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An Introduction To Fermatean Neutrosophic Hypersoft Topological Spaces

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Abstract: In this paper, we introduce the new concept namely fermatean neutrosophic hypersoft set by the extension of fermatean neutrosophic set. We derived the basic operations on fermatean neutrosophic hypersoft set. Also we introduce the fermatean neutrosophic hypersoft topological spaces and their properties and theorems are analyzed.

***Index Terms* – fermatean neutrosophic set, neutrosophic hypersoft, fermatean neutrosophic hypersoft, topological spaces**

I. INTRODUCTION

Zadeh [15] in 1965 presented the idea of fuzzy set theory, which has a very important role in solving problems by providing a suitable way for the expression of vague concepts by having membership. Computer scientists and mathematicians have studied and developed fuzzy set theory with widened applications in fuzzy logic, fuzzy topology, fuzzy control systems, etc. Also theories such as fuzzy probability, soft and rough set theories are used to solve these problems. Intuitionistic fuzzy sets introduced by Atanassov[2] is appropriate for such a situation. The intuitionistic fuzzy sets can only handle the incomplete information considering both the truth-membership (or simply membership) and falsity-membership (or non-membership) values. It does not handle the indeterminate and inconsistent information which exists in belief system.

Smarandache[11,13] introduced the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. Shabir and Naz [10] introduced soft topological spaces and defined some notions of soft sets. By the extension of soft set[4], hypersoft set[12] structure has attracted the attention of researchers because it is more suitable than soft set structure in decision making problems. Although it is a new concept, many studies have been done and the field of study continues to expand [5-8].

Fermatean fuzzy sets (FFSs), proposed by Senapati and Yager [9], can handle uncertain information more easily in the process of decision making. They defined basic operations over the Fermatean fuzzy sets. Recently C.A.C Sweety and R.Jansi[1] proposed a notion namely Fermatean neutrosophic set by the extention of Pythagorean neutrosophic set.

In this paper, we introduce fermatean neutrosophic hypersoft set by the combination of fermatean neutrosophic set[1] and neutrosophic hypersoft set[8]. Also, fermatean neutrosophic hypersoft topological spaces are proposed with suitable theorems and examples.

II. PRELIMINARIES

In this section, we recall the basic concepts for the development of desired set.

Definition 2.1. [15] (Fuzzy set)

Let X be a non-empty set. A fuzzy set A in X is characterized by its membership function $\mu_A : X \rightarrow [0, 1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in fuzzy set A, for each $x \in X$. It is clear that A is completely determined by the set of tuples $A = \{(x, \mu_A(x)) : x \in X\}$.

Definition 2.2. [2] (Intuitionistic fuzzy set)

The Intuitionistic fuzzy sets are defined on a non-empty set X as objects having the form $I = \{\langle x, T(x), F(x)\rangle : x \in X\}$, where $T(x) : X \rightarrow [0, 1]$ and $F(x) : X \rightarrow [0, 1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set I, respectively, and $0 \leq T(x) + F(x) \leq 1$, for all $x \in X$.

Definition 2.3 [4] (Soft Set)

Let U be the universal set and E be the set of attributes with respect to U. Let P(U) be the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U and its mapping is given as $F: A \rightarrow P(U)$. It is also defined as, $(F, A) = \{F(e) \in P(U): e \in E, F(e) = \emptyset \text{ if } e \neq A\}$

Definition 2.4[12] (Hypersoft Set)

Let U be the universal set and P(U) be the power set of U. Consider $e_1, e_2, \ldots e_n$ for n≥1, be n well-defined attributes, whose corresponding attributive values are respectively the set $E_1, E_2, \ldots E_n$ with $E_i \cap E_j = \emptyset$ for $i \neq j$ and i, j ∈ {1,2,...,n}, then the pair $(\mathcal{F}, E_1 \times E_2 \times \ldots \times E_n)$ is said to be Hypersoft set over U where $F: E_1 \times E_2 \times \ldots \times E_n \to P(U)$.

Definition 2.5 [13] (Neutrosophic Set)

A neutrosophic set A on the universe of discourse X is defined as A = {< x, T (x), I(x), F (x) >, x ∈ X}, where T(x), I(x), F(x) represents truth membership function, indeterminate membership function and a falsity membership function of the element x ∈ X to the set A respectively and the condition $^{-}0 \leq T(x) + I(x) + F(x) \leq 3^{+}$.

Definition 2.6 [8] (Neutrosophic Hypersoft Set)

Let U be the universal set and P(U) be the power set of U. Consider $e_1, e_2, \ldots e_n$ for n≥1, be n well-defined attributes, whose corresponding attributive values are respectively the set $E_1, E_2, \ldots E_n$ with $E_i \cap E_j = \emptyset$ for i≠j and i, j ∈ {1,2,...,n}, and their relation $E_1 \times E_2 \times \ldots \times E_n = S$, then the pair $(\mathcal{F}, S)$ is said to be Neutrosophic Hypersoft set over U where $(\mathcal{F}, E_1 \times E_2 \times \ldots \times E_n \to P(U)$ and $\mathcal{F}(E_1 \times E_2 \times \ldots \times E_n) = \{< x, T(\mathcal{F}(S)), I(\mathcal{F}(S)), F(\mathcal{F}(S)) >, x \in U\}$ where T is the membership value of truthfulness, I is the membership value of indeterminacy and F is the membership value of falsity such that T, I, F: U → [0,1] also $0 \leq T(\mathcal{F}(S)) + I(\mathcal{F}(S)) + F(\mathcal{F}(S)) \leq 3$.

Definition 2.7 [14] (Pythagorean fuzzy set)

The Pythagorean fuzzy sets are defined on a non-empty set X as objects having the form P = {⟨x, T (x), F (x)⟩ : x ∈ X}, where T (x) : X → [0, 1] and F (x) : X → [0, 1] denote the degree of membership and the degree of non-membership of each element x ∈ X to the set P, respectively, and $0 \leq (T(x))^2 + (F(x))^2 \leq 1$, for all x ∈ X.

Definition 2.8 [9] (Fermatean fuzzy set)

Let X be the non-empty set in universe of discourse. A Fermatean set is of the form, $F = \{< x, T(x), F(x) >: x \in X\}$ whose truth membership and false membership functions with the condition $0 \leq (T(x))^3 + (F(x))^3 \leq 1$, for all x ∈ X.

Definition 2.9 [3] **(Pythagorean Neutrosophic Set)**

Let X be a non-empty set (universe). A Pythagorean Neutrosophic set [PN Set] A on X is an object of the form: *A* = {⟨ x, T (x), I(x), F (x)⟩ : x ∈ X }, Where $T(x), I(x), F(x) \in [0,1]$. Then $0 \leq (T(x))^2 + (I(x))^2 + (F(x))^2 \leq 2$, for all x in *X*. $T(x)$ is the degree of membership, $I(x)$ is the degree of inderminancy and F(x) is the degree of non-membership. Here T(x) and F(x) are dependent components and I(x) is an independent components.

Definition 2.10 [1] **(Fermatean Neutrosophic Set)**

Let X be a non-empty set (universe). A Fermatean Neutrosophic set [FN Set] A on X is an object of the form: $A = \{(x, T(x), I(x), F(x)) / x \in X\}$, Where T(x) , I(x) , F(x) ∈ [0,1], $0 \leq ((T(x))^3 + (F(x))^3 \leq 1$ and $0 \leq (I(x))^3 \leq 1$. Then $0 \leq (T(x))^3 + (I(x))^3 + (F(x))^3 \leq 2$, for all x in *X*. T(x) is the degree of membership, I(x) is the degree of inderminancy and F(x) is the degree of non-membership. Here T(x) and F(x) are dependent components and I(x) is an independent components.

Definition 2.11 [1]

Let X be a nonempty set and I the unit interval [0,1]. A Fermatean Neutrosophic sets M and N of the form:

$M = \{(x, T_M(x), I_M(x), F_M(x)) / x \in X\}$ and $N = \{(x, T_N(x), I_N(x), F_N(x)) / x \in X\}$. Then

1) $M^C = \{(x, F_M(x), 1 - I_M(x), T_M(x)) : x \in X\}$
2) $M \cup N = \{(x, \max(T_M(x), T_N(x)), \min(I_M(x), I_N(x)), \min(F_M(x), F_N(x)) : x \in X\}$
3) $M \cap N = \{(x, \min(T_M(x), T_N(x)), \max(I_M(x), I_N(x)), \max(F_M(x), F_N(x)) : x \in X\}$

III. Some Basic Operations on Fermatean Neutrosophic Hypersoft Sets

In this section, operations of subset, null set, absolute set, complement, union, intersection, AND, OR on Fermatean neutrosophic hypersoft sets are defined. We also presented basic properties of these operations.

Definition 3.1:

Let U be the universal set and P(U) be the power set of U. Consider $e^1, e^2, \ldots, e^n$ for n ≥ 1 be n well defined attributes, whose corresponding attribute values are respectively the set $E^1, E^2, \ldots, E^n$ with $E^i \cap E^j = \varphi$, for i ≠ j and i, j ϵ {1,2,…,n} and their relation $E^1 \times E^2 \times \ldots \times E^n = \rho$, then the pair (F, ρ) is said to be Fermatean Neutrosophic Hypersoft (FNH in short) set over U where, f: $E^1 \times E^2 \times \ldots \times E^n \to P(U)$ and

$$f(E^1 \times E^2 \times \ldots \times E^n) = \{<x, T(f(\rho)), I(f(\rho)), F(f(\rho)) >, x \in U\}$$

where,

Here, T is the membership value, I is the indeterminate value and F is the non-membership value such that T,I,F: U→[0,1], $0 \leq ((T(x))^3 + (F(x))^3 \leq 1$ and $0 \leq (I(x))^3 \leq 1$. Then $0 \leq (T(f(\rho)))^3 + (I(f(\rho)))^3 + (F(f(\rho)))^3 \leq 2$

Example 3.2

Let $\ddot{U} = \{x_1, x_2, x_3\}$ be an initial universe and $\ddot{E}_1, \ddot{E}_2, \ddot{E}_3$ be sets of attributes. Attributes are given as;

$$\ddot{E}_1 = \{\alpha_1, \alpha_2, \alpha_3\},\ \ddot{E}_2 = \{\beta_1, \beta_2\},\ \ddot{E}_3 = \{\gamma_1, \gamma_2\}$$

Suppose that

$$\ddot{B}_1 = \{\alpha_1\},\ \ddot{B}_2 = \{\beta_1, \beta_2\},\ \ddot{B}_3 = \{\gamma_1, \gamma_2\}$$
$$\ddot{C}_1 = \{\alpha_1, \alpha_3\},\ \ddot{C}_2 = \{\beta_1, \beta_2\},\ \ddot{C}_3 = \{\gamma_2\}$$

are subset of $\ddot{E}_i$ for each i=1,2,3. Then the fermatean neutrosophic hypersoft set $(\ddot{G}_1, \ddot{A}_1)$ over the universe $\ddot{U}$ can be written as follows:

$$(\ddot{G}_1,\ddot{A}_1)=\left\{\begin{array}{l}\left\langle(\alpha_1,\beta_1,\gamma_1),\left\{\frac{x_1}{[0.3,0.6,0.5]},\frac{x_2}{[0.7,0.6,0.8]},\frac{x_3}{[0.4,0.5,0.6]}\right\}\right\rangle,\\ \left\langle(\alpha_1,\beta_1,\gamma_2),\left\{\frac{x_1}{,[0.6,0.5,0.7]},\frac{x_2}{[0.5,0.3,0.2]},\frac{x_3}{[0.7,0.8,0.5]}\right\}\right\rangle,\\ \left\langle(\alpha_1,\beta_2,\gamma_1),\left\{\frac{x_1}{[0.3,0.2,0.6]},\frac{x_2}{[0.6,0.2,0.8]},\frac{x_3}{[0.6,0.9,0.3]}\right\}\right\rangle,\\ \left\langle(\alpha_1,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.5,0.4,0.3]},\frac{x_2}{[0.5,0.2,0.9]},\frac{x_3}{[0.1,0.4,0.6]}\right\}\right\rangle,\end{array}\right\}$$

Definition 3.3

Let $\ddot{U}$ be the universal set and $(\ddot{G}_1,\ddot{A}_1)$, $(\ddot{G}_2,\ddot{A}_2)$ be two fermatean neutrosophic hypersoft set over $\ddot{U}$. Then $(\ddot{G}_1,\ddot{A}_1)$ is the fermatean neutrosophic hypersoft subset of $(\ddot{G}_2,\ddot{A}_2)$ if

1. $\ddot{A}_1 \subseteq \ddot{A}_2$,

2.for $\forall\alpha \in \ddot{A}_1$ and $\forall x \in \ddot{U}$, $T_{\ddot{G}_1(\alpha)}(x) \leq T_{\ddot{G}_2(\alpha)}(x)$, $I_{\ddot{G}_1(\alpha)}(x) \geq I_{\ddot{G}_2(\alpha)}(x)$, $F_{\ddot{G}_1(\alpha)}(x) \geq F_{\ddot{G}_2(\alpha)}(x)$. It is denoted by $(\ddot{G}_1,\ddot{A}_1) \subseteq (\ddot{G}_2,\ddot{A}_1)$.

Definition 3.4

Let $\ddot{U}$ be the universal set and $(\ddot{G},\ddot{A})$ be fermatean neutrosophic hypersoft set over $\ddot{U}$. Then $(\ddot{G},\ddot{A})^c$ is the complement of fermatean neutrosophic hypersoft set of $(\ddot{G},\ddot{A})$ if

$$T^c_{\gamma(\alpha)}(x) = F_{\gamma(\alpha)}(x)$$
$$I^c_{\gamma(\alpha)}(x) = 1\text{-}\ I_{\gamma(\alpha)}(x)$$
$$F^c_{\gamma(\alpha)}(x) = T_{\gamma(\alpha)}(x)$$

where $\forall\alpha \in \ddot{\tau}$ and $\forall$ x $\in \ddot{U}$. It is clear that $\left((\ddot{G},\ddot{A})^c\right)^c = (\ddot{G},\ddot{A})$.

Definition 3.5

1. A fermatean neutrosophic hypersoft set $(\ddot{G},\ddot{A})$ over the universe set $\ddot{U}$ is said to be null fermatean neutrosophic hypersoft set if $T_{\ddot{G}(\alpha)}(x) = 0$, $I_{\ddot{G}(\alpha)}(x) = 1$ and $F_{\ddot{G}(\alpha)}(x) = 1$ where $\forall\alpha \in \ddot{\tau}$ and $\forall x \in \ddot{U}$. It is denoted by $0_{(\ddot{U}_{FNH},\ddot{E})}$.

2. A fermatean neutrosophic hypersoft set $(\ddot{G},\ddot{A})$ over the universe set $\ddot{U}$ is said to be absolute fermatean neutrosophic hypersoft set if $T_{\ddot{G}(\alpha)}(x) = 1$, $I_{\ddot{G}(\alpha)}(x) = 0$ and $F_{\ddot{G}(\alpha)}(x) = 0$ where $\forall\alpha \in \ddot{\tau}$ and $\forall x \in \ddot{U}$. It is denoted by $1_{(\ddot{U}_{FNH},\ddot{E})}$.

Clearly, $0^c_{(\ddot{U}_{FNH},\ddot{E})} = 1_{(\ddot{U}_{FNH},\ddot{E})}$ and $1^c_{(\ddot{U}_{FNH},\ddot{E})} = 0_{(\ddot{U}_{FNH},\ddot{E})}$

Definition 3.6

Let $\ddot{U}$ be the universal set and $(\ddot{G}_1,\ddot{A}_1)$ and $(\ddot{G}_2,\ddot{A}_2)$ be fermatean neutrosophic hypersoft set over $\ddot{U}$. Extended union $(\ddot{G}_1,\ddot{A}_1) \cup (\ddot{G}_2,\ddot{A}_2)$ is defined as

$$T((\ddot{G}_1,\ddot{A}_1) \cup (\ddot{G}_2,\ddot{A}_2)) = \begin{cases} T_{\ddot{G}_1(\alpha)}(x), & \text{if } \alpha \in \ddot{A}_1 - \ddot{A}_2 \\ T_{\ddot{G}_2(\alpha)}(x), & \text{if } \alpha \in \ddot{A}_2 - \ddot{A}_1 \\ \max\{T_{G_1(\alpha)}(x), T_{\ddot{G}_2(\alpha)}(x)\}, & \text{if } \alpha \in \ddot{A}_1 \cap \ddot{A}_2 \end{cases}$$

$$I((\ddot{G}_1,\ddot{A}_1) \cup (\ddot{G}_2,\ddot{A}_2)) = \begin{cases} I_{\ddot{G}_1(\alpha)}(x), & \text{if } \alpha \in \ddot{A}_1 - \ddot{A}_2 \\ I_{\ddot{G}_2(\alpha)}(x), & \text{if } \alpha \in \ddot{A}_2 - \ddot{A}_1 \\ \min\{I_{\ddot{G}_1(\alpha)}(x), I_{\ddot{G}_2(\alpha)}(x)\}, & \text{if } \alpha \in \ddot{A}_1 \cap \ddot{A}_2 \end{cases}$$

$$F((\ddot{G}_1,\ddot{A}_1) \cup (\ddot{G}_2,\ddot{A}_2)) = \begin{cases} F_{\ddot{G}_1(\alpha)}(x), & \text{if } \alpha \in \ddot{A}_1 - \ddot{A}_2 \\ F_{\ddot{G}_2(\alpha)}(x), & \text{if } \alpha \in \ddot{A}_2 - \ddot{A}_1 \\ \min\{F_{\ddot{G}_1(\alpha)}(x), F_{\ddot{G}_2(\alpha)}(x)\}, & \text{if } \alpha \in \ddot{A}_1 \cap \ddot{A}_2 \end{cases}$$

Definition 3.7

Let $\ddot{U}$ be the universal set and $(\ddot{G}_1,\ddot{A}_1)$ and $(\ddot{G}_2,\ddot{A}_2)$ be fermatean neutrosophic hypersoft set over $\ddot{U}$. Extended intersection $(\ddot{G}_1,\ddot{A}_1) \cap (\ddot{G}_2,\ddot{A}_2)$ is defined as

$$T((\ddot{G}_1,\ddot{A}_1) \cap (\ddot{G}_2,\ddot{A}_2)) = \begin{cases} T_{\ddot{G}_1(\alpha)}(x), & \text{if } \alpha \in \ddot{A}_1 - \ddot{A}_2 \\ T_{\ddot{G}_2(\alpha)}(x), & \text{if } \alpha \in \ddot{A}_2 - \ddot{A}_1 \\ \min\{T_{\ddot{G}_1(\alpha)}(x), T_{\ddot{G}_2(\alpha)}(x)\}, & \text{if } \alpha \in \ddot{A}_1 \cap \ddot{A}_2 \end{cases}$$

$$I((\ddot{G}_1,\ddot{A}_1) \cap (\ddot{G}_2,\ddot{A}_2)) = \begin{cases} I_{\ddot{G}_1(\alpha)}(x), & \text{if } \alpha \in \ddot{A}_1 - \ddot{A}_2 \\ I_{\ddot{G}_2(\alpha)}(x), & \text{if } \alpha \in \ddot{A}_2 - \ddot{A}_1 \\ \max\{I_{\ddot{G}_1(\alpha)}(x), I_{\ddot{G}_2(\alpha)}(x)\}, & \text{if } \alpha \in \ddot{A}_1 \cap \ddot{A}_2 \end{cases}$$

$$F((\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_2,\ddot{A}_2)) = \begin{cases} F_{\ddot{G}_1(\alpha)}(x), & \text{if } \alpha \in \ddot{A}_1 - \ddot{A}_2 \\ F_{\ddot{G}_2(\alpha)}(x), & \text{if } \alpha \in \ddot{A}_2 - \ddot{A}_1 \\ \max\{F_{\ddot{G}_1(\alpha)}(x), F_{\ddot{G}_2(\alpha)}(x)\}, & \text{if } \alpha \in \ddot{A}_1 \cap \ddot{A}_2 \end{cases}$$

Definition 3.8

Let $\{(\ddot{G}_i,\ddot{A}_i)\}| i \in I$ be a family of fermatean neutrosophic hypersoft sets over the universe set Ü. Then

$$\cup_{i\in I}(\ddot{G}_i,\ddot{A}_i) = \left\{\left\langle x, \sup[T_{G_i(\alpha)}(x)]_{i\in I}, \inf[I_{G_i(\alpha)}(x)]_{i\in I}, \inf[F_{G_i(\alpha)}(x)]_{i\in I}\right\rangle | x \in \ddot{U}\right\}$$

$$\cap_{i\in I}(\ddot{G}_i,\ddot{A}_i) = \left\{\left\langle x, \inf[T_{G_i(\alpha)}(x)]_{i\in I}, \sup[I_{G_i(\alpha)}(x)]_{i\in I} \sup[F_{G_i(\alpha)}(x)]_{i\in I}\right\rangle | x \in \ddot{U}\right\}$$

Definition 3.9

Let Ü be the universal set and $(\ddot{G}_1,\ddot{A}_1)$ and $(\ddot{G}_2,\ddot{A}_2)$ be fermatean neutrosophic hypersoft set over Ü. The "AND" operator $(\ddot{G}_1,\ddot{A}_1)\wedge(\ddot{G}_2,\ddot{A}_2) = G(\ddot{A}_1\times\ddot{A}_2)$ is defined as

$$T((\ddot{G}_1,\ddot{A}_1)\wedge(\ddot{G}_2,\ddot{A}_2)) = \min\{T_{\ddot{G}_1}(x), T_{\ddot{G}_2}(x)\},$$
$$I((\ddot{G}_1,\ddot{A}_1)\wedge(\ddot{G}_2,\ddot{A}_2)) = \max\{I_{\ddot{G}_1}(x), I_{\ddot{G}_2}(x)\},$$
$$F((\ddot{G}_1,\ddot{A}_1)\wedge(\ddot{G}_2,\ddot{A}_2)) = \max\{F_{\ddot{G}_1}(x), F_{\ddot{G}_2}(x)\},$$

Definition 3.10

Let Ü be the universal set and $(\ddot{G}_1,\ddot{A}_1)$ and $(\ddot{G}_2,\ddot{A}_2)$ be fermatean neutrosophic hypersoft set over Ü. The "OR" operator $(\ddot{G}_1,\ddot{A}_1)\vee(\ddot{G}_2,\ddot{A}_2) = G(\ddot{A}_1\times\ddot{A}_2)$ is defined as

$$T((\ddot{G}_1,\ddot{A}_1)\vee(\ddot{G}_2,\ddot{A}_2)) = \max\{T_{\ddot{G}_1}(x), T_{\ddot{G}_2}(x)\},$$
$$I((\ddot{G}_1,\ddot{A}_1)\vee(\ddot{G}_2,\ddot{A}_2)) = \min\{I_{\ddot{G}_1}(x), I_{\ddot{G}_2}(x)\},$$
$$F((\ddot{G}_1,\ddot{A}_1)\vee(\ddot{G}_2,\ddot{A}_2)) = \min\{F_{\ddot{G}_1}(x), F_{\ddot{G}_2}(x)\}$$

Proposition 3.11

Let $(\ddot{G}_1,\ddot{A}_1)$, $(\ddot{G}_2,\ddot{A}_2)$ and $(\ddot{G}_3,\ddot{A}_3)$ be fermatean neutrosophic hypersoft set over the universe set Ü. Then

1. $(\ddot{G}_1,\ddot{A}_1)\cup[(\ddot{G}_2,\ddot{A}_2)\cup(\ddot{G}_3,\ddot{A}_3)] = [(\ddot{G}_1,\ddot{A}_1)\cup(\ddot{G}_2,\ddot{A}_2)]\cup(\ddot{G}_3,\ddot{A}_3)$ and
   $(\ddot{G}_1,\ddot{A}_1)\cap[(\ddot{G}_2,\ddot{A}_2)\cap(\ddot{G}_3,\ddot{A}_3)] = [(\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_2,\ddot{A}_2)]\cap(\ddot{G}_3,\ddot{A}_3)$
2. $(\ddot{G}_1,\ddot{A}_1)\cup[(\ddot{G}_2,\ddot{A}_2)\cap(\ddot{G}_3,\ddot{A}_3)] = [(\ddot{G}_1,\ddot{A}_1)\cup(\ddot{G}_2,\ddot{A}_2)]\cap[(\ddot{G}_1,\ddot{A}_1)\cup(\ddot{G}_3,\ddot{A}_3)]$ and
   $(\ddot{G}_1,\ddot{A}_1)\cap[(\ddot{G}_2,\ddot{A}_2)\cup(\ddot{G}_3,\ddot{A}_3)] = [(\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_2,\ddot{A}_2)]\cup[(\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_3,\ddot{A}_3)]$
3. $(\ddot{G}_1,\ddot{A}_1)\cup 0_{(\ddot{U}_{FNH},\ddot{E})} = (\ddot{G}_1,\ddot{A}_1)$ and $(\ddot{G}_1,\ddot{A}_1)\cap 0_{(\ddot{U}_{FNH},\ddot{E})} = 0_{(\ddot{U}_{FNH},\ddot{E})}$
4. $(\ddot{G}_1,\ddot{A}_1)\cup 1_{(\ddot{U}_{FNH},\ddot{E})} = 1_{(\ddot{U}_{FNH},\ddot{E})}$ and $(\ddot{G}_1,\ddot{A}_1)\cap 1_{(\ddot{U}_{FNH},\ddot{E})} = (\ddot{G}_1,\ddot{A}_1)$

Proof. Straightforward

Proposition 3.12

Let $(\ddot{G}_1,\ddot{A}_1)$ and $(\ddot{G}_2,\ddot{A}_2)$ be two fermatean neutrosophic hypersoft sets over the universe set Ü. Then

1. $[(\ddot{G}_1,\ddot{A}_1)\cup(\ddot{G}_2,\ddot{A}_2)]^c = (\ddot{G}_1,\ddot{A}_1)^c\cap(\ddot{G}_2,\ddot{A}_1)^c$;
2. $[(\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_2,\ddot{A}_2)]^c = (\ddot{G}_1,\ddot{A}_1)^c\cup(\ddot{G}_2,\ddot{A}_1)^c$;

Proof. 1. for all x∈ Ü,

$$(\ddot{G}_1,\ddot{A}_1)\cup(\ddot{G}_2,\ddot{A}_2) = \left\{\left\langle \begin{matrix} x, \max\{T_{\ddot{G}_1(\alpha)}(x), T_{\ddot{G}_2(\alpha)}(x)\}, \min\{I_{\ddot{G}_1(\alpha)}(x), I_{\ddot{G}_2(\alpha)}(x)\} \\ \min\{F_{\ddot{G}_1(\alpha)}(x), F_{\ddot{G}_2(\alpha)}(x)\} \end{matrix}\right\rangle\right\},$$

$$[(\ddot{G}_1,\ddot{A}_1)\cup(\ddot{G}_2,\ddot{A}_2)]^c = \left\{\left\langle \begin{matrix} x, \min\{T_{\ddot{G}_1(\alpha)}(x), T_{\ddot{G}_2(\alpha)}(x)\}, 1-\min\{I_{\ddot{G}_1(\alpha)}(x), I_{\ddot{G}_2(\alpha)}(x)\} \\ \max\{F_{\ddot{G}_1(\alpha)}(x), F_{\ddot{G}_2(\alpha)}(x)\} \end{matrix}\right\rangle\right\}$$

Now,

$(\ddot{G}_1,\ddot{A}_1)^c = \{\langle x, F_{\ddot{G}_1(\alpha)}(x), 1 - I_{\ddot{G}_1(\alpha)}(x), T_{\ddot{G}_1(\alpha)}(x)\rangle\}$;

$(\ddot{G}_2,\ddot{A}_2)^c = \{\langle x, F_{\ddot{G}_2(\alpha)}(x), 1 - I_{\ddot{G}_2(\alpha)}(x), T_{\ddot{G}_2(\alpha)}(x)\rangle\}$;

Then, $$(\ddot{G}_1,\ddot{A}_1)^c\cap(\ddot{G}_2,\ddot{A}_2)^c = \left\{\left\langle \begin{matrix} x, \min\{F_{\ddot{G}_1(\alpha)}(x), F_{\ddot{G}_2(\alpha)}(x)\}, \max\{(1-I_{\ddot{G}_1(\alpha)}(x)), (1-I_{\ddot{G}_2(\alpha)}(x))\} \\ \max\{T_{\ddot{G}_1(\alpha)}(x), T_{\ddot{G}_2(\alpha)}(x)\} \end{matrix}\right\rangle\right\}$$

$$= \left\{ \left\langle \begin{matrix} x, \min\{F_{\ddot{G}_1(\alpha)}(x), F_{\ddot{G}_2(\alpha)}(x)\}, 1 - \min\{I_{\ddot{G}_1(\alpha)}(x), I_{\ddot{G}_2(\alpha)}(x)\}, \\ \max\{T_{\ddot{G}_1(\alpha)}(x), T_{\ddot{G}_2(\alpha)}(x)\} \end{matrix} \right\rangle \right\}$$

Therefore, $[(\ddot{G}_1, \ddot{A}_1) \cup (\ddot{G}_2, \ddot{A}_2)]^c = (\ddot{G}_1, \ddot{A}_1)^c \cap (\ddot{G}_2, \ddot{A}_2)^c$.

2. It is obtained similarly.

Proposition 3.13

Let $(\ddot{G}_1, \ddot{A}_1)$ and $(\ddot{G}_2, \ddot{A}_1)$ be fermatean neutrosophic hypersoft sets over the universe set $\ddot{U}$. Then

1. $[(\ddot{G}_1, \ddot{A}_1) \vee (\ddot{G}_2, \ddot{A}_2)]^c = (\ddot{G}_1, \ddot{A}_1)^c \wedge (\ddot{G}_2, \ddot{A}_2)^c$;
2. $[(\ddot{G}_1, \ddot{A}_1) \wedge (\ddot{G}_2, \ddot{A}_2)]^c = (\ddot{G}_1, \ddot{A}_1)^c \vee (\ddot{G}_2, \ddot{A}_2)^c$;

Proof. 1. for all $x \in \ddot{U}$,

$$(\ddot{G}_1, \ddot{A}_1) \vee (\ddot{G}_2, \ddot{A}_2) = \left\{ \left\langle \begin{matrix} x, \max\{T_{\ddot{G}_1(\alpha)}(x), T_{\ddot{G}_2(\alpha)}(x)\}, \min\{I_{\ddot{G}_1(\alpha)}(x), I_{\ddot{G}_2(\alpha)}(x)\} \\ \min\{F_{\ddot{G}_1(\alpha)}(x), F_{\ddot{G}_2(\alpha)}(x)\} \end{matrix} \right\rangle \right\}$$

$$[(\ddot{G}_1, \ddot{A}_1) \vee (\ddot{G}_2, \ddot{A}_2)]^c = \left\{ \left\langle \begin{matrix} x, \min\{F_{\ddot{G}_1(\alpha)}(x), F_{\ddot{G}_2(\alpha)}(x)\}, 1 - \min\{I_{\ddot{G}_1(\alpha)}(x), I_{\ddot{G}_2(\alpha)}(x)\} \\ \max\{T_{\ddot{G}_1(\alpha)}(x), T_{\ddot{G}_2(\alpha)}(x)\} \end{matrix} \right\rangle \right\}$$

On the other hand,

$(\ddot{G}_1, \ddot{A}_1)^c = \{\langle x, F_{\ddot{G}_1(\alpha)}(x), 1 - I_{\ddot{G}_1(\alpha)}(x), T_{\ddot{G}_1(\alpha)}(x)\rangle\}$

$(\ddot{G}_2, \ddot{A}_1)^c = \{\langle x, F_{\ddot{G}_2(\alpha)}(x), 1 - I_{\ddot{G}_1(\alpha)}(x), T_{\ddot{G}_2(\alpha)}(x)\rangle\}$

Then,

$$(\ddot{G}_1, \ddot{A}_1)^c \cap (\ddot{G}_2, \ddot{A}_2)^c = \left\{ \left\langle \begin{matrix} x, \min\{F_{\ddot{G}_1(\alpha)}(x), F_{\ddot{G}_2(\alpha)}(x)\}, \max\{(1 - I_{\ddot{G}_1(\alpha)}(x)), (1 - I_{\ddot{G}_2(\alpha)}(x))\} \\ \max\{T_{\ddot{G}_1(\alpha)}(x), T_{\ddot{G}_2(\alpha)}(x)\} \end{matrix} \right\rangle \right\}$$

$$= \left\{ \left\langle \begin{matrix} x, \min\{F_{\ddot{G}_1(\alpha)}(x), F_{\ddot{G}_2(\alpha)}(x)\}, 1 - \min\{\hat{I}_{\ddot{G}_1(\alpha)}(x), \hat{I}_{\ddot{G}_2(\alpha)}(x)\} \\ \max\{T_{\ddot{G}_1(\alpha)}(x), T_{\ddot{G}_2(\alpha)}(x)\} \end{matrix} \right\rangle \right\}$$

Hence, $[(\ddot{G}_1, \ddot{A}_1) \vee (\ddot{G}_2, \ddot{A}_1)]^c = (\ddot{G}_1, \ddot{A}_1)^c \wedge (\ddot{G}_2, \ddot{A}_1)^c$.

Example 3.14

Let $\ddot{U} = \{x_1, x_2, x_3\}$ be an initial universe and $\ddot{E}_1, \ddot{E}_2, \ddot{E}_3$ be sets of attributes. Attributes are given as;

$$\ddot{E}_1 = \{\alpha_1, \alpha_2, \alpha_3\},\ \ddot{E}_2 = \{\beta_1, \beta_2\},\ \ddot{E}_1 = \{\gamma_1, \gamma_2\}$$

Suppose that

$$\ddot{B}_1 = \{\alpha_1\},\ \ddot{B}_2 = \{\beta_1, \beta_2\},\ \ddot{B}_3 = \{\gamma_1, \gamma_2\}$$
$$\ddot{C}_1 = \{\alpha_1, \alpha_3\},\ \ddot{C}_2 = \{\beta_1, \beta_2\},\ \ddot{C}_3 = \{\gamma_2\}$$

are subset of $\ddot{E}_i$ for each i =1, 2, 3. Then the fermatean neutrosophic hypersoft sets $(\ddot{G}_1, \ddot{A}_1)$ and $(\ddot{G}_2, \ddot{A}_2)$ over the universe $\ddot{U}$ as follows.

$$(\ddot{G}_1, \ddot{A}_1) = \left\{ \begin{matrix} \left\langle (\alpha_1, \beta_1, \gamma_1), \left\{ \frac{x_1}{[0.3,0.6,0.5]}, \frac{x_2}{[0.7,0.6,0.8]}, \frac{x_3}{[0.4,0.5,0.6]} \right\} \right\rangle, \\ \left\langle (\alpha_1, \beta_1, \gamma_2), \left\{ \frac{x_1}{[0.6,0.5,0.7]}, \frac{x_2}{[0.5,0.3,0.2]}, \frac{x_3}{[0.7,0.8,0.5]} \right\} \right\rangle, \\ \left\langle (\alpha_1, \beta_2, \gamma_1), \left\{ \frac{x_1}{[0.3,0.2,0.6]}, \frac{x_2}{[0.6,0.2,0.8]}, \frac{x_3}{[0.6,0.9,0.3]} \right\} \right\rangle, \\ \left\langle (\alpha_1, \beta_2, \gamma_2), \left\{ \frac{x_1}{[0.5,0.4,0.3]}, \frac{x_2}{[0.5,0.2,0.9]}, \frac{x_3}{[0.1,0.4,0.6]} \right\} \right\rangle, \end{matrix} \right\}$$

$$(\ddot{G}_2, \ddot{A}_2) = \left\{ \begin{matrix} \left\langle (\alpha_1, \beta_1, \gamma_2), \left\{ \frac{x_1}{[0.1,0.8,0.8]}, \frac{x_2}{[0.4,0.8,0.6]}, \frac{x_3}{[0.9,0.6,0.5]} \right\} \right\rangle, \\ \left\langle (\alpha_1, \beta_2, \gamma_2), \left\{ \frac{x_1}{[0.4,0.7,0.6]}, \frac{x_2}{[0.5,0.1,0.3]}, \frac{x_3}{[0.1,0.7,0.7]} \right\} \right\rangle, \\ \left\langle (\alpha_3, \beta_1, \gamma_2), \left\{ \frac{x_1}{[0.7,0.8,0.4]}, \frac{x_2}{[0.2,0.8,0.4]}, \frac{x_3}{[0.1,0.7,0.5]} \right\} \right\rangle, \\ \left\langle (\alpha_3, \beta_2, \gamma_2), \left\{ \frac{x_1}{[0.4,0.8,0.3]}, \frac{x_2}{[0.2,0.4,0.6]}, \frac{x_3}{[0.3,0.1,0.6]} \right\} \right\rangle, \end{matrix} \right\}$$

The union, intersection operation of these sets are follows:

$$(\ddot{G}_1,\ddot{A}_1)\cup(\ddot{G}_2,\ddot{A}_2)=\left\{\begin{array}{l}\left\langle(\alpha_1,\beta_1,\gamma_1),\left\{\frac{x_1}{[0.3,0.6,0.5]},\frac{x_2}{[0.7,0.6,0.8]},\frac{x_3}{[0.4,0.5,0.6]}\right\}\right\rangle,\\ \left\langle(\alpha_1,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.6,0.5,0.7]},\frac{x_2}{[0.5,0.3,0.2]},\frac{x_3}{[0.9,0.6,0.5]}\right\}\right\rangle,\\ \left\langle(\alpha_1,\beta_2,\gamma_1),\left\{\frac{x_1}{[0.3,0.2,0.6]},\frac{x_2}{[0.6,0.2,0.8]},\frac{x_3}{[0.6,0.9,0.3]}\right\}\right\rangle,\\ \left\langle(\alpha_1,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.5,0.4,0.3]},\frac{x_2}{[0.5,0.1,0.3]},\frac{x_3}{[0.1,0.4,0.6]}\right\}\right\rangle,\\ \left\langle(\alpha_3,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.7,0.8,0.4]},\frac{x_2}{[0.2,0.8,0.4]},\frac{x_3}{[0.1,0.7,0.5]}\right\}\right\rangle,\\ \left\langle(\alpha_3,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.4,0.8,0.3]},\frac{x_2}{[0.2,0.4,0.6]},\frac{x_3}{[0.3,0.1,0.6]}\right\}\right\rangle\end{array}\right\}$$

$$(\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_2,\ddot{A}_2)=\left\{\begin{array}{l}\left\langle(\alpha_1,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.1,0.8,0.8]},\frac{x_2}{[0.4,0.8,0.6]},\frac{x_3}{[0.7,0.8,0.5]}\right\}\right\rangle,\\ \left\langle(\alpha_1,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.4,0.7,0.6]},\frac{x_2}{[0.5,0.2,0.9]},\frac{x_3}{[0.1,0.7,0.7]}\right\}\right\rangle\end{array}\right\}$$

IV. Fermatean Neutrosophic Hypersoft Topological Spaces

Definition 4.1 Let $FNHSS(\ddot{U},\ddot{E})$ be the family of all fermatean neutrosophic hypersoft sets over the universe set $\ddot{U}$ and $\ddot{\tau}\subseteq FNHSS(\ddot{U},\ddot{E})$. Then $\ddot{\tau}$ is said to be a fermatean neutrosophic hypersoft topology on $\ddot{U}$ if

1. $0_{(\ddot{U}_{FNH},\ddot{E})}, 1_{(\ddot{U}_{FNH},\ddot{E})}$ belongs to $\ddot{\tau}$
2. the union of any number of fermatean neutrosophic hypersoft sets in $\ddot{\tau}$ belongs to $\ddot{\tau}$
3. the intersection of finite number of fermatean neutrosophic hypersoft sets in $\ddot{\tau}$ belongs to $\ddot{\tau}$.

Then $(\ddot{U},\ddot{E},\ddot{\tau})$ is said to be a fermatean neutrosophic hypersoft topological space over $\ddot{U}$. Each members of $\ddot{\tau}$ is said to be fermatean neutrosophic hypersoft open set.

Definition 4.2 Let $(\ddot{U},\ddot{E},\ddot{\tau})$ be a fermatean neutrosophic hypersoft topological space over $\ddot{U}$ and $(\ddot{G},\ddot{A})$ be a fermatean neutrosophic hypersoft set over $\ddot{U}$. Then $(\ddot{G},\ddot{A})$ is said to be fermatean neutrosophic hypersoft closed set iff its complement is a fermatean neutrosophic hypersoft open set.

Proposition 4.3 Let $(\ddot{U},\ddot{E},\ddot{\tau})$ be a fermatean neutrosophic hypersoft topological space over $\ddot{U}$. Then

1. $0_{(\ddot{U}_{FNH},\ddot{E})}, 1_{(\ddot{U}_{FNH},\ddot{E})}$ are fermatean neutrosophic hypersoft closed sets over $\ddot{U}$.
2. The intersection of any number of fermatean neutrosophic hypersoft closed sets is a fermatean neutrosophic hypersoft closed set over $\ddot{U}$.
3. The union of finite number of fermatean neutrosophic hypersoft closed sets is a fermatean neutrosophic hypersoft closed set over $\ddot{U}$.

Proof.

It is clear that the definition of fermatean neutrosophic hypersoft topological space.

Definition 4.4 Let $FNHSS(\ddot{U},\ddot{E})$ be the family of all fermatean neutrosophic hypersoft sets over the universe set $\ddot{U}$.

1. If $\ddot{\tau}=\{0_{(\ddot{U}_{FNH},\ddot{E})}, 1_{(\ddot{U}_{FNH},\ddot{E})}\}$ then $\ddot{\tau}$ is said to be the fermatean neutrosophic hypersoft indiscrete topology and $(\ddot{U},\ddot{E},\ddot{\tau})$ is said to be a fermatean neutrosophic hypersoft indiscrete topological space over $\ddot{U}$.
2. If $\ddot{\tau}=FNHSS(\ddot{U},\ddot{E})$ then $\ddot{\tau}$ is said to be the fermatean neutrosophic hypersoft discrete topology and $(\ddot{U},\ddot{E},\ddot{\tau})$ is said to be a fermatean neutrosophic hypersoft discrete topological space over $\ddot{U}$.

Proposition 4.5 Let $(\ddot{U},\ddot{E},\ddot{\tau}_1)$ and $(\ddot{U},\ddot{E},\ddot{\tau}_2)$ be two fermatean neutrosophic hypersoft topological spaces over the same universe set. Then $(\ddot{U},\ddot{E},\ddot{\tau}_1\cap\ddot{\tau}_2)$ is fermatean neutrosophic hypersoft topological space over $\ddot{U}$.

Proof.

1. Since $0_{(\ddot{U}_{FNH},\ddot{E})}, 1_{(\ddot{U}_{FNH},\ddot{E})}\in\ddot{\tau}_1$ and $0_{(\ddot{U}_{FNH},\ddot{E})}, 1_{(\ddot{U}_{FNH},\ddot{E})}\in\ddot{\tau}_2$ then $0_{(\ddot{U}_{FNH},\ddot{E})}, 1_{(\ddot{U}_{FNH},\ddot{E})}\in\ddot{\tau}_1\cap\ddot{\tau}_2$.
2. Suppose that $\{(G_i,A_i)|\, i\in I\}$ be a family of fermatean neutrosophic hypersoft sets in $\ddot{\tau}_1\cap\ddot{\tau}_2$. Then $(G_i,A_i)\in\ddot{\tau}_1$ and $(G_i,A_i)\in\ddot{\tau}_2$ for all $i\in I$, so $\cup_{i\in I}(G_i,A_i)\in\ddot{\tau}_1$ and $\cup_{i\in I}(G_i,A_i)\in\ddot{\tau}_2$. Thus, $\cup_{i\in I}(G_i,A_i)\in\ddot{\tau}_1\cap\ddot{\tau}_2$.
3. Let $\{(G_i,A_i)|\, i=\overline{1,k}$ be a family of the finite number of fermatean neutrosophic hypersoft sets in $\ddot{\tau}_1\cap\ddot{\tau}_2$. Then $(G_i,A_i)\in\ddot{\tau}_1$ and $(G_i,A_i)\in\ddot{\tau}_2$ for $i=\overline{1,k}$, so $\cap_{i\in I}^{n}(G_i,A_i)\in\ddot{\tau}_1$ and $\cap_{i\in I}^{n}(G_i,A_i)\in\ddot{\tau}_2$. Thus $\cap_{i\in I}^{n}(G_i,A_i)\in\ddot{\tau}_1\cap\ddot{\tau}_2$.

Remark 4.6 The union of two fermatean neutrosophic hypersoft topologies over $\ddot{U}$ may not be a fermatean neutrosophic hypersoft topology on $\ddot{U}$.

Example 4.7 Let $\ddot{U}=\{x_1,x_2,x_3\}$ be an initial universe and $\ddot{E}_1,\ddot{E}_2,\ddot{E}_3$ be sets of attributes. Attributes are given as;

$$\ddot{E}_1=\{\alpha_1,\alpha_2,\alpha_3\},\ \ddot{E}_2=\{\beta_1,\beta_2\},\ \ddot{E}_3=\{\gamma_1,\gamma_2\}$$

Suppose that

$$\ddot{B}_1=\{\alpha_1\},\ \ddot{B}_2=\{\beta_1,\beta_2\},\ \ddot{B}_3=\{\gamma_1,\gamma_2\}$$
$$\ddot{C}_1=\{\alpha_1,\alpha_3\},\ \ddot{C}_2=\{\beta_1,\beta_2\},\ \ddot{C}_3=\{\gamma_2\}$$

are subset of $\ddot{E}_i$ for each i=1,2,3.

$$\ddot{\tau}_1=\{0_{(\ddot{U}_{FNH},\ddot{E})}, 1_{(\ddot{U}_{FNH},\ddot{E})}, (\ddot{G}_1,\ddot{A}_1), (\ddot{G}_2,\ddot{A}_2)\}\text{ and}$$
$$\ddot{\tau}_2=\{0_{(\ddot{U}_{FNH},\ddot{E})}, 1_{(\ddot{U}_{FNH},\ddot{E})}, (\ddot{G}_3,\ddot{A}_3), (\ddot{G}_4,\ddot{A}_4)\}$$

be two fermatean neutrosophic hypersoft topologies over Ü. Here, the fermatean neutrosophic hypersoft sets $(\ddot{G}_1,\ddot{A}_1),(\ddot{G}_2,\ddot{A}_2)(\ddot{G}_3,\ddot{A}_3)$ and $(\ddot{G}_4,\ddot{A}_4)$ over Ü are defined as following:

$$(\ddot{G}_1,\ddot{A}_1)=\left\{\begin{array}{l}\langle(\alpha_1,\beta_1,\gamma_1),\left\{\frac{x_1}{[0.1,0.7,0.8]},\frac{x_2}{[0.4,0.6,0.9]},\frac{x_3}{[0.2,0.6,0.5]}\right\}\rangle,\\ \langle(\alpha_1,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.3,0.7,0.6]},\frac{x_2}{[0.5,0.5,0.9]},\frac{x_3}{[0.7,0.5,0.7]}\right\}\rangle,\\ \langle(\alpha_1,\beta_2,\gamma_1),\left\{\frac{x_1}{[0.5,0.6,0.8]},\frac{x_2}{[0.7,0.4,0.5]},\frac{x_3}{[0.7,0.8,0.6]}\right\}\rangle,\\ \langle(\alpha_1,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.6,0.9,0.8]},\frac{x_2}{[0.3,0.9,0.8]},\frac{x_3}{[0.2,0.9,0.6]}\right\}\rangle,\end{array}\right\}$$

$$(\ddot{G}_2,\ddot{A}_2)=\left\{\begin{array}{l}\langle(\alpha_1,\beta_1,\gamma_1),\left\{\frac{x_1}{[0.3,0.6,0.6]},\frac{x_2}{[0.7,0.6,0.5]},\frac{x_3}{[0.4,0.5,0.2]}\right\}\rangle,\\ \langle(\alpha_1,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.6,0.5,0.4]},\frac{x_2}{[0.7,0.2,0.8]},\frac{x_3}{[0.8,0.4,0.3]}\right\}\rangle,\\ \langle(\alpha_1,\beta_2,\gamma_1),\left\{\frac{x_1}{[0.5,0.4,0.3]},\frac{x_2}{[0.8,0.2,0.3]},\frac{x_3}{[0.8,0.7,0.5]}\right\}\rangle,\\ \langle(\alpha_1,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.9,0.7,0.6]},\frac{x_2}{[0.5,0.7,0.7]},\frac{x_3}{[0.4,0.7,0.5]}\right\}\rangle,\end{array}\right\}$$

$$(\ddot{G}_3,\ddot{A}_3)=\left\{\begin{array}{l}\langle(\alpha_2,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.1,0.6,0.9]},\frac{x_2}{[0.3,0.7,0.8]},\frac{x_3}{[0.3,0.9,0.7]}\right\}\rangle,\\ \langle(\alpha_2,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.2,0.7,0.8]},\frac{x_2}{[0.8,0.3,0.6]},\frac{x_3}{[0.4,0.9,0.6]}\right\}\rangle,\\ \langle(\alpha_3,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.7,0.5,0.6]},\frac{x_2}{[0.5,0.6,0.7]},\frac{x_3}{[0.2,0.3,0.8]}\right\}\rangle,\\ \langle(\alpha_3,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.7,0.8,0.5]},\frac{x_2}{[0.4,0.7,0.9]},\frac{x_3}{[0.3,0.8,0.8]}\right\}\rangle,\end{array}\right\}$$

$$(\ddot{G}_4,\ddot{A}_4)=\left\{\begin{array}{l}\langle(\alpha_2,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.1,0.5,0.6]},\frac{x_2}{[0.4,0.6,0.5]},\frac{x_3}{[0.3,0.6,0.4]}\right\}\rangle,\\ \langle(\alpha_2,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.3,0.5,0.4]},\frac{x_2}{[0.8,0.1,0.3]},\frac{x_3}{[0.5,0.4,0.1]}\right\}\rangle,\\ \langle(\alpha_3,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.8,0.4,0.3]},\frac{x_2}{[0.6,0.5,0.2]},\frac{x_3}{[0.7,0.2,0.8]}\right\}\rangle,\\ \langle(\alpha_3,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.8,0.7,0.4]},\frac{x_2}{[0.7,0.4,0.3]},\frac{x_3}{[0.6,0.7,0.8]}\right\}\rangle,\end{array}\right\}$$

Since $(\ddot{G}_1,\ddot{A}_1)\cup(\ddot{G}_4,\ddot{A}_4)\notin\ddot{\tau}_1\cup\ddot{\tau}_2$, then $\ddot{\tau}_1\cup\ddot{\tau}_2$ is not a fermatean neutrosophic hypersoft topology over Ü.

Definition 4.8 Let $(\ddot{U},\ddot{E},\ddot{\tau})$ be a fermatean neutrosophic hypersoft topological space over Ü and $(\ddot{G},\ddot{B})\in FNHSS(\ddot{U},\ddot{E})$ be a fermatean neutrosophic hypersoft set. Then, the fermatean neutrosophic hypersoft interior of $(\ddot{G},\ddot{B})$, denoted $FNHSint(\ddot{G},\ddot{B})$, is defined as the fermatean neutrosophic hypersoft union of all fermatean neutrosophic hypersoft open subsets of $(\ddot{G},\ddot{B})$.

Clearly, $FNHSint(\ddot{G},\ddot{B})$ is the biggest fermatean neutrosophic hypersoft open set that is contained by $(\ddot{G},\ddot{B})$.

Example 4.9 Let us consider the fermatean neutrosophic hypersoft topology $\ddot{\tau}_1$ given in Example 4.7. Suppose that any $(\ddot{G},\ddot{A})\in FNHSS(\ddot{U},\ddot{E})$ is defined as following:

$$\ddot{\tau}_1=\{0_{(\ddot{U}_{FNH},\ddot{E})},1_{(\ddot{U}_{FNH},\ddot{E})},(\ddot{G}_1,\ddot{A}_1),(\ddot{G}_2,\ddot{A}_2)\}$$

be a fermatean neutrosophic hypersoft topologies over Ü. Here, the fermatean neutrosophic hypersoft sets $(\ddot{G}_1,\ddot{A}_1),(\ddot{G}_2,\ddot{A}_2)$ over Ü are defined as following:

$$(\ddot{G}_1,\ddot{A}_1)=\left\{\begin{array}{l}\langle(\alpha_1,\beta_1,\gamma_1),\left\{\frac{x_1}{[0.1,0.7,0.8]},\frac{x_2}{[0.4,0.6,0.9]},\frac{x_3}{[0.2,0.6,0.5]}\right\}\rangle,\\ \langle(\alpha_1,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.3,0.7,0.6]},\frac{x_2}{[0.5,0.5,0.9]},\frac{x_3}{[0.7,0.5,0.7]}\right\}\rangle,\\ \langle(\alpha_1,\beta_2,\gamma_1),\left\{\frac{x_1}{[0.5,0.6,0.8]},\frac{x_2}{[0.7,0.4,0.5]},\frac{x_3}{[0.7,0.8,0.6]}\right\}\rangle,\\ \langle(\alpha_1,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.6,0.9,0.8]},\frac{x_2}{[0.3,0.9,0.8]},\frac{x_3}{[0.2,0.9,0.6]}\right\}\rangle,\end{array}\right\}$$

$$(\ddot{G}_2,\ddot{A}_2)=\left\{\begin{array}{l}\langle(\alpha_1,\beta_1,\gamma_1),\left\{\frac{x_1}{[0.3,0.6,0.6]},\frac{x_2}{[0.7,0.6,0.5]},\frac{x_3}{[0.4,0.5,0.2]}\right\}\rangle,\\ \langle(\alpha_1,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.6,0.5,0.4]},\frac{x_2}{[0.7,0.2,0.8]},\frac{x_3}{[0.8,0.4,0.3]}\right\}\rangle,\\ \langle(\alpha_1,\beta_2,\gamma_1),\left\{\frac{x_1}{[0.5,0.4,0.3]},\frac{x_2}{[0.8,0.2,0.3]},\frac{x_3}{[0.8,0.7,0.5]}\right\}\rangle,\\ \langle(\alpha_1,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.9,0.7,0.6]},\frac{x_2}{[0.5,0.7,0.7]},\frac{x_3}{[0.4,0.7,0.5]}\right\}\rangle,\end{array}\right\}$$

$$(\ddot{G},\ddot{A})=\left\{\begin{array}{l}\langle(\alpha_1,\beta_1,\gamma_1),\left\{\frac{x_1}{[0.4,0.5,0.3]},\frac{x_2}{[0.8,0.4,0.4]},\frac{x_3}{[0.5,0.4,0.1]}\right\}\rangle,\\ \langle(\alpha_1,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.7,0.3,0.2]},\frac{x_2}{[0.7,0.1,0.6]},\frac{x_3}{[0.9,0.2,0.2]}\right\}\rangle,\\ \langle(\alpha_1,\beta_2,\gamma_1),\left\{\frac{x_1}{[0.6,0.2,0.2]},\frac{x_2}{[0.8,0.1,0.2]},\frac{x_3}{[0.8,0.3,0.4]}\right\}\rangle,\\ \langle(\alpha_1,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.9,0.5,0.6]},\frac{x_2}{[0.6,0.5,0.6]},\frac{x_3}{[0.5,0.6,0.5]}\right\}\rangle,\end{array}\right\}$$

Then, $0_{(\ddot{U}_{FNH},\ddot{\Sigma})}$, $(\ddot{G}_1,\ddot{A}_1)$ $(\ddot{G}_2,\ddot{A}_2)\subseteq(\ddot{G},\ddot{A})$.

Therefore, $\text{FNHSint}(\ddot{G},\ddot{A})=0_{(\ddot{U}_{FNH},\ddot{E})}\cup(\ddot{G}_1,\ddot{A}_1)\cup(\ddot{G}_2,\ddot{A}_2)=(\ddot{G}_2,\ddot{A}_2)$.

Theorem 4.10 Let $(\ddot{U},\ddot{E},\ddot{\tau})$ be a fermatean neutrosophic hypersoft topological space over $\ddot{U}$ and $(\ddot{G},\ddot{B})\in \text{FNHSS}(\ddot{U},\dot{E})$. $(\ddot{G},B)$ is a fermatean neutrosophic hypersoft open set iff $(\ddot{G},\ddot{B})=\text{FNHSint}(\ddot{G},\ddot{B})$

Proof.

Let $(\ddot{G},\ddot{B})$ be a fermatean neutrosophic hypersoft open set. Then the biggest fermatean neutrosophic hypersoft open set that is contained by $(\ddot{G},\ddot{B})$ is equal to $(\ddot{G},\ddot{B})$. Hence, $(\ddot{G},\ddot{B})=\text{FNHSint}(\ddot{G},\ddot{B})$.

Conversely, it is known that $\text{FNHSint}(\ddot{G},\ddot{B})$ is a fermatean neutrosophic hypersoft open set and if $(\ddot{G},\ddot{B})=\text{FNHSint}(\ddot{G},\ddot{B})$, then $(\ddot{G},\ddot{B})$ is a fermatean neutrosophic hypersoft open set.

Theorem 4.11 Let $(\ddot{U},\ddot{E},\ddot{\tau})$ be a fermatean neutrosophic hypersoft topological space over $\ddot{U}$ and $(\ddot{G}_1,\ddot{A}_1),(\ddot{G}_2,\ddot{A}_2)\in \text{FNHSS}(\ddot{U},\ddot{E})$. Then,

1. $\text{FNHSint}(\text{FNHSint}(\ddot{G}_1,\ddot{A}_1))=\text{FNHSint}(\ddot{G}_1,\ddot{A}_1)$,
2. $\text{FNHSint}(0_{(\ddot{U}_{FNHS},\ddot{E})})=0_{(\ddot{U}_{FNHS},\ddot{E})}$ and $\text{FNHSint}(1_{(\ddot{U}_{FNHS},\ddot{E})})=1_{(\ddot{U}_{FNHS},\ddot{E})}$,
3. $(\ddot{G}_1,\ddot{A}_1)\subseteq(\ddot{G}_2,\ddot{A}_2)\Rightarrow \text{FNHSint}(\ddot{G}_1,\ddot{A}_1)\subseteq \text{FNHSint}(\ddot{G}_2,\ddot{A}_2)$,
4. $\text{FNHSint}[(\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_2,\ddot{A}_2)]=\text{FNHSint}(\ddot{G}_1,\ddot{A}_1)\cap \text{FNHSint}(\ddot{G}_2,\ddot{A}_2)$,
5. $\text{FNHSint}(\ddot{G}_1,\ddot{A}_1)\cup \text{FNHSint}(\ddot{G}_2,\ddot{A}_2)\subseteq \text{FNHSint}[(\ddot{G}_1,\ddot{A}_1)\cup(\ddot{G}_2,\ddot{A}_2)]$

Proof.

1. Let $\text{FNHSint}(\ddot{G}_1,\ddot{A}_1)=(\ddot{G}_2,\ddot{A}_2)$. Then $(\ddot{G}_2,\ddot{A}_2)\in\ddot{\tau}$ iff $(\ddot{G}_2,\ddot{A}_2)=\text{FNHSSint}(\ddot{G}_2,\ddot{A}_2)$. So, $\text{FNHSint}[\text{FNHSint}(\ddot{G}_1,\ddot{A}_1)]=\text{FNHSint}(\ddot{G}_1,\ddot{A}_1)$.

2. Straightforward.

3. It is known that, $\text{FNHSint}(\ddot{G}_1,\ddot{A}_1)\subseteq(\ddot{G}_1,\ddot{A}_1)\subseteq(\ddot{G}_2,\ddot{A}_2)$ and $\text{FNHSint}(\ddot{G}_2,\ddot{A}_2)\subseteq(\ddot{G}_2,\ddot{A}_2)$. Since $\text{FNHSint}(\ddot{G}_2,\ddot{A}_2)$ is the biggest fermatean neutrosophic hypersoft open set contained in $(\ddot{G}_2,\ddot{A}_2)$ and so, $\text{FNHSint}(\ddot{G}_1,\ddot{A}_1)\subseteq \text{FNHSint}(\ddot{G}_2,\ddot{A}_2)$.

4. Since $(\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_2,\ddot{A}_2)\subseteq(\ddot{G}_1,\ddot{A}_1)$ and $(\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_2,\ddot{A}_2)\subseteq(\ddot{G}_2,\ddot{A}_2)$, then $\text{FNHSint}[(\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_2,\ddot{A}_2)]\subseteq \text{FNHSint}(\ddot{G}_1,\ddot{A}_1)$ and $\text{FNHSint}[(\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_2,\ddot{A}_2)]\subseteq \text{FNHSint}(\ddot{G}_2,\ddot{A}_2)$ and so, $\text{FNHSint}[(\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_2,\ddot{A}_2)]\subseteq \text{FNHSint}(\ddot{G}_1,\ddot{A}_1)\cap$ $\text{FNHSint}(\ddot{G}_2,\ddot{A}_2)$.

On the other hand, since $\text{FNHSint}(\ddot{G}_1,\ddot{A}_1)\subseteq(\ddot{G}_1,\ddot{A}_1)$ and $\text{FNHSint}(\ddot{G}_2,\ddot{A}_2)\subseteq(\ddot{G}_2,\ddot{A}_2)$, then $\text{FNHSint}(\ddot{G}_1,\ddot{A}_1)\cap \text{FNHSint}(\ddot{G}_2,\ddot{A}_2)\subseteq(\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_2,\ddot{A}_2)]$. Besides, $\text{FNHSint}[(\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_2,\ddot{A}_2)]\subseteq(\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_2,\ddot{A}_2)]$ and it is the biggest fermatean neutrosophic hypersoft open set. Therefore, $\text{FNHSint}(\ddot{G}_1,\ddot{A}_1)\cap \text{FNHSint}(\ddot{G}_2,\ddot{A}_2)\subseteq \text{FNHSint}[(\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_2,\ddot{A}_2)]$. Thus, $\text{FNHSint}[(\ddot{G}_1,\ddot{A}_1)\cap(\ddot{G}_2,\ddot{A}_2)]=\text{FNHSint}(\ddot{G}_1,\ddot{A}_1)\cap \text{FHSint}(\ddot{G}_2,\ddot{A}_2)$.

5. Since $(\ddot{G}_1,\ddot{A}_1)\subseteq(\ddot{G}_1,\ddot{A}_1)\cup(\ddot{G}_2,\ddot{A}_2)$ and $(\ddot{G}_2,\ddot{A}_2)\subseteq(\ddot{G}_1,\ddot{A}_1)\cup(\ddot{G}_2,\ddot{A}_2)$, then, $\text{FNHSint}(\ddot{G}_1,\ddot{A}_1)\subseteq \text{FNHSint}[(\ddot{G}_1,\ddot{A}_1)\cup(\ddot{G}_2,\ddot{A}_2)]$ and $\text{FNHSint}(\ddot{G}_2,\ddot{A}_2)\subseteq \text{FNHSint}[(\ddot{G}_1,\ddot{A}_1)\cup(\ddot{G}_2,\ddot{A}_2)]$. Therefore, $\text{FNHSint}(\ddot{G}_1,\ddot{A}_1)\cup \text{FNHSint}(\ddot{G}_2,\ddot{A}_2)\subseteq \text{FNHSint}[(\ddot{G}_1,\ddot{A}_1)\cup(\ddot{G}_2,\ddot{A}_2)]$.

Definition 4.12 Let $(\ddot{U},\ddot{E},\ddot{\tau})$ be a fermatean neutrosophic hypersoft topological space over $\ddot{U}$ and $(\ddot{G},\ddot{B})\in \text{FNHSS}(\ddot{U},\dot{E})$ be a fermatean neutrosophic hypersoft set. Then, the fermatean neutrosophic hypersoft closure of $(\ddot{G},\ddot{B})$, denoted $\text{FNHSScl}(\ddot{G},\ddot{B})$, is defined as the fermatean neutrosophic hypersoft intersection of all fermatean neutrosophic hypersoft closed supersets of $(\ddot{G},\ddot{B})$.

Clearly, $\text{FNHSScl}(\ddot{G},\ddot{B})$ is the smallest fermatean neutrosophic hypersoft closed set that containing $(\ddot{G},\ddot{B})$.

Example 4.13 Let us consider the fermatean neutrosophic hypersoft topology $\ddot{\tau}_2$ given in Example 4.7. Suppose that any $(\ddot{G},\ddot{A})\in \text{FNHSS}(\ddot{U},\ddot{E})$ is defined as following:

$$(\ddot{G},\ddot{A})=\left\{\begin{array}{l}\langle(\alpha_1,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.4,0.6,0.7]},\frac{x_2}{[0.3,0.7,0.6]},\frac{x_3}{[0.2,0.5,0.4]}\right\}\rangle,\\ \langle(\alpha_1,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.3,0.7,0.9]},\frac{x_2}{[0.2,0.9,0.8]},\frac{x_3}{[0.1,0.7,0.6]}\right\}\rangle,\\ \langle(\alpha_3,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.2,0.6,0.8]},\frac{x_2}{[0.1,0.7,0.7]},\frac{x_3}{[0.3,0.8,0.8]}\right\}\rangle,\\ \langle(\alpha_3,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.3,0.5,0.8]},\frac{x_2}{[0.2,0.6,0.7]},\frac{x_3}{[0.1,0.4,0.6]}\right\}\rangle,\end{array}\right\}$$

Obviously, ${0^c}_{(\ddot{U}_{FNHS},\Sigma)}$, ${1^c}_{(\ddot{U}_{FNHS},\Sigma)}$, $(\ddot{G}_3,\ddot{A}_3)^c$, $(\ddot{G}_4,\ddot{A}_4)^c$ are all fermatean neutrosophic hypersoft closed sets over $(\ddot{U},\ddot{E},\ddot{\tau})$. They are given as following:

${0^c}_{(\ddot{U}_{FNHS},\ddot{E})}=1_{(\ddot{U}_{FNHS},\ddot{E})}$ and ${1^c}_{(\ddot{U}_{FNHS},\ddot{E})}=0_{(\ddot{U}_{FNHS},\ddot{E})}$

$$(\ddot{G}_3,\ddot{A}_3)^c=\left\{\begin{array}{l}\left\langle(\alpha_1,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.9,0.4,0.1]},\frac{x_2}{[0.8,0.3,0.3]},\frac{x_3}{[0.7,0.1,0.3]}\right\}\right\rangle,\\ \left\langle(\alpha_1,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.8,0.3,0.2]},\frac{x_2}{[0.6,0.7,0.8]},\frac{x_3}{[0.6,0.1,0.4]}\right\}\right\rangle,\\ \left\langle(\alpha_3,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.6,0.5,0.7]},\frac{x_2}{[0.7,0.4,0.5]},\frac{x_3}{[0.8,0.7,0.2]}\right\}\right\rangle,\\ \left\langle(\alpha_3,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.5,0.2,0.7]},\frac{x_2}{[0.9,0.3,0.4]},\frac{x_3}{[0.8,0.2,0.3]}\right\}\right\rangle,\end{array}\right\}$$

$$(\ddot{G}_4,\ddot{A}_4)^c=\left\{\begin{array}{l}\left\langle(\alpha_1,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.6,0.5,0.1]},\frac{x_2}{[0.5,0.4,0.4]},\frac{x_3}{[0.4,0.4,0.3]}\right\}\right\rangle,\\ \left\langle(\alpha_1,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.4,0.5,0.3]},\frac{x_2}{[0.3,0.9,0.8]},\frac{x_3}{[0.1,0.6,0.5]}\right\}\right\rangle,\\ \left\langle(\alpha_3,\beta_1,\gamma_2),\left\{\frac{x_1}{[0.3,0.6,0.8]},\frac{x_2}{[0.2,0.5,0.6]},\frac{x_3}{[0.8,0.8,0.7]}\right\}\right\rangle,\\ \left\langle(\alpha_3,\beta_2,\gamma_2),\left\{\frac{x_1}{[0.4,0.3,0.8]},\frac{x_2}{[0.3,0.6,0.7]},\frac{x_3}{[0.8,0.3,0.6]}\right\}\right\rangle,\end{array}\right\}$$

Then, ${0^c}_{(\ddot{U}_{FNHS},\Sigma)}, (\ddot{G}_3,\ddot{A}_3)^c, (\ddot{G}_4,\ddot{A}_4)^c \supseteq (\ddot{G},\ddot{A})$.

Therefore, $FNHScl(\ddot{G},\ddot{A}) = {0^c}_{(\ddot{U}_{FNHS},\Sigma)} \cap (\ddot{G}_3,\ddot{A}_3)^c \cap (\ddot{G}_4,\ddot{A}_4)^c = (\ddot{G}_4,\ddot{A}_4)^c$.

Theorem 4.14 Let $(\ddot{U},\ddot{E},\ddot{\tau})$ be a fermatean neutrosophic hypersoft topological space over $\ddot{U}$ and $(\ddot{G},\ddot{B}) \in FNHSS(\ddot{U},\ddot{E})$. $(\ddot{G},\ddot{B})$ is fermatean neutrosophic hypersoft closed set iff $(\ddot{G},\ddot{B}) = FNHScl(\ddot{G},\ddot{B})$.

Proof. Straightforward.

Theorem 4.15 Let $(\ddot{U},\Sigma,\ddot{\tau})$ be a fermatean neutrosophic hypersoft topological space over $\ddot{U}$ and $(\ddot{G}_1,\ddot{B}_1),(\ddot{G}_2,\ddot{B}_2) \in FNHSS(\ddot{U},\ddot{E})$. Then,

1. $FNHScl[FNHScl(\ddot{G}_1,\ddot{B}_1)] = FNHScl(\ddot{G}_1,\ddot{B}_1)$,
2. $FNHScl(0_{(\ddot{U}_{FNHS},\ddot{E})}) = 0_{(\ddot{U}_{FNHS},\ddot{E})}$ and $FNHScl(1_{(\ddot{U}_{FNHS},\ddot{E})}) = 1_{(\ddot{U}_{FNHS},\ddot{E})}$,
3. $(\ddot{G}_1,\ddot{B}_1) \subseteq (\ddot{G}_2,\ddot{B}_2) \Rightarrow FNHScl(\ddot{G}_1,\ddot{B}_1) \subseteq FNHScl(\ddot{G}_2,\ddot{B}_2)$,
4. $FNHScl[(\ddot{G}_1,\ddot{B}_1) \cup (\ddot{G}_2,\ddot{B}_2)] = FNHScl(\ddot{G}_1,\ddot{B}_1) \cup FNHScl(\ddot{G}_2,\ddot{B}_2)$,
5. $FNHScl[(\ddot{G}_1,\ddot{B}_1) \cap (\ddot{G}_2,\ddot{B}_2)] \subseteq FNHScl(\ddot{G}_1,\ddot{B}_1) \cap FNHScl(\ddot{G}_2,\ddot{B}_2)$

Proof.

1. Let $FNHScl(\ddot{G}_1,\ddot{B}_1) = (\ddot{G}_2,\ddot{B}_2)$. Then, $(\ddot{G}_2,\ddot{B}_2)$ is a fermatean neutrosophic hypersoft closed set. Hence, $(\ddot{G}_2,\ddot{B}_2)$ and $FNHScl(\ddot{G}_2,\ddot{B}_2)$ are equal. So, $FNHScl[FNHScl(\ddot{G}_1,\ddot{B}_1)] = FNHScl(\ddot{G}_1,\ddot{B}_1)$.

2. Straightforward.

3. It is known that $(\ddot{G}_1,\ddot{B}_1) \subseteq FNHScl(\ddot{G}_1,\ddot{B}_1)$ and $(\ddot{G}_2,\ddot{B}_2) \subseteq FNHScl(\ddot{G}_2,\ddot{B}_2)$ and so, $(\ddot{G}_1,\ddot{B}_1) \subseteq (\ddot{G}_2,\ddot{B}_2) \subseteq FNHScl(\ddot{G}_2,\ddot{B}_2)$. Since $FNHScl(\ddot{G}_1,\ddot{B}_1)$ is the smallest fermatean neutrosophic hypersoft closed set contained $(\ddot{G}_1,\ddot{B}_1)$, then, $FNHScl(\ddot{G}_1,\ddot{B}_1) \subseteq FNHScl(\ddot{G}_2,\ddot{B}_2)$.

4. Since $(\ddot{G}_1,\ddot{B}_1) \subseteq (\ddot{G}_1,\ddot{B}_1) \cup (\ddot{G}_2,\ddot{B}_2)$ and $(\ddot{G}_2,\ddot{B}_2) \subseteq (\ddot{G}_1,\ddot{B}_1) \cup (\ddot{G}_2,\ddot{B}_2)$, then $FNHScl(\ddot{G}_1,\ddot{B}_1) \subseteq FNHScl[(\ddot{G}_1,\ddot{B}_1) \cup (\ddot{G}_2,\ddot{B}_2)]$ and $FNHScl(\ddot{G}_2,\ddot{B}_2) \subseteq FNHScl[(\ddot{G}_1,\ddot{B}_1) \cup (\ddot{G}_2,\ddot{B}_2)]$ and so, $FNHScl(\ddot{G}_1,\ddot{B}_1) \cup FNHScl(\ddot{G}_2,\ddot{B}_2) \subseteq FNHScl[(\ddot{G}_1,\ddot{B}_1) \cup (\ddot{G}_2,\ddot{B}_2)]$.

Conversely, since $(\ddot{G}_1,\ddot{B}_1) \subseteq FNHScl(\ddot{G}_1,\ddot{B}_1)$ and $(\ddot{G}_2,\ddot{B}_2) \subseteq FNHScl(\ddot{G}_2,\ddot{B}_2)$, then $[(\ddot{G}_1,\ddot{B}_1) \cup (\ddot{G}_2,\ddot{B}_2)] \subseteq FNHScl(\ddot{G}_1,\ddot{B}_1) \cup FHScl(\ddot{G}_2,\ddot{B}_2)$.

Besides, $FNHScl[(\ddot{G}_1,\ddot{B}_1) \cup (\ddot{G}_2,\ddot{B}_2)]$ is the smallest fermatean neutrosophic hypersoft closed set that containing $(\ddot{G}_1,\ddot{B}_1) \cup (\ddot{G}_2,\ddot{B}_2)$.

Therefore, $FNHScl[(\ddot{G}_1,\ddot{B}_1) \cup (\ddot{G}_2,\ddot{B}_2)] \subseteq FNHScl(\ddot{G}_1,\ddot{B}_1) \cup FNHScl(\ddot{G}_2,\ddot{B}_2)$.

Thus, $FNHScl[(\ddot{G}_1,\ddot{B}_1) \cup (\ddot{G}_2,\ddot{B}_2)] = FNHScl(\ddot{G}_1,\ddot{B}_1) \cup FNHScl(\ddot{G}_2,\ddot{B}_2)$.

5. Since $(\ddot{G}_1,\ddot{B}_1) \cap (\ddot{G}_2,\ddot{B}_2) \subseteq FNHScl(\ddot{G}_1,\ddot{B}_1) \cap FNHScl(\ddot{G}_2,\ddot{B}_2)$ and $FNHScl\left[(\ddot{G}_1,\ddot{B}_1) \cap (\ddot{G}_2,\ddot{B}_2)\right]$ is the smallest fermatean neutrosophic hypersoft closed set that containing $(\ddot{G}_1,\ddot{B}_1) \cap (\ddot{G}_2,\ddot{B}_2)$, then $FNHScl\left[(\ddot{G}_1,\ddot{B}_1) \cap (\ddot{G}_2,\ddot{B}_2)\right] \subseteq FNHScl(\ddot{G}_1,\ddot{B}_1) \cap FHScl(\ddot{G}_2,\ddot{B}_2)$.

Theorem 4.16 Let $(\ddot{U},\ddot{E},\ddot{\tau})$ be a fermatean neutrosophic hypersoft topological space over $\ddot{U}$ and $(\ddot{G},\ddot{B}) \in FNHSS(\ddot{U},\dot{E})$. Then,

1. $\left[FNHScl(\ddot{G},\ddot{B})\right]^c = FNHSint\left[(\ddot{G},\ddot{B})^c\right]$
2. $\left[FNHSint(\ddot{G},\ddot{B})\right]^c = FNHScl\left[(\ddot{G},\ddot{B})^c\right]$

Proof. 1. $FNHScl(\ddot{G},\ddot{B}) = \cap\left\{(\ddot{G}_1,\ddot{B}_1) \in \ddot{\tau}^c ☹ \ddot{G}_1,\ddot{B}_1) \supseteq (\ddot{G},\ddot{B})\right\}$

$$\Rightarrow \left[FNHScl(\ddot{G},\ddot{B})\right]^c = \left[\cap\left\{(\ddot{G}_1,\ddot{B}_1) \in \ddot{\tau}^c: (\ddot{G}_1,\ddot{B}_1) \supseteq (\ddot{G},\ddot{B})\right\}\right]^c$$
$$\cup\left\{(\ddot{G}_1,\ddot{B}_1)^c \in \ddot{\tau}: (\ddot{G}_1,\ddot{B}_1)^c \subseteq (\ddot{G},\ddot{B})^c\right\} = FNHSint\left[(\ddot{G},\ddot{B})^c\right].$$

2. $FNHSint(\ddot{G},\ddot{B}) = \cup\left\{(\ddot{G}_1,\ddot{B}_1) \in \ddot{\tau}: (\ddot{G}_1,\ddot{B}_1) \subseteq (\ddot{G},\ddot{B})\right\}$

$$\Rightarrow \left[FNHSint(\ddot{G},\ddot{B})\right]^c = \left[\cup\left\{(\ddot{G}_1,\ddot{B}_1) \in \ddot{\tau}: (\ddot{G}_1,\ddot{B}_1) \subseteq (\ddot{G},\ddot{B})\right\}\right]^c$$
$$\cap\left\{(\ddot{G}_1,\ddot{B}_1)^c \in \ddot{\tau}^c: (\ddot{G}_1,\ddot{B}_1)^c \supseteq (\ddot{G},\ddot{B})^c\right\} = FNHScl\left[(\ddot{G},\ddot{B})^c\right].$$

V. CONCLUSIONS

In this paper, we have defined fermatean neutrosophic hypersoft operations such as complement, null set, absolute set, union, intersection, "AND", "OR". Providing De-Morgan's laws and other properties of these operations are demonstrated with various proofs and examples. Thus, operations on the fermatean neutrosophic hypersoft set structure are well defined. After that, by using these operations, we introduced fermatean neutrosophic hypersoft topological spaces, fermatean neutrosophic hypersoft open(closed) sets, fermatean neutrosophic hypersoft interior and fermatean neutrosophic hypersoft closure. Various properties of them have been studied. These are illustrated with some appropriate examples. In future, many topics such as compactness, continuity, connectedness and separation axioms can be studied on fermatean neutrosophic hypersoft topological spaces by considering this study.

CONFLICT OF INTEREST

The author declares no conflict of interest.

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