



KALMAN FILTER DESIGN FOR BALLISTIC MISSILE DEFENCE APPLICATIONS

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Abstract: The inappropriate identification of a menace, such as ballistic missiles, poses a serious danger to defence system analysts. Thus, it poses a huge risk and puts the defensive capability of fighter aircrafts under test. Due to the surrounding environment, Radar provides noisy measurements.

We propose the utilization of the Kalman Filter to estimate and track the location of missile for capture attempt by terminating countermeasures. The Kalman Filter produces estimates of hidden variables based on inaccurate and uncertain measurements. It also provides a prediction of the future system state based on past estimations.

The Extended Kalman Filter is a broadened version of the Kalman Filter where non-linearity is approximated utilizing the first or second order derivative. Both the filters utilize similar methods however Extended Kalman Filter beats the constraints of Kalman filter.

The aim is to estimate the states (position, velocity) of the ballistic missile. In this paper a mathematical model for the target will be developed, simulated and the noise corrupted data will be filtered using Extended Kalman Filter. The performance of the filters will be shown in the results.

Index Terms – Ballistic Missile, Kalman Filter, Extended Kalman Filter.

I. INTRODUCTION

Incoming ballistic missiles are a serious danger to the country. They can be detected with the assistance of tracking radars. They are utilized to compute the objective's (Ballistic missile) relative position in range, azimuth angle, elevation angle, and speed. It is the important part of both military and civilian radar systems particularly for missile guidance. Missile guidance is exceedingly difficult without target tracking, as a matter of fact. The issue is that the radar estimation contains specific vulnerability in the measurement of current position of the rocket. In radar, the objective is to estimate the location of targets (ballistic missiles, aircrafts.) by examining the two-way transit timing of received echoes of transmitted signals. Since the reflected heartbeats are unavoidably embedded in noise, their measured values are randomly distributed. The measurements which contain data in regards of interest are often associated with a noisy signal. The Kalman filter has been regarded as the optimal solution to many tracking and data prediction tasks. A Kalman filter is an algorithm used to estimate states of a system from indirect and uncertain measurements. Kalman filters are great for systems which are constantly changing. They make use of their benefit that they are light on memory (they don't have to keep any history other than the preceding state), and they are fast, making them appropriate for real time problems.

II. EXTENDED KALMAN FILTER

In Extended Kalman Filter(EKF), it uses the method called first order Taylor expansion to obtain linear approximation of the polar coordinate measurements in the update. In this process, a Jacobian matrix is produced, which represents the linear mapping from polar to Cartesian coordinate, applied at the update step.

To apply extended Kalman-filtering techniques, it is first necessary to describe the real world by a set of nonlinear differential equations.

$$\dot{x}=f(x) + w$$

where x is a vector of the system states, $f(x)$ is a nonlinear function of those states, and w is a random zero-mean process.

The process-noise matrix describing the random process w for the preceding model is given by $Q=E (ww^T)$.

Finally, the measurement equation, required for the application of extended Kalman filtering, is considered to be a nonlinear function of the states according to

$$z = h(x) + v$$

where v is a zero-mean random process described by the measurement noise matrix R , which is defined as

$$R=E (vv^T)$$

For systems in which the measurements are discrete, the nonlinear measurement equation can be rewritten as

$$Z_k = h (x_k) + v_k$$

The matrices are related to the nonlinear system and measurement equations according to

$$F = \frac{\partial f(x)}{\partial x} \Big|_{x=\hat{x}}$$

$$H = \frac{\partial h(x)}{\partial x} \Big|_{x=\hat{x}}$$

The fundamental matrix, required for the discrete Riccati equations, can be approximated by the Taylor-series expansion for exp(FT_s) and is given by

$$\Phi_{T_s} = I + FT_s \frac{F^2 T_s^2}{2!} + \frac{F^3 T_s^3}{3!}$$

where T_s is the sampling time and I is the identity matrix.

Often the series is approximated by only the first two terms or

$$\Phi_k \sim I + FT_s$$

$$M_k = \Phi_k P_{k-1} \Phi_k^T + Q_k$$

$$K_k = M_k H^T (H M_k H^T + R_k)^{-1}$$

$$P_k = (I - K_k H) M_k$$

where P_k is a covariance matrix representing errors in the state estimates after an update and M_k is the covariance matrix representing errors in the state estimates before an update.

As was already mentioned, the preceding approximations for the fundamental and measurement matrices only have to be used in the computation of the Kalman gains. The actual extended Kalman-filtering equations do not have to use those approximations but instead can be written in terms of the nonlinear state and measurement equations. With an extended Kalman filter the new state estimate is the old state estimate projected forward to the new sampling or measurement time plus a gain times a residual or

$$\hat{x}_k = \bar{x}_k + K_k [z_k - h(\bar{x}_k)]$$

In the preceding equation the residual is the difference between the actual measurement and the nonlinear measurement equation. The old estimates that have to be propagated forward do not have to be done with the fundamental matrix but instead can be propagated directly by integrating the actual nonlinear differential equations forward at each sampling interval. For example, Euler integration can be applied to the nonlinear system differential equations yielding

$$\bar{x}_k = \hat{x}_{k-1} + \hat{x}_{k-1} T_s$$

where the derivative is obtained from

$$\dot{\hat{x}}_{k-1} = f(x_{k-1})$$

In the preceding equation the sampling time T_s is used as an integration interval. In problems where the sampling time is large, T_s would have to be replaced by a smaller integration interval, or possibly a more accurate method of integration has to be used. The best way to show how an extended Kalman filter is implemented and performs is by doing an example.

III. RESULTS AND DISCUSSION

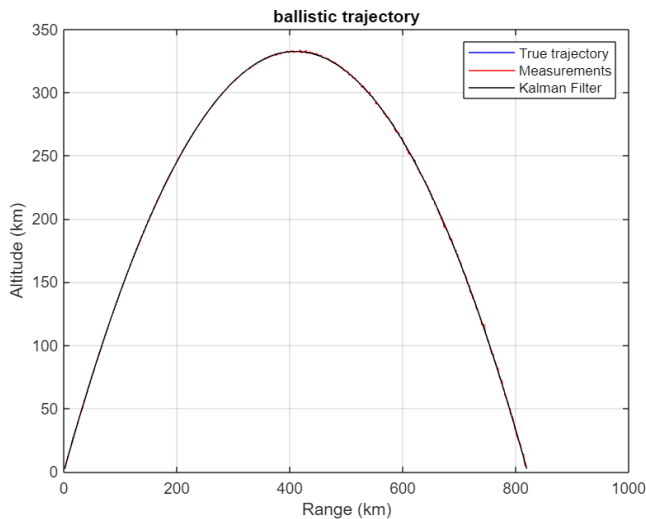


Fig 1. Ballistic Trajectory

Figure 1 displays the true ballistic trajectory, radar measurements and the estimated projectile trajectory based on the raw radar measurements. The blue colour in the graph indicates the true ballistic trajectory, the red colour in the graph indicates the radar measurements and the black colour indicates the EKF estimate of the trajectory. The figure depicts the trajectory of a Short Range Ballistic Missile (SRBM).

The range of the missile is 819.5 km, the altitude is 324.6 km and the total time of flight is 520sec. The initial launch velocity of the missile is 3000m/s and launch angle is 45 degrees.

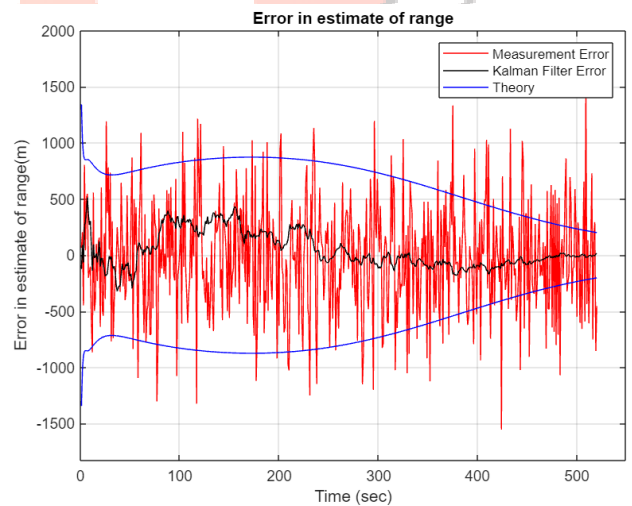


Fig 2. Error in estimate of range

Figure 2 displays the error in estimate of range of the ballistic missile. The red colour in the graph indicates the measurement error while the black colour in the graph indicates the EKF error. The error in measurement which is up to 500m is reduced to 0 metres by the end of the time of flight as seen in the figure.

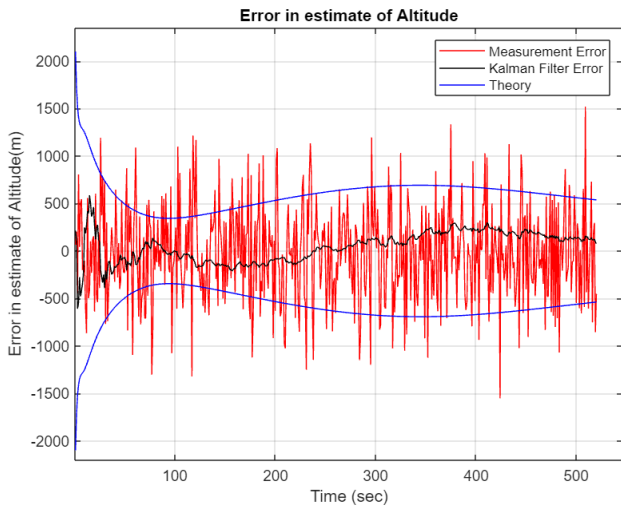
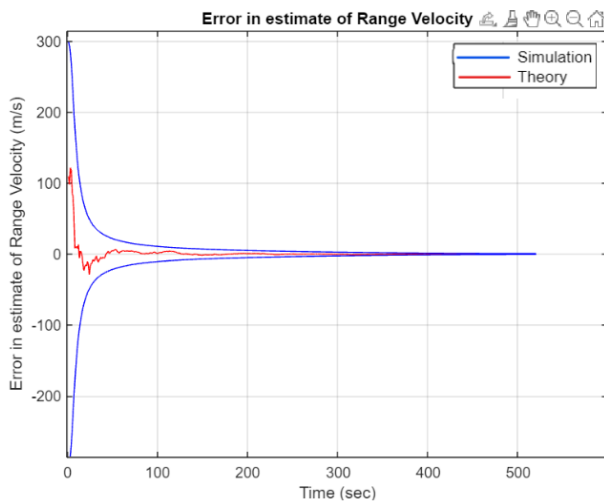


Fig 3. Error in estimate of altitude

Figure 3 displays the error in estimate of altitude of the ballistic missile. The red color in the graph indicates the measurement error while the black color in the graph indicates the EKF error. The error in measurement error which is up to 500m is reduced to around 50 meters by the end of the time of flight as seen in the figure.



IV. CONCLUSION

Noise is removed very precisely using EKF. The efficiency is also increased. The error in Range and Altitude was 500 metres which was reduced to 0 metres and 50 metres respectively. The error in Range Velocity and Altitude Velocity is reduced completely to zero towards the finish of the time of flight.

EKF handles the Kalman gain rather than the state and measurement noise covariance. The elements of EKF helps to achieve the actual position of the moving target. It works by linearizing the nonlinear states. There is not really any difference between the real position and estimated position. The actual and estimated graphs are overlying on each other providing the precise data.

The states (position, velocity) of the ballistic missile have been estimated. In this paper, a mathematical model for the target has been developed and the noise corrupted data is filtered using Extended Kalman Filter. The performance of the filter has been shown in the results.

V. FUTURE SCOPE

When the state transition and observation state space models – the predict and update functions f and h are highly non-linear, the EKF cannot give up to the mark performance because the linearization of the underlying non-linear model propagates the covariance. Although EKF is straightforward and simple it suffers from instability due to linearization and erroneous parameters, costly calculation of Jacobian matrices, and the biased nature of its estimates. The EKF can be improved to deal with highly nonlinear functions.

Similarly, there arises the case of getting more than one measurement sample for a tracking radar when an object other than the desirable. The filter prediction can be used to select and proceed with the correct measurement.

Later, the Extended Kalman Filter can be further developed where it can track multiple objects at a time and with much more accuracy.

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Fig 4. Error in estimate of Range Velocity

Figure 4 shows how the single simulation run errors in the estimates of velocity compare with the theoretical predictions of the Riccati equation covariance matrix (i.e., square root P_{22}).

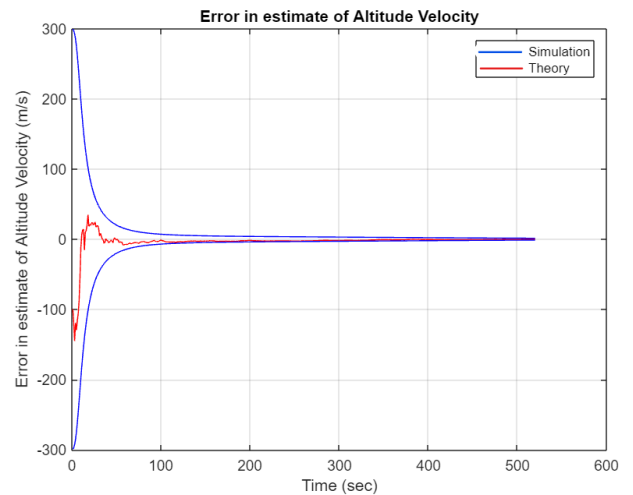


Fig 4. Error in estimate of Altitude Velocity

Figure 4 shows how the single-run simulation errors in the estimates of the projectile's altitude velocity compare with the theoretical predictions of the covariance matrix (i.e., square root of P_{44}). Again, we can see that the single-run simulation results lie within the theoretical bounds 68% of the time, giving us a good indication that the filter is performing properly.

V. REFERENCES

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