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A New class of Binary Open Sets in Binary Topological Space

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Abstract: In this paper, we introduced new type of binary open sets namely binary s_α-open sets in binary topological space. Also, some of the properties have been discussed.

Keywords: ${}^{b}\mathbb{S}\alpha$- closed set, ${}^{b}\mathbb{S}\alpha$-open set.

I. Introduction

In 2011, S.Nithyanantha Jothi and P.Thangavelu [2] introduced topology between two sets and also studied some of their properties. Topology between two sets is the binary structure from X to Y which is defined to be the ordered pairs (A,B) where $A \subseteq X$ and $B \subseteq Y$. . In 2000, G.B.Navalagi, proposed semi-α open sets in topological spaces. In 2014, S.N.Jothi and P.Thangavelu [5] introduced generalized binary closed sets in binary topological spaces. Also, S.N.Jothi and P.Thangavelu [3] introduced binary semiopen open sets and discussed some of their properties in binary topological spaces. In continuation, we have found ${}^{b}\mathbb{S}\alpha$- closed set in binary topological spaces and analyzed some of their properties and also explored its relationship with other existing sets.

II. Preliminaries

Definition 2.1.[2] Let X and Y be any two nonempty sets. A binary topology is a binary structure $M \subseteq P(X) \times P(Y)$ from X to Y which satisfies the following axioms:

(i) $(\emptyset, \emptyset) \in M$; $(X, Y) \in M$.

(ii) $(A_1 \cap A_2, B_1 \cap B_2) \in M$ where $A_1, A_2, B_1, B_2 \in M$

(iii) If $(A_\alpha, B_\alpha : \alpha \in A)$ is a family of members of M, then $(\cup_{\alpha\in A} A_\alpha, \cup_{\alpha\in A} B_\alpha) \in M$.

If M is a binary topology from X to Y, then the triplet (X, Y, M) is called binary topological space and the members of M are called the binary open sets of the binary topological space (X, Y, M).

Definition 2.2.[2] Let X and Y be any two nonempty sets and let (A, B) and (C, D) $\in P(X) \times P(Y)$. If $A \subseteq C$ and $B \subseteq D$, then $(A, B) \subseteq (C, D)$.

Definition 2.3.[2] Let (X, Y, M) be a binary topological space and $(A, B) \subseteq (X, Y, M)$.

$(A, B)^{1^\circ} = \cup\{A_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$

$(A, B)^{2^\circ} = \cup\{B_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$.

Definition 2.4.[2] The ordered pair $((A, B)^{1^\circ}, (A, B)^{2^\circ})$ is called the binary interior of (A, B) and it is denoted by b-int(A,B).

Definition 2.5.[2] Let (X, Y, M) be a binary topological space and $(A, B) \subseteq (X, Y, M)$.

$(A, B)^{1^\star} = \cap\{A_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A_\alpha, B_\alpha) \supseteq (A, B)$ and

$(A, B)^{2^\star} = \cap\{B_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A_\alpha, B_\alpha) \supseteq (A, B)$.

Definition 2.6.[2] The ordered pair $((A, B)^{1^\star}, (A, B)^{2^\star})$ is called the binary closure of (A, B). The binary closure of (A, B) is denoted by $b - cl(A, B)$.

Definition 2.7.[2] A subset (A, B) of a binary topological space (X, Y, M) is called

(i) binary regular open if (A, B) = b -int(b -cl(A, B)) and binary regular closed if (A, B) = b -cl(b-int(A, B)).

(ii) binary semi open set if $(A, B) \subseteq$ b- int(b- cl(A, B)). The compliment of binary semiopen set is binary semi closed set.

Definition 2.8[3]. A subset (A, B) of a binary topological space (X, Y, M) is called

(i) binary pre closed if $b - cl(b - int(A, B)) \subseteq (A, B)$

(ii) binary semi pre closed (or binary β closed if $b-cl(b-int(b-cl(A, B))) \subseteq (A, B)$

(iii) binary α closed if $b - int(b - cl(b - int(A, B))) \subseteq (A, B)$.

Definition 2.9[4]. In a topological space (X,τ), the subset A of X is said to be semi-α-open if there exists a α-open set U in X such that $U \subseteq A \subseteq cl(U)$. The family of all semi-α-open sets of X is denoted by $S_\alpha(X)$.

Definition 2.10[3]. Let (X, Y, M) be a binary topological space. Let $(A, B) \subseteq (X, Y)$.Then (A, B) is called binary semi open if there exists a binary open set (U, V) such that $(U, V) \subseteq (A, B) \subseteq$ b-cl(U, V)

III. ON BINARY SEMI-α-OPEN SETS

Definition 3.1. Let (X,Y,M) be a binary topological space and $(A,B) \subseteq (X,Y)$. The subset (A,B) is said to be binary semi α -open ($_bS_\alpha O$)if there exists an binary α-open set (U,V) in X such that $(U,V) \subseteq (A,B) \subseteq cl(U,V)$.

Theorem 3.2. In a binary topological space (X,Y,M), if the subset $(A,B) \in {}_b\alpha o(U,V)$ iff there exists a binary open set (C,D) such that $(C,D) \subseteq (A,B) \subseteq {}_bint({}_bcl(C,D))$.

IV. Proof, Let (A,B) be a binary α-open set in binary topological space. Then, $(A,B) \subseteq {}_bint({}_bcl({}_bint(A,B)))$. We have ${}_bint(A,B) \subseteq (A,B) \subseteq {}_bint({}_bcl({}_bint(A,B)))$. Let $(C,D) = {}_bint(A,B)$. Then there exists a open set ${}_bint(A,B)$ such that $(C,D) \subseteq (A,B) \subseteq {}_bint({}_bcl(C,D))$. Conversely, suppose there exists a binary open set (C,D) such that $(C,D) \subseteq (A,B) \subseteq {}_bint({}_bcl(C,D))$. Since, ${}_bint(A,B)$ is the largest binary open set contained in (A,B), then $(C,D) \subseteq {}_bint(A,B)$ which implies ${}_bcl(C,D) \subseteq {}_bcl({}_bint(A,B))$. Hence, ${}_bint({}_bcl(C,D)) \subseteq {}_bint(\subseteq {}_bcl({}_bint(A,B)))$. But we have $(C,D) \subseteq (A,B) \subseteq {}_bint({}_bcl(C,D))$. Therfore, $(A,B) \subseteq {}_bint({}_bcl({}_bint(A,B)))$. Hence $(A,B) \in {}_b\alpha o(U,V)$.

Theorem 3.3. In a binary topological space, union of any family of binary S_α- open set is binary S_α- open set.

V. Proof. Let $\{(A_i, B_i)\}$ be a family of binary S_α- open set in a binary topological space. To prove, $\cup_{i\in\Delta}(A_i, B_i)$ is a binary S_α- open set. Since, $(A_i, B_i) \in {}_bS_\alpha O(X)$, then there exists a binary α-open set (U_i, V_i) such that $(U_i, V_i) \subseteq (A_i, B_i) \subseteq {}_bcl(U_i, V_i)$ which implies $\cup_{i\in\Delta}(U_i, V_i) \subseteq \cup_{i\in\Delta}(A_i, B_i) \subseteq \cup_{i\in\Delta}({}_bcl(U_i, V_i)) \subseteq {}_bcl(\cup_{i\in\Delta}(U_i, V_i))$. Since, arbitrary union of binary α-open set is binary α-open, $\cup_{i\in\Delta}(U_i, V_i)$ is also binary α-open set. Hence, $\cup_{i\in\Delta}(A_i, B_i)$ is binary S_α- open set.

Remark 3.4.

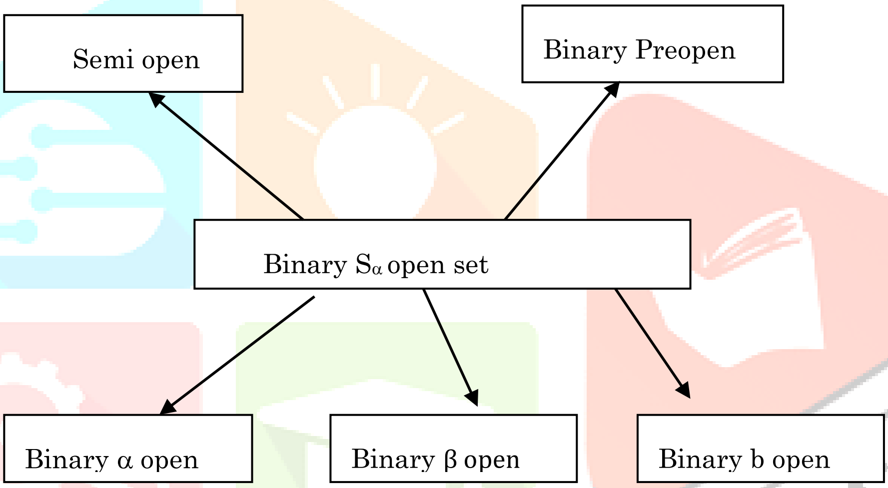
1. Every binary S_α open set is binary semi open set.

2. Every binary S_α open set is binary α open set.

3. Every binary S_α open set is binary pre open set.

4. Every binary S_α open set is binary β open set.

5. Every binary S_α open set is binary b open set.



Following example shows that converse of the above remarks need not be true.

Example3.5

X={a,b,c},Y={1,2}

M={(∅,∅),(X,Y),(∅,{2}),({b},{1}),({b},Y),(X,{2}),({b,c},{2}),({b},∅),({b},{2}),({b,c},Y)}

S_α open set ={(∅,∅),(X,Y),(∅,{2}),({b},{1}),(X,{2}),({b,c},{2}),({b},∅),({b},{2})}

Semi open set={(∅,∅),(X,Y),(∅,{2}),({b},{1}),(X,{2}),({b,c},{2}),({b},∅),({b},{2}),({b,c},Y),({a,c},{2})}

α open set={(∅,∅),(X,Y),(∅,{2}),({b},{1}),({b},Y),(X,{2}),({b,c},{2}),({b},∅),({b},{2}),({b,c},Y)}

Pre open set={(∅,∅),(X,Y),(∅,{2}),({b},{1}),({b},Y),(X,{2}),({b,c},{2}),({b},∅),({b},{2})}

β open set={(∅,∅),(X,Y),(∅,{2}),({b},{1}),({b},Y),(X,{2}),({b,c},{2}),({b},∅),({b},{2}),(X,{1}),({a,c},{2})}

b open set={(∅,∅),(X,Y),(∅,{2}),({b},{1}),({b},Y),(X,{2}),({b,c},{2}),({b},∅),({b},{2}),({a,c},{2})}

the set ({b,c},Y) is binary semi open and binary α open but not binary S_α open set the set ({b},Y) is binary pre open , binary β open set and binary b open set but not binary S_α open set

VI. REFERENCES

[1] S. Jayalakshmi and A. Manonmani, Binary Pre Generalized Regular Beta Closed Sets in Binary Topological spaces, International Journal of Mathematics Trends and Technology (IJMTT), Volume 66 Issue 7, July 2020.

[2] S. Nithyanantha Jothi and P. Thangavelu, Topology between two sets, Journal of Mathematical Sciences and Computer Applications,1 (3): 95–107, 2011.

[3] S. Nithyanantha Jothi, Binary semiopen sets in binary topological spaces, International Journal of Mathematical Archive-7(9), 2016.

[4] S. Nithyanantha Jothi, P. Thangavelu, Generalized Binary Regular Closed Sets, IRA-International Journal of Applied Sciences, Vol.04, Issue 02 (2016).

[5] Nithyanantha Jothi.S and Thangavelu.P, Generalized binary closed sets in binary topological spaces, Ultra Scientist Vol.26 (1) A, (2014)25-30.

[6] S. Nithyanantha Jothi and P.Thangavelu, On binary topological spaces, Pacific-Asian Journal of Mathematics, 5(2)(2011), 133-138.

[7] S. Nithyanantha Jothi and P.Thangavelu, On binary continuity and binary separation axioms, Ultra Scientist Of Physical Sciences, Vol. 24(1)A, (2012), 121 -126.

[8] C.Santhini and T. Dhivya, New notion of generalised binary closed sets in binary topological space, International Journal of Mathematical Archive-9(10), 2018.

[9] D. Savithiri and C. Janaki, Binary Regular Generalized Closed Sets In Binary Topological Spaces, IJSRSET, Volume 6, Issue 3, 2019.