



A STUDY ON ESTIMATES OF VALUE AT RISK (VaR) BASED ON SIMULATION MODELS, GARCH (1,1) AND VARIANCE BASED MODEL FOR EXPORTERS AND IMPORTERS IN INDIA

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Abstract : Businesses the around the globe are now concerned with risk as much as they care about profitability. Importer and exporters are exposed to loss arising out of exchange rate fluctuations. One of the important approaches to quantify risk is by estimating the Value at Risk (VaR). VaR is the maximum loss that can occur over a certain period of time. This paper estimated the VaR for exporters and importers using the Simulation models, GARCH (1,1) model and variance based model. Based on the results of Back testing it was found that Historical simulation model among these models provides the best estimates and it is therefore recommended for use by exporters and importers in India.

Introduction

The top management of businesses around the world are now focussed on risk besides profitability. Questions like, what is the maximum loss that can occur on a transaction? What are the chances this loss would occur? While risk has many dimensions to it, attempts have been made to *quantify* risk. One such approach to quantify risk is to estimate the *Value at Risk (VaR)*. The VaR may be defined as the maximum amount of loss that is not likely to get exceeded over a certain period of time. It may be represented, say, as VaR (99%, 1 day). The arguments in the brackets show the confidence level and the time interval. E.g. if is estimated that Rs. 5,00,000 is VaR (99%, 1 day), it may be said that there is 99% chance that the loss over one day will not exceed Rs. 5,00,000 and there is only one percent chance that it may be exceeded. This kind of measure helps the management to know the maximum possible risk and plan accordingly to face any eventuality in the future. This kind of measure can be helpful to importers and exporters in knowing what risk they could face if they are dealing in transactions involving foreign currency. Knowing about the VaR involved in the export-import transaction they can make suitable hedging strategies too. This measure would also let them know the maximum risk they could face if the foreign currency exposure is left unhedged. With ever increasing cross-border trade, it is important to estimate the losses that might crop up due to foreign exchange rate fluctuations. The exporters-importers would stand to gain if they are in a position to quantify the maximum possible losses due to these fluctuations. Several approaches and models have been suggested to estimate the VaR. This

research utilizes the Historical Simulation approach, Monte Carlo simulation, Variance based model and GARCH (1,1) model to estimate and analyse the VaR for exporters and importers of India. Extremely scarce literature was found available on this topic when an online Google search and Jstor search was carried out. No specific research work mentioning the estimates of VaR for Indian exporters and importers was found. This points to a research gap that this study attempts to fill.

Objectives of the Research

The goals contemplated to be achieved were:

- Estimate the VaR (99%, 30 days) for exporters and importers based on a notional outstanding of US\$ (USD) 1,00,000. This would also highlight the percentage loss possible on a foreign exchange exposure
- To evaluate the estimates of VaR (99%, 30 days) obtained using Historical Simulation, Monte Carlo Simulation, Variance based model and GARCH (1,1) by subjecting them to *Back Testing*. Based on the results obtained in Back testing, suggest the best possible model that could be utilized by the exporters and importers to estimate their risk in foreign currency transactions.

Data Collection

The data on daily exchange rates for the currency pair of USD(US\$) – INR (Indian Rupees) was collected from the archives of the Reserve Bank of India (RBI) from its website. The data used for building the models was taken from 10th July, 2018 till 22nd January, 2021. A total of 615 daily exchange rates were available.

Historical Simulation Model

Percentage change in the daily exchange rates was calculated based on the data collected and 614 observations (i.e. percentage change in daily exchange rates were obtained. An exposure of USD 1,00,000 due to a foreign currency transaction on 22-Jan-2021 was assumed. This exposure meant that for an exporter, USD 1,00,000 would be received in the next 30 days (one month). As corollary this exposure for an importer meant that he would pay USD 1,00,000 after a month. The exchange rate on 22-Jan-2021 was Rs.73.038 translating into Rs. 73,03,800 as foreign currency exposure on this day. A rise in the exchange rate, over the next 30 days, would mean a loss for the importer and a fall in the exchange rate would put the exporter to disadvantage. Scenarios of possible changes in the INR value of the foreign currency exposure were generated based on the historical daily percentage changes. The change was calculated as:

$$u_i = (S_t - S_{t-1}) / S_{t-1}$$

S_{t-1} - the exchange rate of the previous day

S_t - exchange rate for current day / today

u_i – percentage change in the exchange rate (in decimals)

e.g. the exchange rate on 10-July-2018 was Rs. 68.7942 this changed to Rs. 68.829 on 11-July-2018. The u_i for 11-July-2018 was therefore 0.0005 (0.05%). This change in exchange rate was applied to the exchange rate of 22-Jan-2021 to find the exchange rate of the next day i.e. 23-Jan-2021. Based on the exchange rate of 23-Jan-21, INR value of the foreign currency exposure was obtained as Rs.73,07,494.68. It therefore implies

that exporter would gain Rs. 3694.68 (Rs.73,07,494.68 - Rs. 73,03,800) and as a corollary the importer would stand to lose Rs. 3694.8. This gain/loss is a scenario generated based on historical change in the exchange rate. The same process was followed for all 615 days' exchange rates and 614 scenarios or observations were generated. The losses and gains for exporters and importers were sorted in the ascending order and ranked from 1 till 614. So observations with negative sign (losses) would get lower ranks. The VaR (99%, 1day) was found by taking one percentile of the sorted data. One percentile was taken as the rank that equalled 1% of 614 ranked data (i.e. 6.14, that was rounded off to six). Therefore, the VaR (99%, 1day) obtained based on Historical Simulation was the six worst loss in the ranked data. The N -days VaR is calculated as $\sqrt{N} * \text{VaR}$ (99%, 1day) i.e. a 30-day VaR would equal $\sqrt{30} * \text{VaR}$ (99%, 1day).

Table 1

VaR Obtained Based on Historical Changes in The Exchanges Rates

	Exporter	Importer
VAR (99%, 1 day) - Rs.	83,331	78,600
VAR (99%, 30 days) - Rs.	4,56,421	4,30,510
VAR (99%, 30 days) - as a percentage of total receivable	6.25%	5.89%

Model based on Variance

Variance of percentage changes in the exchange rate was calculated for the 614 observations and the standard deviation (i.e. square root of variance) was found to be 0.0037. This was considered to be the volatility in the exchange rate changes per day.

$$\text{VaR (99\%, 1 day)} = \sigma_{\text{daily}} * 2.58 * \text{FCE}$$

σ_{daily} - Standard deviation of daily changes in the USD-INR exchange rates

FCE – Value of the foreign currency exposure.

$$\text{The VaR (99\%, 30 days)} = \sqrt{30} * \text{VaR (99\%, 1day)}.$$

$$\text{Therefore, the VaR (99\%, 30 days)} = 0.0037 * 2.58 * \text{Rs. } 73,03,800 = \text{Rs. } 3,86,921.44$$

Monte Carlo Simulation based VaR

The change in the USD-INR exchange rate is assumed to follow a Generalized Weiner Process, based on which a model known as *Geometric Brownian motion* model is:

$$\Delta S = \mu S \Delta t + \sigma \varepsilon \sqrt{\Delta t}$$

ΔS – percentage change in the exchange rate compared to previous day's rate

μ - mean of daily percentage changes of the USD-INR exchange rates (here data from 10th

July, 2018

till 22nd January, 2021)

σ – Standard deviation of the changes in the exchange rates

ε - has standard normal distribution (mean of zero and standard deviation of 1)

Δt – small time interval, equal to 1 for daily change in exchange rates

The mean of changes in the daily exchange rates for the sample was 0.0001 and the Standard deviation was 0.0037. Therefore, the model for daily change in the exchange rate was:

$$\Delta S = 0.0001 S + 0.0037\varepsilon$$

To simulate the next day's (23-Jan-2021) change in the exchange rate and hence obtain a scenario for the next day's rate, the value of ε was generated using Random numbers between 0 and 1. Five hundred random numbers were generated using Excel to generate equal number of forecast of the exchange rates. Based on these scenarios, VaR(99%,1day) for the exporters and importers were found to be Rs. 62,412.53 and Rs. 58,477.98 respectively. The VaR (99%, 30 days) were Rs. 3,41,847.53 and Rs. 3,20,297.07 for exporters and importer respectively.

Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model

GARCH (1,1) model was used to forecast the daily volatility of the next day i.e. 23-Jan-21. The model for predicting the next day's variance was:

$$\sigma^2_n = \gamma V_L + \alpha u^2_{n-1} + \beta \sigma^2_{n-1}$$

u^2_{n-1} - observed variance in the exchange rates (considered equal to the square of the percentage change in exchange rate) on the previous day i.e. n-1 day

α - weight assigned to the observed variance on the previous day

σ^2_{n-1} - Forecast of the variance for the previous day (n-1)

β - weight assigned to the Forecast of the variance for the previous day (n-1)

V_L - Long range variance observed for the sample data

$\gamma = (1 - \alpha - \beta)$ - weight assigned to the Long range variance

σ^2_n - Forecast of the variance for the next day (n)

The GARCH (1,1) parameters, α and β were estimated using the Maximum Likelihood Method. The function to be maximized for the estimation was:

$$-\ln(\sigma^2_n) - (u^2_n / \sigma^2_n)$$

where u^2_n is the observed value of the variance on day 'n'

The Long range variance, V_L , is 0.000014042. The α and β were determined using an iterative process. Initially, 'Solver' function in Microsoft Excel was tried for maximizing the function using the constraints:

$$\alpha + \beta + \gamma = 1 ; \alpha + \beta < 1 ; \alpha, \beta > 0$$

Success in generating the GARCH parameters was not achieved using the Solver function as it gave a negative weight to V_L . Iterating manually and observing the inflection point (point from which value decreased from the previous level) for the function, the Maximum Likelihood estimates for α , β and γ were 0.16320392673, 0.75873523, 0.07806084477 respectively. Therefore, the GARCH (1,1) model for the estimate of next day's variance:

$$\sigma^2_n = 0.07806084477 V_L + 0.16320392673 u^2_{n-1} + 0.75873523 \sigma^2_{n-1}$$

The variance observed on 22-Jan-21 was 0.000000781 and the forecast for the day was 0.000006502. Based on the GARCH (1,1) model the forecast of variance for 23-Jan-21 was 0.000006157 translating into a daily volatility (Standard deviation, σ) of 0.002481.

The, VaR for 30days is equal to $\sqrt{30} * \text{VaR}(99\%, 1\text{day})$, this implies that $\text{VaR}(99\%, 30\text{days}) = 2.58 * \sigma * \sqrt{30} * \text{Rs. } 73,03,800 = \text{Rs. } 2,56,103.9694$

Back Testing of the Models

The reliability and the accuracy of the models was tested using the past ten years' data. A model would be considered reliable if the VaR estimate obtained for 30-days would not be exceeded more than one percent of the times. The VaR (99%, 30-days) was tested against the actual change in the INR value of USD 1,00,000 exposure over 30 days. Daily exchange rate for USD-INR from 01-01-2010 till 22-01-2021 was obtained from RBI's website. The actual percentage change was calculated between the exchange rates with a gap of 30 days. e.g. For calculating this, the difference between the first observation and the thirtieth observation was taken. First observation was Rs. 46.65 (on 01-01-2010) and thirtieth observation from this was Rs. 46.38 (15-02-2010). This does not correspond to an exact difference of 30 days but the gap of 30 observations was considered for calculating the change. The next change was considered between the second observation and the 31st observation. This led to generating 2635 number of data points for changes in value of USD 1 lakh exposure over 30 days.

Analysis and Findings of the Back Testing

The models were tested for exporter and importer separately. The summary of test for exporter VaR (99%, 30 days):

Table 2

Results of back test for various models for an exporter having a foreign currency exposure of USD 1,00,000

	Simulation - Historical data	Simulation - Monte Carlo	GARCH(1,1)	Model based on Volatility
VaR estimate of the model (Rs.)	456420.59	341847.53	256103.97	386921.44
No. of times VaR exceeded	5	32	77	20
No. of Observations for Back test	2635	2635	2635	2635
% of times VaR was exceeded	0.19%	1.21%	2.92%	0.76%

The summary of test for importer VaR (99%, 30 days):

Table 3

Results of back test for various models for an importer having a foreign currency exposure of USD 1,00,000

	Simulation Historical data (1-Day change)	Simulation - Monte Carlo	GARCH (1,1)	Model based on Volatility	Simulation Historical data (30 day Change)
VaR estimate of the model (Rs.)	4030509.51	341847.5	256104	386921.4	474129.23
No. of times VaR exceeded	50	109	238	79	24
No. of Observations for Back test	2635	2635	2635	2635	2635
% of times VaR was exceeded	1.90%	4.14%	9.03%	3.00%	0.91%

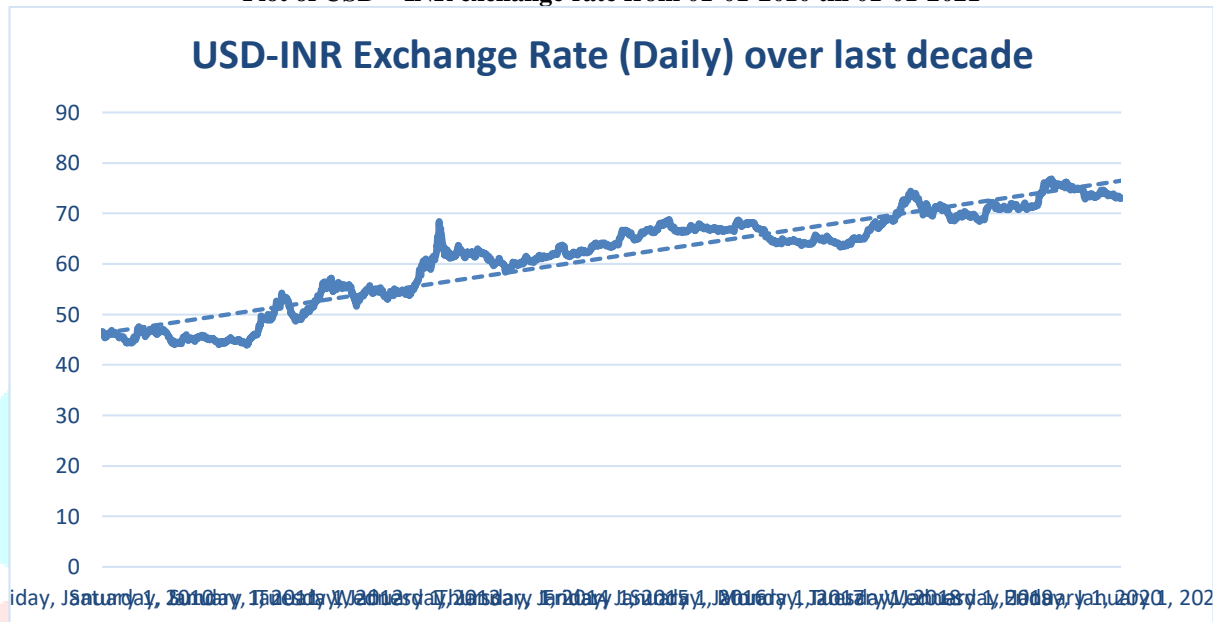
The results of the back testing show that the VaR estimates obtained for using Historical simulation and the model based on variance meet the desired criteria of not getting exceeded for than one percent of the times in the test period. Whereas the VaR estimates based on Monte Carlo simulation and GARCH (1,1) fail to meet this criterion. The best results are obtained using the Historical Simulation approach.

The results are different for importers. The VaR estimates using the models used for an exporter fail to meet the VaR criterion. A search for improved model was accomplished when historical simulation was carried out for the data between 10th July, 2018 till 22nd January, 2021 using a 30-day change in the exchange rate versus using the daily change in the exchange rate to calculate the loss/gain to the importer. The VaR estimate using a 30-day change was found to be Rs. 474129.23 versus Rs. 4030509.51 obtained based on one day changes in the exchange rate. With this improved estimate, the criterion for VaR of not getting exceeded for more than one percent of the times, was met.

One plausible reason why GARCH (1,1) failed could be the presence of a long term trend of rising USD-INR exchange rates. This seems to affect the affect even the 30-day estimates of VaR for importers. GARCH could be more successful, if the values of variable (here, the exchange rate) would be mean reverting. As seen in the Figure 1 this is not the case.

Figure 1

Plot of USD – INR exchange rate from 01-01-2010 till 01-01-2021



Conclusion and Suggestion

The best estimates for VaR (99%, 30 days) for importer and exporter were obtained using the Historical simulation approach when compared with Monte Carlo Simulation, GARCH (1,1) and model based on the variance. As a percentage of the exposure the estimates obtained were 6.25% and 6.49% of the INR value of foreign currency exposure for the exporter and importer respectively. This implies that if the foreign currency exposures left unhedged the business entities could be 99% sure that the loss on foreign currency exposure may not exceed 6.49% of the INR value of the exposure over a 30 days' period. However, there are occasions when a loss of 6.5% in revenue could completely wipe out the profit and get the firms into a loss. This also hints to the fact the VaR for longer periods like three months would be higher and it would be preferable to hedge.

Historical simulation model requires simple spreadsheet workings and therefore relatively simple to implement even by firms who may not be equipped with deep academic knowledge about concepts of estimating risk. This method gives an objective risk estimate of the foreign currency risk and assists the importers and exporters to make an informed decision about strategies to be employed for mitigating this risk. The limitation of this research was that Stress Testing was not carried out for the VaR estimates. In periods of severe financial turmoil like the one during the Sub-prime crisis in 2008, the change in the exchange rates may be well beyond 2.33 or 2.58 standard deviations. The extent of loss could, in these severe conditions may

well be beyond the VaR estimates and it is not estimated in this research by how much would the VaR be exceeded.

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