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Quit-Based Separation in the Job-Search Model—A Theoretical Analysis

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Abstract: This paper introduces quit-based job separation in the canonical DMP model. Here, we assume that a job is destroyed when worker quits the job and this can happen when wage paid to the labour is less than the reservation wage. This implies that job separation is endogenized by linking it to the reservation wage. In this quit-based job separation model we find that higher match productivity decreases steady state unemployment rate, but higher unemployment income and larger labour share increase this. Although we get similar results in the DMP model where separation occurs due to productivity shocks, our results are less calibrated in the regime of higher productivity and larger labour share.

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Key Words: Quit, job-destruction, job-search.

1. Introduction:

Job creation and job destruction are the two key concepts in the matching theory of unemployment. In reality, both are flexible[1] and most of the labour policies influence steady state unemployment via these two channels.[2] Empirically, we find inverse relation between vacancy and unemployment which is represented by the Beveridge curve. Often the two flows :job creation and job destruction conflict each other and thus leads

[1]See Mortensen and Pissarides (1994).
[2]See Pissarides (2000).

to ambiguous effect of labour policies on unemployment. In this situation job creation effect on unemployment dominates the job destruction effect which gives the usual shape of the Beveridge curve.

In the matching theory of unemployment, both firm and labour play crucial role on the existence of match. A match may be dissolved either by firm or by labour or by both. In the literature, we find that match separation depends on factors like job tenure, level and changes of wage, productivity shocks. Jovanovic (1979) shows that separation between firm and labour occurs due to labour turnover which mostly depends on the tenures of the match. Jovanovic and Mincer (1978) argue that people with a lower propensity to change jobs will tend to have longer job tenures and vice versa. Jovanovic (1979) mentions that all separations involve imperfect information. A match may be dissolved because of either the arrivals of information about the current match or the new information about alternative prospective match.

The turnover cost model of Stiglitz (1974) and Salop (1979) show that labour quits depend on the level of wage. A firm pays high wage to reduce the number of quits because hiring and training new workers is costly. Campbell III(1994) has shown that quits also depends on the change in the wage as well as on its level. He argues that current wage is determined by the initial wage and by the change in the wage over the tenure at the firm. He also shows that change in the wage has negative effect on the quit rate and quits are more affected by the change in the wage than by the level of current wage.

The DMP model assumes that match separation occurs due to productivity shocks. They have shown that each match has a critical value of idiosyncratic productivity which is known as the reservation productivity. If productivity falls below this level firm dissolves match. Thus, in the DMP model it is the firm who decides whether to continue or destroy job, although at this level worker also quit the match involuntarily.

In the literature, we find two types of match separation: separation due to labour quits and separation due to productivity shocks, leading to firm quits. Empirically, we find quit-based separation in the European economy[3] where as productivity-based separation in the USA economy. We can hardly find any model which incorporates quit-based job destruction in the context of the DMP model. This gap in the research actually motivated us to make this study.

Our goal is to find out the implications of the labour quits for job creation, job destruction and unemployment in a search theoretic model. We also examine the consequences of labour policies on job creation, job destruction and on unemployment in the steady state through the lens of the DMP model.

[3] See Rebitzer and Taylor (1995), Portugal and Cardoso (2006), Pierre and David (2012), Dube et al. (2012).

2. The Model:

We assume the basic structure of the DMP model where production requires one firm and one labour. Both worker and firm are risk-neutral, live infinitely and discount future income at the same rate (r). Every firm and every labour are equally productive at any point of time. Each firm searches labour and each labour searches job. Once they are matched, production starts and each firm-labour pair produces y units of output per unit of time.

Matching is not an instantaneous process. It takes time andresources. Match frictions cause unemployment in the labour market. Like the DMP model we assume a constant return to scale matching function $m = m(u,v)$ where matching (m) depends positively on unemployment rate (u) and vacancy rate (v) with $m_u, m_v > 0$. Total flow of match is $m = au$ and total flow of job is $m = vq$. So, $\frac{m}{u} = a$ is the job arrival rate and $\frac{m}{v} = q$ is the job offer rate. We may write $q = q(\theta), a = \frac{m}{u} = \frac{m}{v}\frac{v}{u} = \theta q(\theta)$ where $\theta = \frac{v}{u}$ is thelabour market tightness with $q'(\theta) < 0, \left|e_\theta^q\right| < 1.$

Workers are either employed or unemployed. Only unemployed workers search for jobs and get unemployment benefit (b). An employed worker gets wage w which is determined through the Nash-bargaining game between matched firm and matched labour. We assume that there exists wage dispersion which is represented by the wage function $w\varepsilon$ where w the general wage is and ε is the idiosyncratic component of wage. We assume that the wage shock changes the wage from initial value ε to ε' and the general distribution of ε is $G(\varepsilon)$ in the range $0 \le \varepsilon \le 1$. We also assume that the distribution is free of holes.

We assume that the idiosyncratic wage shocks arrive to jobs at Poisson rate λ. The idiosyncratic wage is independent of initial wage and is irreversible. We assume that each worker has reservation wage w_R, which is equal to the values of unemployment (rU).Worker has the choice either to continue match at the new wage or to quit the match. The optimal decision is that worker continues match and produce output if $W(\varepsilon) \ge U$ and quits if $W(\varepsilon) < U$. Thus, the worker's separation rule is $W(\varepsilon) < U$ and it satisfies the reservation property that $W(\varepsilon) = U$. In terms of the reservation wage, a worker continues match if $\varepsilon > w_R$ and quits it if $\varepsilon < w_R$. This property makes the wage a stochastic Poisson process with initial value w and the terminal value $w\varepsilon$.We may also explain match separation in terms of the values of job. At $\varepsilon > w_R$, job is continued and so $J(\varepsilon) \ge 0$ and at $\varepsilon < w_R$, job is destroyed and so $J(\varepsilon) < 0$.Therefore, we may write

$J(\varepsilon)<0$ as the job separation rule and $J(\varepsilon)=0$ as the reservation property. Forsimplicity we consider only the steady state situation where all aggregate variables are stationary over time.

The value of unemployment satisfies the following Bellman equation:

$$rU = b + \theta q(\theta)\left[W(\varepsilon) - U\right] \quad (1)$$

Value of employment is given by:

$$rW(\varepsilon) = w\varepsilon + \lambda G\left(w_R\right)U - \lambda W(\varepsilon) + \lambda \int_{w_R}^{1} W(\varepsilon')dG(\varepsilon') \quad (2)$$

Equation (2) states that worker gets wage $w\varepsilon$ and his expected income is $W(\varepsilon)$. When a shock arrives, he has to giveup this income. He again joins the new jobs if the new job pays new wage in the range $w_R \le \varepsilon' \le 1$. Outside this range, he becomes unemployed for expected return U .

The steady state value of vacancy may be written as:

$$rV = -C + q(\theta)(J - V) \quad (3)$$

Where C is the cost of vacancy.

The steady state equation for the value of filled job is:

$$rJ(\varepsilon) = y - w\varepsilon - \lambda J(\varepsilon) + \lambda \int_{w_R}^{1} J(\varepsilon')dG(\varepsilon') \quad (4)$$

In Equation (4), a firm has to give up the value of job $J(\varepsilon)$ for new value $J(\varepsilon')$ when wage shock arrives and this is feasible provided worker joins the new job which happens if the new wage falls in the range $w_R \le \varepsilon' \le 1$.

As search is expensive, the search cost can be saved after matching. So, match generates surplus which is shared by both the matched firm and the matched worker. The surplus-sharing rule assumed is that surplus is divided between firm and worker in fixed proportion at all ε. This follows from the Nash-bargaining game. Thus we may write the sharing rule as:

$$(1-\beta)\left[W(\varepsilon) - U\right] = \beta\left[J(\varepsilon) - V\right] \quad (5)$$

It is assumed that a firm creates vacancy up to the point where $V = 0$. Putting this condition into (3) we get the job vacancy creation condition as

$$J(\varepsilon) = \frac{C}{q(\theta)} \quad (6)$$

Using (6) into (5) we may write

$$W(\varepsilon)-U=\frac{\beta}{(1-\beta)}\frac{C}{q(\theta)} \tag{7}$$

From Equations (1) and (7) we get

$$rU=b+\theta q(\theta).\frac{\beta}{(1-\beta)}\frac{C}{q(\theta)}=b+\frac{\beta C\theta}{(1-\beta)} \tag{8}$$

It is assumed that the reservation wage is equal to the value of unemployment. So, we have

$$w_R=rU=b+\frac{\beta C\theta}{(1-\beta)} \tag{9}$$

This is the reservation wage equation which shows that reservation wage depends positively on b,β,C,θ. The slope of this curve is: $\frac{dw_R}{d\theta}=\frac{\beta C}{(1-\beta)}>0$. So, the reservation wage is positively sloped in the $w_R-\theta$ space. This is one of the key equations in this model.

Using (2)-(5) we get the Nash-wage equation as:[4]

$$w\varepsilon=(1-\beta)b+\beta(y+C\theta) \tag{10}$$

Using (9) from (10) we get

$$w\varepsilon=(1-\beta)w_R+\beta y \tag{10a}$$

From Equations (4) and (10) we get the job creation curve as:[5]

$$\frac{C}{q(\theta)}=\frac{(1-\beta)}{\beta}.\frac{w}{(r+\lambda)}(\varepsilon-w_R) \tag{11}$$

It is assumed that at all new jobs worker gets maximum idiosyncratic wage, $\varepsilon=1$ and the new job creation condition is:

$$\frac{C}{q(\theta)}=\frac{(1-\beta)}{\beta}.\frac{w}{(r+\lambda)}(1-w_R) \tag{11a}$$

This is another key equation in our model.

The slope of this curve is:

$$\frac{dw_R}{d\theta}=\frac{q'(\theta)(1-w_R)w}{q(\theta)\left[w-(1-\beta)(1-w_R)\right]}<0,\left(\because q'(\theta)<0\right)$$

[4] See the Appendix A.1.
[5] See the Appendix A.2.

Now, solving Equations (9) and (11a) we get the steady state values of w_R, θ.

The model is closed by the steady state rate of unemployment:

$$u = \frac{\lambda G(w_R)}{\lambda G(w_R) + \theta q(\theta)} \tag{12}$$

This is the Beveridge curve. The shape of this curve is influenced by both w_R, θ. As θ increases, vacancy also increases (given unemployment). This increases matching and so the Beveridge curve is negatively sloped. On the other hand, when θ increases, w_R also increases (see Equation (9)) and so also wage. The worker's outside opportunities increase and so more labour quits and more jobs are destroyed. This implies that the Beveridge curve is positively sloped. However, under the assumption that the job matching effect on unemployment dominates the job destruction effect, the Beveridge curve is negatively sloped.[6] Now, once w_R, θ are obtained, we get steady state value of unemployment from Equation (12).

3. The Comparative Static Effects:

Taking total differentials of Equations (11a) and (8) and then solving by Cramer's rule we get the following results:[7]

$$\frac{dw_R}{d\beta} = \frac{(A_2 B_3 + B_2 \beta C)}{\Delta} < 0 \text{ if } \left|e_q^\theta\right| = \beta \tag{13}$$

$$\frac{dw_R}{db} = \frac{(1-\beta) A_2}{\Delta} > 0 \tag{14}$$

$$\frac{dw_R}{dy} = -\frac{B_1 \beta C}{\Delta} > 0 \tag{15}$$

$$\frac{d\theta}{d\beta} = \frac{(A_1 B_3 + (1-\beta) B_2)}{\Delta} < 0 \tag{16}$$

$$\frac{d\theta}{db} = \frac{(1-\beta) A_1}{\Delta} < 0 \tag{17}$$

$$\frac{d\theta}{dy} = -\frac{(1-\beta) B_1}{\Delta} > 0 \tag{18}$$

Now, differentiating Equation (12) we get

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[6]See Pissarides (2000).

[7]See Appendix A.3.

$$du = \left[-\frac{u^2 q(\theta)}{\lambda G(w_R)} \left(1 - \left|e_q^{\theta}\right|\right) \right] d\theta + \left[\frac{G'(w_R) u(1-u)}{G(w_R)} \right] dw_R \tag{19}$$

This equation shows that changes in the parameters of the model influence the steady state unemployment rate through the matching function and the labour quit. The conflict arises when the two effects move in opposite directions. In those cases, the empirically supported assumption the matching effect on unemployment dominates the job destruction effect determine the net effects on unemployment rate.

On the basis of the results obtained above we can make the following propositions:

Proposition 1: An increase in the match productivity increases both job creation rate and job destruction rate. But under the assumption that the job matching effect on unemployment dominates the job destruction effect, the steady state unemployment rate falls.

In the canonical DMP model job creation rate increases but the job destruction rate decreases with the rise in the general productivity and so the steady state unemployment rate falls more. Thus, the effect of an increase in match productivity on unemployment rate is less calibrated in our model.

We may give an intuitive explanation of the Proposition 1. Higher match productivity shifts the job creation curve up and to the right. The reservation wage curve does not shift as it is independent of productivity. This increases both market tightness and reservation wage. At given unemployment, both the job creation rate as well as the job destruction rate increase. However, the former effect dominates the latter which is empirically supported. So, unemployment must fall until the job creation rate reduces to the lower rate of job destruction. As a result, finally, unemployment rate falls.

Proposition 2: Larger unemployment benefit decreases job creation and increases job destruction. Under the assumption that the job matching effect on unemployment dominates the job destruction effect, larger unemployment income ultimately increases steady state unemployment rate.

In the DMP model where separation occurs due to idiosyncratic shocks that hit match productivity below the reservation level, a rise in the unemployment income increases job destruction via higher reservation productivity and higher bargained wage and decreases job creation by reducing market tightness and finally, it increases steady state rate of unemployment.

The explanation of the above proposition is as follows. An increase in the unemployment income shifts the reservation curve up and to the left. The job creation curve does not shift as the job creation condition does not depend on the unemployment income. At steady state equilibrium, market tightness falls and reservation wage rises and so also the bargained wage. Thus, at given unemployment, job creation falls and job destruction rises and this leads to higherrate of unemployment in the steady state.

Proposition 3: Larger labour share decreases job creation and job destruction if $\left|e_q^{\theta}\right| = \beta$. This leads to higher rate of unemployment in the steady state if matching effect on unemployment dominates the job destruction effect.

In the DMP model larger labour share decreases job creation but it has no effect on job destruction under the bench mark condition $\left|e_q^{\theta}\right| = \beta$. Thus, unemployment rate increases in the steady state. In our model as job destruction falls, the effect of larger labour share on steady state unemployment is less calibrated.

When labour share increases the reservation wage curve rotates anti clock-wise and the job creation curve shifts down and to the left. This leads to lower market tightness and lower reservation wage if $\left|e_q^{\theta}\right| = \beta$. Thus, at given unemployment, job creation falls and so also job destruction. This produces larger unemployment rate in the steady state as the matching effect is assumed to dominate the job destruction effect. Note that as job destruction falls in our model the steady state rate of unemployment increases less following a larger labour share. Thus, our result is less calibrated than the DMP model.

4. Conclusions:

In this paper we have introduced quit-based job separation in the canonical DMP model.It has been motivated by the fact that in the European economy separations in the labour market are mostly occurred due to the labour quits. We explain quits in terms of idiosyncratic wage which is drawn from a distributional wage function $G(\varepsilon)$ in the range of $0 \le \varepsilon \le 1$. We assume that each work has a reservation wage w_R which is equal to the value of unemployment. If wage falls below w_R due to idiosyncratic shocks, workers quit the jobs. In this situation $J(w_R) < 0$ and so firm also destroys job involuntarily. This implies that separation is mutual, not contradictory.

We have studied the consequences of high match productivity, high unemployment income and larger labour share on unemployment rate in the steady state situation. We find that higher labour productivity lowers unemployment rate but high unemployment income and larger labour share increase unemployment rate under the some plausible assumptions. We also get the same type of effects in the DMP model where separation occurs due to productivity shocks, rather than wage shocks. However, in the case of higher general productivity and larger labour share the effects on unemployment rate are less calibrated in our quit-based job separation model.This is the implication of labour quit in the context of the DMP type job search model.

Appendices

Appendix A.1: Derivation of the Nash-wage equation:

The Nash-bargaining problem is:

$$\underset{w}{\text{Max.}}\ \Omega = \left[W(\varepsilon) - U\right]^{\beta} \left[J(\varepsilon) - V\right]^{(1-\beta)} \quad \text{(A.1)}$$

Where β is the bargaining power of the labourwith $0 < \beta < 1$.

The first order condition for the Nash problem is:

$$\beta J(\varepsilon) \frac{\partial}{\partial w}\left[W(\varepsilon) - U\right] + (1-\beta)\left[W(\varepsilon) - U\right] \frac{\partial}{\partial w}\left[J(\varepsilon) - V\right] = 0 \quad \text{(A.2)}$$

Now from Equations (2) and (4) we get

$$\begin{aligned} \frac{\partial}{\partial w}\left[J(\varepsilon) - V\right] &= -\frac{\varepsilon}{(r+\lambda)} \\ \frac{\partial}{\partial w}\left[W(\varepsilon) - U\right] &= \frac{\varepsilon}{(r+\lambda)} \end{aligned} \quad \text{(A.3)}$$

Using (A.3) into (A.2) we get

$$\beta J(\varepsilon) = (1-\beta)\left[W(\varepsilon) - U\right] \quad \text{(A.4)}$$

This is the surplus-sharing rule in this model. Now multiplying (4) by β and (2) by $(1-\beta)$ and then subtracting the later from the former and using the rent-sharing rule and after simplifying we get the Nash-wage as

$$w\varepsilon = \beta y + (1-\beta) rU \quad \text{(A.5)}$$

Putting $rU = w_R$ and using (8) into (A.5) we may write

$$\begin{aligned} w\varepsilon &= (1-\beta) w_R + \beta y \\ \therefore w\varepsilon &= (1-\beta) b + \beta (y + C\theta) \end{aligned} \quad \text{(A.6)}$$

Appendix A.2: Derivation of the job creation condition:

Substituting (A.6) into (4) we get

$$(r+\lambda) J(\varepsilon) = \left(\frac{1-\beta}{\beta}\right) w\varepsilon - \left(\frac{1-\beta}{\beta}\right) b - C\theta + \lambda \int_{w_R}^{1} J(\varepsilon') dG(\varepsilon') \quad \text{(A.7)}$$

Putting $\varepsilon = w_R$ with $J(W_R) = 0$ into (A.7) we get,

$$0=\left(\frac{1-\beta}{\beta}\right)w.w_R-\left(\frac{1-\beta}{\beta}\right)b-C\theta+\int_{w_R}^{1} J(\varepsilon')dG(\varepsilon') \quad \text{(A.8)}$$

Subtracting (A.8) from (A.7) and using the zero-profit condition and assuming that all jobs are created offering maximum idiosyncratic wage $\varepsilon=1$, we get

$$\frac{C}{q(\theta)}=\left(\frac{1-\beta}{\beta}\right)w\frac{(1-w_R)}{(r+\lambda)} \quad \text{(A.9)}$$

This is the new job creation condition in this model.

Appendix A.3: Effects of changes in y,β,b on u :

Total differentials of Equations (11a) and (8) are

$$A_1dw_R-A_2d\theta=B_1dy-B_2d\beta \quad \text{(A.10)}$$

$$(1-\beta)dw_R-\beta Cd\theta=B_3d\beta+(1-\beta)db \quad \text{(A.11)}$$

Where

$$A_1=(1-\beta)q(\theta)\left[w-(1-\beta)(1-w_R)\right]>0 \quad \text{(A.12)}$$

$$A_2=(1-\beta)q'(\theta)(1-w_R)w<0 \quad \text{(A.13)}$$

$$B_1=(1-\beta)q(\theta)(1-w_R)\beta>0 \quad \text{(A.14)}$$

$$B_2=q(\theta)(1-w_R)\left[w-(1-\beta)(y-w_R)\right]+(r+\lambda)C>0 \quad \text{(A.15)}$$

$$B_3=(w_R-b+C\theta)>0 \quad \text{(A.16)}$$

Solving (A.10) and (A.11) by Cramer's rule we get

$$dw_R=\frac{1}{\Delta}\left[(A_2B_3+B_2\beta C)d\beta+(1-\beta)A_2-B_1\beta Cdy\right] \quad \text{(A.17)}$$ [8]

$$d\theta=\frac{1}{\Delta}\left[\{A_1B_3+(1-\beta)B_2\}d\beta+(1-\beta)A_1db-(1-\beta)B_1dy\right] \quad \text{(A.18)}$$

Where

[8] Using $\beta=\left|e_q^\theta\right|$ and Equation (11a) we can write $A_2B_3+B_2\beta C=\frac{(r+\lambda)\beta C^2}{w}\left[w-(y-w_R)\right]$. An important property of the model is that the match productivity is less than the reservation wage. This happens because each job has a positive option value which implies some labour hoarding by the firm. So, from Equation (4) we find that at $w_R, y<w_R$. This implies $(A_2B_3+B_2\beta C)>0$.

$$\Delta = \underset{(-)}{(1-\beta) A_2} - \underset{(+)}{A_1 \beta C} < 0 \qquad \text{(A.19)}$$

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