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Order Level Inventory System For Deteriorating Items With Declining Demand, Shortages And Life Time

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ABSTRACT:

In the present paper, a deterministic inventory model has been developed. In this model, the demand rate of an item decreases negative exponentially with time and inventory is depleted not only by demand but also by deterioration. The items are deteriorating at constant rate and it starts after a time interval which may be called the life period of the item in inventory. The model is developed by considering shortages and instantaneous delivery. It is shown that the developed model can be related to Aggarwal's model(2), Sharma et al model (8) and standard model without deterioration.

KEY WORDS:

Demand, deterioration, life time, shortages.

INTRODUCTION :

First Inventory models for deteriorating items have been undertaken by many researcher in recent years. When the items are kept in the stock to meet the future demand, there may be deterioration of items in inventory due to many factors like storage condition, weather condition, insect biting etc. So, deterioration character of the items has the significant impact on inventory system. The rate of deterioration may be a constant or variable proportion of the on hand inventory when the item is produced or purchased, it is fresh and new and deterioration starts after a certain period, this certain period is called life time of that particular item and it is different for different items. Fruits have small life period while drugs have a large life period.

Chowdhury and Chaudhuri(3) developed an order level inventory model for deteriorating items with finite rate of replenishment. Goel and Aggarwal(4) developed, an order level inventory system with power demand pattern for deteriorating items. Kumar and Sharma (5,6) developed models for deteriorating items.

Sharma et.al (10) developed order level inventory models for deteriorating items with declining demand and Weibull distributed deterioration.

Shah and Jaiswal (9) carried out a study for an order level inventory model for deteriorating items with constant rate of deterioration. Aggarwal(2) developed an order level inventory model by rectifying errors in Shah and Jaiswal's model(9).

In the present paper, an attempt has been made to study a situation in which demand decreases exponentially over a fix time horizon and deterioration of items starts after a time u (u≥0) from the instant of the arrival of stock. The proposed model allows for shortages and are completely backlogged as well.

ASSUMPTIONS AND NOTATIONS :-

The proposed inventory model is developed under the following assumptions and notations.

(i) Demand rate, D(t) is known and decrease exponentially i.e. at time t, t≥ 0, $D(t) = Ae^{-\lambda t}$, A is initial demand and λ is constant governing the decreasing rate of the demand.

(ii) The prescribed schedule period 'T' is constant.

(iii) The units in the system deteriorates at a constant rate 'θ'(say) of the on hand inventory per unit time only after the expiry of the life period 'u' of the item.

Hence, the deterioration fraction can be taken in the following form :-

θ=θH(t-u) , 0 < θ<1

u>0

Where H(t-u) is heavisides function defined as follow :-

H(t-u)=1 t≥ u

=0 t<u

(iv) Replenishment rate is infinite. Replenishment size is constant and lead time is zero.

(v) The lot size '*q*' raises the initial inventory level in each schedule period to the order level *S*.

(vi) Shortages, if any are allowed and backlogged as soon as a fresh stock arrives.

(vii) The unit cost *C* per unit, inventory holding cost C_1 and shortage cost C_2 per unit per unit time is known and constant during the period under consideration.

MATHEMATICAL MODELLING AND ANALYSIS :-

At t=0 of the period, the lot size *'q'* enters the system from which *(q-S)* units are delivered towards backorders leaving *'S'* units (*S*>0) as the initial inventory. Thereafter during (0,u) the inventory level gradually decreases due to market demand and during (u,t_1) (t_1<T) due to demand and partly due to deterioration. Shortages occur during time period (t_1, T) that are fully backlogged. Let $t_2 = T - t_1$

Let *I(t)* be the inventory level of the system at time '*t' ($0 \le t \le T$).*

The differential equations governing the stock status over the cycle time *(0,T)* can be written as

$$\frac{dI(t)}{dt} = -Ae^{-\lambda t} \qquad 0 \le t \le u \qquad (1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -Ae^{-\lambda t} \qquad u \le t \le t_1 \qquad (2)$$

$$\frac{dI(t)}{dt} = -Ae^{-\lambda t} \qquad t_1 \le t \le T \qquad (3)$$

Using boundary conditions i.e. *I(t) = S at t = 0 and I(t)=I(u) at t=u*

I(t)=0 at $t=t_1$

The solutions of equation (1) and (3) are obtained using boundary conditions and after adjusting constant of integration.

$$I(t) = S + \frac{A}{\lambda}\left[\exp.(-\lambda t) - 1\right] \qquad (0 \le t \le u) \qquad (4)$$

$$I(t) = \frac{A\exp.(-\lambda t)}{(\lambda-\theta)} + \left[S + \frac{A}{\lambda}(\exp.(-\lambda u - 1) - \frac{A\exp.(-\lambda u - 1)}{(\lambda-\theta)}\right]\exp.(-\theta(t-u))(u \le t \le t_1) \qquad (5)$$

$$I(t) = \frac{A}{\lambda}\left(\exp.(-\lambda t) - \exp.(-\lambda t_1)\right) \qquad (t_1 \le t \le T) \qquad (6)$$

Also $I(t_1)=0$ given by (5) as

$$t_1 = \frac{1}{-(\lambda-\theta)}\log\left[-\frac{(\lambda-\theta)}{A}\exp.(\theta u)\left\{S + \frac{A}{\lambda}(\exp.(-\lambda u)) - 1 - \frac{A}{(\lambda-\theta)}\exp.(-\lambda u)\right\}\right]$$

$$= \frac{1}{-(\lambda-\theta)}\log\left[1 - \left\{\frac{S(\lambda-\theta)}{A} + \frac{Su\lambda\theta}{A}\right\}\right] \qquad (7)$$

Lot size θ entering the system becomes

$$Q = \frac{A}{\lambda}\left(\exp.(-\lambda t_1) - \exp.(-\lambda T)\right) + S \qquad (8)$$

Inventory holding cost over the period (0,T) is given by

$$= C_1\left[\int_0^u I(t)dt + \int_u^{t_1} I(t)dt\right]$$

$$= C_1\left[\left\{Su - \frac{A\exp.(-\lambda u)}{\lambda^2} - \frac{Au}{\lambda} + \frac{A}{\lambda^2}\right\} + \left\{\begin{array}{l}\dfrac{A(\exp(-\lambda u) - \exp(-\lambda t_1))}{\lambda(\lambda-\theta)} \\ + \dfrac{1}{\theta}S + \dfrac{A}{\lambda}(\exp(-\lambda u) - 1) - \dfrac{A}{\lambda-\theta}\end{array}\right\}\right]$$

$$= C_1\int_0^{t_1} I(t)dt$$

$$= C_1\int_0^{t_1}\left[\frac{Ae^{-\lambda t}}{\lambda-\theta} + \left(S - \frac{A}{\lambda-\theta}\right)e^{-\theta t}\right]dt$$

$$= C_1\left[\frac{A(1-e^{-\lambda t_1})}{\lambda(\lambda-\theta)}+\frac{1}{\theta}\left(S-\frac{A}{\lambda-\theta}\right)\left(1-e^{-\theta t_1}\right)\right] \tag{9}$$

Total amount of deteriorated units

$$= S-\int_0^{t_1} Ae^{-\lambda t}dt$$

$$= S-\frac{A}{\lambda}\left(1-e^{-\lambda t_1)}\right)$$

Unit cost over the period *(0,T)* is given by

$$= C\left[S-\frac{A}{\lambda}(1-e^{-\lambda t_1})\right] \tag{10}$$

Shortage cost is given by

$$= C_2\int_{t_1}^{T}\left[-I(t))\right]dt$$

$$= -C_2\int_{t_1}^{T}\frac{A}{\lambda}\left(e^{-\lambda t}-e^{-\lambda t_1}\right)dt$$

$$= \frac{C_2 A}{\lambda}\left[\frac{e^{-\lambda T}-e^{-\lambda t_1}}{\lambda}+e^{-\lambda t_1}(T-t_1)\right] \tag{11}$$

Hence the total average cost *K(S)* per unit time is given by

K(S)= Average unit cost + Average holding cost + Average shortage cost.

$$K(S)=\frac{C}{T}\left[S-\frac{A}{\lambda}\left\{1-e^{-\lambda t_1}\right\}\right]+\frac{C_1}{T}\left[\frac{A(1-e^{-\lambda t_1})}{\lambda(\lambda-\theta)}+\left(S-\frac{A}{\lambda-\theta}\right)\frac{1}{\theta}\left(1-e^{-\theta t_1}\right)\right]$$

$$+\frac{C_2 A}{\lambda T}\left[\frac{e^{-\lambda T}-e^{-\lambda t_1}}{\lambda}+e^{-\lambda t_1}(T-t_1)\right] \tag{12}$$

<u>THE APPROXIMATE SOLUTION PROCEDURE</u> :-

According to equation (12), finding optimal solution of the model is very difficult. In most of the cases λ and θ are small. Hence, here Maclaurin series for approximation can be used.

$$e^{-xt}=1-xt+\frac{x^2t^2}{2}$$

By using Maclaurin series for approximation and (7), *K(S)* reduces to

$$K(S)=\frac{C}{T}[S-At_1]+\frac{C_1}{T}\left[\frac{At_1}{\lambda-\theta}-\frac{S}{\lambda-\theta}\right]+\frac{C_2A(T-t_1)}{\lambda T}\left[\frac{\lambda T}{2}-\frac{\lambda t_1}{2}\right]$$

$$=\frac{C}{T}[S-At_1]+\frac{C_1}{T}\left[\frac{At_1}{\lambda-\theta}-\frac{S}{\lambda-\theta}\right]+\frac{C_2A(T-t_1)^2}{2T}$$

Now substituting value of t_1 from equation (7), we get

$$K(S) = \frac{C}{T}\left[S + \frac{A}{\lambda-\theta}\log\left\{1-\frac{S(\lambda-\theta)}{A}\right\}\right] + \frac{C_1}{T}\left[-\frac{S}{\lambda-\theta} - \frac{A}{(\lambda-\theta)^2}\log\left\{1-\frac{S(\lambda-\theta)}{A}\right\}\right]$$

$$+\frac{C_2 A}{2T}\left[T + \frac{1}{\lambda-\theta}\log\left\{1-\frac{S(\lambda-\theta)}{A}\right\}\right]^2 \qquad (13)$$

Now using series for logarithmic terms and ignoring terms of second and higher powers of θ and λ as θ and λ are very small and $\theta << T, (\lambda\text{-}\theta) < A$

The correct total cost equation of the system then becomes

$$K(S) = \frac{-CS^2(\lambda-\theta)}{2AT} + \frac{C_1 S^2}{2AT}\left[1+\frac{2S(\lambda-\theta)}{3A}\right] + \frac{C_2 A}{2T}\left[T-\frac{S}{A}\right]^2 - \frac{C_2(\lambda-\theta)S^2}{2AT}\left[T-\frac{S}{A}\right]$$

(14)

Now for cost K(S) to be minimum, condition is

$$\frac{dK(S)}{dS} = 0$$

$$\Rightarrow \frac{-CS(\lambda-\theta)}{AT} + \frac{C_1 S}{AT} + \frac{C_1 S^2(\lambda-\theta)}{A^2 T} + \frac{C_2 S}{AT} - C_2 - \frac{C_2(\lambda-\theta)S}{A} + \frac{3C_2 S^2(\lambda-\theta)}{2A^2 T} = 0$$

This equation simplifies to the following equation

$$-CS(\lambda-\theta) + C_1 S + C_2 S\left[1+\frac{3(\lambda-\theta)S}{2A}\right] - (\lambda-\theta)S\left[C_2 T - \frac{C_1 S}{A}\right] - C_2 AT = 0 \quad (15)$$

Solution of equation (15) for S gives the optimum order level S_o under condition (17). The optimum lot size and the minimum cost can be obtained by substituting the optimum order level S_o in the equations (16) and (14) respectively.

$$\Rightarrow q = AT - \frac{S_0^2(\lambda-\theta)}{A} \qquad (16)$$

Where S_0 is the solution of (15)

Also

$$\frac{d^2K(S)}{dS^2} = \frac{-C(\lambda-\theta)}{AT} + \frac{C_1}{AT} + \frac{2SC_1(\lambda-\theta)}{A^2 T} + \frac{C_2}{AT} - \frac{C_2(\lambda-\theta)}{A} + \frac{3C_2 S(\lambda-\theta)}{A^2 T}$$

$$\Rightarrow \frac{d^2K(S)}{dS^2} > 0 \qquad \textit{for } S > 0 \qquad (17)$$

SPECIAL CASES :-

Two special cases illustrate the effectiveness of the developed model.

Case (i)

If $\theta = 0$, $\lambda = 0$ and $A=R$ i.e. there is no deterioration and demand rate is constant.

$$S_0 = \frac{C_2}{C_1 + C_2} RT$$

which is the standard formula as given by Naddor (7) for finding the order level S=S_0 for an order level system for non deteriorating items.

Case (ii)

If $\lambda = 0$ and $A=R$, then our model reduces to Aggarwal's Model (2)

$$C_2 S\theta + C_1 S + C_2 S\left(1 - \frac{3S}{2A}\right) + \theta S\left(C_2 T - \frac{C_1 S}{A}\right) - C_2 AT = 0$$

NUMERICAL EXAMPLE :-

The following example illustrates the effectiveness of the developed model. Assume that the demand rate equation as $D(t) = 250e^{-\lambda t}$ whereas the value of the other variables are C = 0.20 Rs., C_1 = 0.30 Rs., C_2 = 1.50 Rs., T = One Year. The optimum values of $t_1, t_2, S, K(S), q$ are calculated numerically for different values of λ and θ of the model. Table 1 lists these values and provides the following necessary information.

(i) When λ increases, t_1 increases $K(S)$, q, S and t_2 decreases.

(ii) When θ increases t_1 decreases, $K(S)$, q, S and t_2 increases.

TABLE- Effect of λ and θ on the order level system.

λ	θ	t_1	t_2	S	Q	K(S)
0.0	0.05	0.83	0.17	210.90	258.90	33.95
	0.10	0.82	0.18	214.08	268.33	36.74
0.30	0.05	0.88	0.12	199.16	210.33	24.69
	0.10	0.87	0.13	200.61	217.80	26.35
	0.15	0.86	0.14	203.17	225.24	27.40
0.35	0.05	0.89	0.11	197.84	203.03	23.16

CONCLUDING REMARKS :-

In this paper, a deterministic inventory model for deteriorating items with an exponential declining demand is developed for a fixed and finite planning horizon considering shortage and excess demand is backlogged as well. Two special cases illustrate the effectiveness of the developed model. The modal is resolved by using Maclaurin series.

The present model provides valuable reference for decision makers in the planning as well as controlling the inventory. A numerical example is also presented to examine the effect of λ and θ. The immediate extension of the model is for variable deterioration rate and stochastic nature of demand.

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