, \mathrm{C}_{M_{QNC}}, \mathrm{U}_{M_{QNC}}, \mathrm{F}_{M_{QNC}} : x \in X >)^C \cup (< x, \mathrm{T}_{N_{QNC}}, \mathrm{C}_{N_{QNC}}, \mathrm{U}_{N_{QNC}}, \mathrm{F}_{N_{QNC}} : x \in X >)^C$$

$$\Rightarrow y_i \in$$

$$< x, \mathrm{F}_{M_{QNC}}, \mathrm{U}_{M_{QNC}}, \mathrm{C}_{M_{QNC}}, \mathrm{F}_{M_{QNC}} : x \in X > \cup < x, \mathrm{F}_{N_{QNC}}, \mathrm{U}_{N_{QNC}}, \mathrm{C}_{N_{QNC}}, \mathrm{F}_{N_{QNC}} : x \in X >$$

$$\Rightarrow y_i \in$$

$$< x, \max(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{N_{QNC}}), \max(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{N_{QNC}}), \min(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{N_{QNC}}), \min(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{N_{QNC}})$$

$$: x \in X >$$

$$\Rightarrow y_i \in (< x, \min(\mathrm{T}_{M_{QNC}}, \mathrm{T}_{N_{QNC}}), \min(\mathrm{C}_{M_{QNC}}, \mathrm{C}_{N_{QNC}}), \max(\mathrm{U}_{M_{QNC}}, \mathrm{U}_{N_{QNC}}),$$

$$\max(\mathrm{F}_{M_{QNC}}, \mathrm{F}_{N_{QNC}}) : x \in X >)^C$$

$$\Rightarrow y_i \in (M_{QNC} \cap N_{QNC})^C$$

$$\Rightarrow {M_{QNC}}^{\mathrm{C}} \cup {N_{QNC}}^{\mathrm{C}} \subseteq (M_{QNC} \cap N_{QNC})^C \text{ ---------------} (16)$$

From (15) and (16), we conclude that

$${M_{QNC}}^{\mathrm{C}} \cap {N_{QNC}}^{\mathrm{C}} = (M_{QNC} \cup N_{QNC})^C$$

4. Quadripartitioned Neutrosophic Chaotic topological Spaces

4.1 Definition

A quadripartitioned neutrosophic chaotic topological space on a non-empty set X is a τ_{QNC} of quadripartitioned neutrosophic chaotic sets satisfying the following axioms.

i) $0_{QNC}, 1_{QNC} \in \tau_{QNC}$

ii) The union of the elements of any sub collection of τ_{QNC} is in τ_{QNC}

iii) The intersection of the elements of any finite sub collection τ_{QNC} is in τ_{QNC}

The pair (X, τ_{QNC}) is called a quadripartitioned neutrosophic chaotic topological space over X**.**

4.2 Note

1. Every member of τ_{QNC} is called a quadripartitioned neutrosophic chaotic open set in X.
2. The set A is called a quadripartitioned neutrosophic chaotic closed set in X if $A \in {\tau_{QNC}}^c$, where ${\tau_{QNC}}^c = \{A^c : A \in \tau_{QNC}\}$.

4.3 Example

Let X = $\{b_1, b_2, b_3\}$ and Let A, B, C be quadripartitioned neutrosophic chaotic sets where
A = $\{< b_1, 0.5,0.1,0.9,0.7,0.2 >< b_2, 0.7,0.5,0.9,0.2,0.1 > < b_3, 0.8,0.2,0.8,0.4,0.8 >\}$
B = $\{< b_1, 0.9,0.7,0.6,0.3,0.2 >< b_2, 0.2,0.3,0.6,0.4,0.7 > < b_3, 0.5,0.6,0.7,0.1,0.3 >\}$
C = $\{< b_1, 0.9,0.7,0.6,0.3,0.2 >< b_2, 0.7,0.5,0.6,0.2,0.1 > < b_3, 0.8,0.6,0.7,0.1,0.3 >\}$
$\tau_{QNC} = \{A, B, C, 0_{QNC}, 1_{QNC}\}$ is a quadripartitioned neutrosophic chaotic topological space on X.

4.4 Definition Let us consider a quadripartitioned neutrosophic chaotic subset X of a quadripartitioned neutrosophic chaotic topological space,(X, τ_{QNC}). Then,the quadripartitioned neutrosophic chaotic closure (cl_{QNC}) of X is the intersection of all quadripartitioned neutrosophic chaotic closed sets containing X and the quadripartitioned neutrosophic chaotic interior (int_{QNC}) of X is the union of all quadripartitioned neutrosophic chaotic open sets contained in X, i.e.

cl_{QNC} (U) = ∩ {W: U ⊆ W and W is a QNCCS in (X, τ_{QNC}) };

int_{QNC} (U) = ∪ {V: V ⊆ U and V is a QNCOS in (X, τ_{QNC})}.

4.5 Remark 3.12. It is clear that $cl_{QNC}(U)$ is the smallest quadripartitioned neutrosophic chaotic closed set in (X, τ_{QNC}) that contains U and $int_{QNC}(U)$ is the largest quadripartitioned neutrosophic chaotic open set in (X, τ_{QNC}) which is contained in U.

Theorem 3.13. If T and R be any two quadripartitioned neutrosophic chaotic subsets of a quadripartitioned neutrosophic chaotic topological spaces(X, τ_{QNC}), then

(i) $int_{QNC}(T) \subseteq T \subseteq cl_{QNC}(T)$;

(ii) $T \subseteq R \Rightarrow cl_{QNC}(T) \subseteq cl_{QNC}(R)$;

(iii) $T \subseteq R \Longrightarrow int_{QNC}(T) \subseteq int_{QNC}(R)$;

(iv) T is an QNCOS iff $int_{QNC}(T) = T$;

(v) T is an QNCCS iff $cl_{QNC}(T) = T$.

Proof. (i) From the previous definition, we have $int_{QNC}(T) = \cup\{R: R$ is a QNCOS in (X, τ_{QNC}) and $R \subseteq T\}$.

Since, each $R \subseteq T$, so $\cup\{R: R$ is a QNCOS in (X, τ_{QNC}) and $R \subseteq T\} \subseteq T$, (i.e). $int_{QNC}(T) \subseteq T$.

Again, $cl_{QNC}(T) = \cap\{Z: Z$ is a QNCCS in (X, τ_{QNC}) and $T \subseteq Z\}$.

Since, each $Z \supseteq T$, so $\cap\{Z: Z$ is a QNCCS in (X, τ_{QNC}) and $T \subseteq Z\} \supseteq T$, (i.e). $cl_{QNC}(T) \supseteq T$.

Therefore, $int_{QNC}(T) \subseteq T \subseteq cl_{QNC}(T)$.

(ii) Assume that T and R be any two quadripartitioned neutrosophic chaotic subsets of a quadripartitioned neutrosophic chaotic topological space (X, τ_{QNC}) such that $T \subseteq R$.

Now, $cl_{QNC}(T) = \cap\{Z: Z$ is a QNCCS in (X, τ_{QNC}) and $T \subseteq Z\}$

$\subseteq \cap\{Z: Z$ is a QNCCS in(X, τ_{QNC}) and $R \subseteq Z\}$ [since $T \subseteq R$]

$= cl_{QNC}(R)$

$\Longrightarrow cl_{QNC}(T) \subseteq cl_{QNC}(R)$.

Therefore, $T \subseteq R \Longrightarrow cl_{QNC}(T) \subseteq cl_{QNC}(R)$.

(iii) Assume that T and R be any two quadripartitioned neutrosophic chaotic subsets of a quadripartitioned neutrosophic chaotic topological space (X, τ_{QNC}) such that $T \subseteq R$.

Now, $\text{int}_{QNC}(T) = \cup\{Z\colon Z$ is a QNCOS in (X, τ_{QNC}) and $Z \subseteq T\}$

$\subseteq \cup\{Z\colon Z$ is a QNCOS in (X, τ_{QNC}) and $Z \subseteq R\}$ [since $T \subseteq R$]

$= \text{int}_{QNC}(R)$

$\Longrightarrow \text{int}_{QNC}(\text{T}) \subseteq \text{int}_{QNC}(\text{R})$.

Therefore, $T \subseteq R \Longrightarrow \text{int}_{QNC}(\text{T}) \subseteq \text{int}_{QNC}(\text{R})$.

(iv) Assume that T be a quadripartitioned neutrosophic chaotic open set in a quadripartitioned neutrosophic chaotic topological space (X, τ_{QNC}). Now, int_{QNC} (T) = $\cup\{Z\colon Z$ is a QNCOS in (X, τ_{QNC}) and $Z \subseteq T\}$. Since, T is a quadripartitioned neutrosophic chaotic open set in (X, τ_{QNC}), so T is the largest quadripartitioned neutrosophic chaotic open set, which is contained in T. Therefore, $\cup\{Z\colon Z$ is a QNCOS in (X, τ_{QNC}) and $Z \subseteq T\} = T$. This implies, $\text{int}_{QNC}(T) = T$.

(v) Assume that T be a quadripartitioned neutrosophic chaotic closed set in a quadripartitioned neutrosophic chaotic topological space (X, τ_{QNC}) . Now, $cl_{QNC}(T) = \cap\{Z : Z$ is a QNCCS in (X, τ_{QNC}) and $T \subseteq Z\}$. Since, T is a quadripartitioned neutrosophic chaotic closed set in (X, τ_{QNC}), so T is the smallest quadripartitioned neutrosophic chaotic closed set, which contains T. Therefore, $\cap\{Z\colon Z$ is a QNCCS in (X, τ_{QNC}) and $T \subseteq Z\} = T$. This implies, $cl_{QNC}(T) = T$.

Theorem 3.14. Let E be a quadripartitioned neutrosophic chaotic subset of a quadripartitioned neutrosophic chaotic closed set (X, τ_{QNC}). Then,

(i) $(int_{QNC}(E))^c = cl_{QNC}(E^c)$;

(ii) $(cl_{QNC}(E))^c = int_{QNC}(E^c)$.

Proof.

(i) Suppose that (X, τ_{QNC}) be a QNCTS and $E=\{(w, T_{E(w)}, C_{E(w)}, U_{E(w)}, F_{E(w)})\colon w \in X\}$ be a quadripartitioned neutrosophic chaotic subset of X. Now, $int_{QNC}(E) = \cup\{Z_i\colon i \in \Delta$ and Z_i is a QNCOS in (X, τ_{QNC})such that $Z_i \subseteq E\} = \{(w, \vee T_{Z_i}(w), \vee C_{Z_i}(w), \wedge U_{Z_i}(w), \wedge F_{Z_i}(w))\colon w \in X\}$, where for all $i \in \Delta$ and Z_i is a QNCOS in (X, τ_{QNC}) such that $Z_i \subseteq E$. This implies, $(int_{QNC}(E))^c = \{(w, \wedge T_{Z_i}(w), \wedge C_{Z_i}(w), \vee U_{Z_i}(w), \vee F_{Z_i}(w))\colon w \in X\}$. Since, $\wedge T_{Z_i}(w) \leq T_{E(w)}$, $\wedge C_{Z_i}(w) \leq C_{E(w)}$, $\vee U_{Z_i}(w) \geq U_{E(w)}$, $\vee F_{Z_i}(w) \geq F_{E(w)}$, for each $i \in \Delta$ and $w \in X$, so $cl_{QNC}(E^c) = \{(w, \wedge T_{Z_i}(w), \wedge C_{Z_i}(w), \vee U_{Z_i}(w), \vee F_{Z_i}(w))\colon w \in X\} = \cap\{Z_i\colon i \in \Delta$ and Z_i is a QNCCS in (X, τ_{QNC}) such that $E^c \subseteq Z_i\}$. Therefore, $(int_{QNC}(E))^c = cl_{QNC}(E^c)$.

(ii) Suppose that (X, τ_{QNC}) be a QNCTS and $E=\{(w, T_{E(w)}, C_{E(w)}, U_{E(w)}, F_{E(w)})\colon w \in X\}$ be a quadripartitioned neutrosophic chaotic subset of X. Now, $cl_{QNC}(E) = \cap\{Z_i\colon i \in \Delta$ and Z_i is a QNCCS in (X, τ_{QNC}) such that $Z_i \supseteq E\} = \{(w, \wedge T_{Z_i}(w), \wedge C_{Z_i}(w), \vee U_{Z_i}(w), \vee F_{Z_i}(w))\colon w \in X\}$, where for all $i \in \Delta$ and Z_i is a QNCCS in (X, τ_{QNC}) such that $Z_i \supseteq E$. This implies, $(cl_{QNC}(E))^c = \{w, \vee T_{Z_i}(w), \vee C_{Z_i}(w), \underline{\wedge U_{Z_i}}(w), \wedge F_{Z_i}(w))\colon w \in X\}$. Since, $\vee T_{Z_i}(w) \geq T_{E(w)}$, $\vee C_{Z_i}(w) \geq C_{E(w)}$, $\wedge U_{Z_i}(w) \leq U_{E(w)}$, $\wedge F_{Z_i}(w) \leq F_{E(w)}$, for each $i \in \Delta$ and $w \in X$, so

$int_{QNC}(E^c)= \{(w, \vee T_{z_i}(w), \vee C_{z_i}(w), \wedge U_{z_i}(w), \wedge F_{z_i}(w)):w\in X\}=\cup\{Z_i: i\in\Delta$ and Z_i is a QNCOS in (X,τ_{QNC}) such that $Z_i \subseteq E^c\}$.Therefore, $(cl_{QNC}(E))^c = int_{QNC}(E^c)$.

5. Conclusion: In this article we have developed the quadripartitioned neutrosophic chaotic set which includes the contradiction and unknown membership functions which clearly defines interminancy component. Thus the newly introduced set is an extension of neutrosophic chaotic set. Various operators have been defined on quadripartitioned neutrosophic chaotic sets and properties verified. The idea of quadripartitioned neutrosophic chaotic topological spaces was introduced and validated with suitable examples. The future scope of this articlemay comprise of the study of different types of operators on quadripartitioned neutrosophic chaotic sets dealing with actual problems and implementing them in decision-making problems [7,12-16].

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