IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

# Applications of Emad-Sara Transform in Handling Population Growth and Decay Problems

Dinkar P. Patil, Priti R. Pardeshi, Rizwana A. R. Shaikh , Harshali M. Deshmukh

Department of Mathematics, K.R.T.Arts, B.H. Commerce and A. M. Science College, Nashik.

## Abstract.

Population growth and decay problems are solved by many researchers using various integral transforms. In this paper we use recently developed Emad-Sara transform to solve the problems on population growth and decay.

**Key Words:** Growth problems, Decay Problems, Integral transforms, Emad Sara Transform.

## 1. Introduction:

Many quantities in the universe grow or decay at a rate proportional to their size. For example a colony of bacteria may double or triple in a hour. If the size of the colony after t hours is given by $y(t)$ , then we can express this information in the mathematical language in the form of a first order differential equation,

$$\frac{dy}{dt} = 2y$$

The quantity $y$ that grows or decays at a rate proportional to its size is governed by a first order differential equation,

$$\frac{dy}{dt} = ky$$

If $k < 0$ then the above equation is called the law of natural decay and if If $k > 0$ then the above equation is called the law of natural growth. This equation is solved by separation of variable method. Integral transforms plays an important role in solving differential equations.

Recently, S.R Kushare and D. P. Patil [1] introduce Kushare transform in September 2021.In October 2021, S.S.Khakale and D. P. Patil [2] introduce Soham transform. As researchers are going to introduce new integral transforms at the same time many researchers are interested to apply these transforms to various types of problems. In January 2022,R. S. Sanap and D. P. Patil [3] used Kushare transform to solve the problems based on Newton's law of cooling. In April 2022 D. P. Patil etc. [4] use Kushare transform to solve the problems on growth and decay. In October 2021 D. P. Patil [5] used Sawi transform in Bessel function. D. P. Patil [6] used Sawi transform of error function for evaluating improper integral further, Lpalce and Shenu transforms are used in chemical science by D. P. Patil [7]. Dr. Patil [8] solved the wave equation by Sawi transform and its convolution theorem. Further Patil [9] also used Mahgoub transform for solving parabolic boundary value problems.

Dr. Dinkar Patil [10] obtains solution of the wave equation by using double Laplace and double Sumudu transform. Dualities between double integral transforms are derived by D. P. Patil [11]. Laplace, Elzaki, and Mahgoub transforms are used for solving system of first order and first degree differential equations by Kushare and Patil [12].Boundary value problems of the system of ordinary differentiable equations are by using Aboodh and Mahgoub transform by D. P. Patil [13]. D. P. Patil [14] study Laplace, Sumudu, Elzaki and Mahgoub transforms comparatively and apply them in Boundary value problems. Parabolic Boundary value problems are also solved by Dinkar Patil [15]. For that he used double Mahgoub transform.

Soham transform is used to obtain the solution of system of differential equations by D. P. Patil et al [16]. D. P. Patil et al also used Soham transform for solving Volterra integral equations of first kind [17]. D. P. Patil et al [19] used Anuj transform to solve Volterra integral equations of first kind. Borse,Kapadi and Patil used Emad Falih transform for solving telegraph equations [20] . Recently Zankar, Kandekar and D. P. Patil used general integral transform of error function for evaluating improper integrals[21]. Recently, Dinkar Patil, Prerana Thakare and Prajakta Patil [22] used double general integral transform for obtaining the solution of parabolic boundary value problems. D. P. Patil et al [23] used emad-Sara transform to obtain the solution of telegraph equation. Shirsath, D. P. Patil et al [24] used the HY integral transform for handling growth and Decay problems. Komal Patil, Snehal Patil and Dinkar Patil [ 25] solved Newton's law of cooling by using " Emad- Falih Transform. D. P. Patil et al[26] used HY transform for solving Newton's law of cooling.

**(1)** The proposed transform is defined for an exponential order function:

$$B = \{f(t) : \exists\, K, m_1, m_2 > 0\,, |f(t)| < Ke^{mj\,|t|} \; if \; t \in (-1)^j \times [\,0\,,\infty\,)\} \qquad (1)$$

Where $f(t)$ a function in the set is $B$ , $K$ is a finite constant number, $m_1$ and $m_2$ may or may not be finite.
The kernel function of Emad-Sara Transform symbolized by $ES(\cdot)$ is defined by the equation:

$$ES\{f(t)\} = T(\propto) = \frac{1}{\propto^2}\int_0^\infty e^{-at}\, f(t)\, dt \qquad (2)$$

Where $t \geq 0\,, m_1 \leq \infty \leq m_2$ and $\infty$ is a variable that is used as a factor to the variable $t$ in the function $f$.
***ES* Transform of some fundamental functions:**

| Sr. No. | Functions | $ES$ Transform |
|---|---|---|
| 1 | $k$ | $\frac{k}{\propto^3}$ |
| 2 | $t$ | $\frac{1}{\propto^4}$ |
| 3 | $t^n$ | $\frac{n!}{\propto^{n+3}}$ |
| 4 | $e^{at}$ | $\frac{1}{\propto^2\,(\propto - a)}$ |
| 5 | $\sin at$ | $\frac{a}{\propto^2\,(\propto^2 + a^2)}$ |
| 6 | $\cos at$ | $\frac{1}{\propto\,(\propto^2 + a^2)}$ |
| 7 | $\sinh at$ | $\frac{a}{\propto^2\,(\propto^2 - a^2)}$ |
| 8 | $\cosh at$ | $\frac{1}{\propto\,(\propto^2 - a^2)}$ |

Let $T(\propto)$ be the Emad-Sara Transform of $\left[ES\big(f(t) = T(\propto)\big)\right]$ , and then transform of Emad-Sara is defined as

$$ES\left[f'^{(t)}\right] = -\frac{f(0)}{\propto^2} + \propto T(\propto)$$

**3) Application of Emad-Sara Transform in Population Growth and Decay Problems:**
In this section we obtain Emad Sara transform of growth problem and decay problem:
**3.1) Emad Sara Transfrom for Growth Problem:**
In this section, we use Emad-Sara Transfrom for population growth problem as follows:
The population growth (growth of plant, or a call, or an organ, or a species) is governed by first order linear ordinary differential equation,

$$\frac{dN}{dt} = PN \qquad (1)$$

With initial condition,

$$N(t_\circ) = N_\circ \qquad (2)$$

Where $P$ is a positive real number, $N$ is the amount of population at time $t$ and $N_\circ$ is the initial population at time $t_\circ$.
Applying Emad-Sara Transform on both side of equation (1)

$$ES\left\{\frac{dN}{dt}\right\} = P \cdot ES\{N(t)\}$$

Now applying the property, Emad Sara Transform of derivative of function, on above equation,

$$-\frac{N}{\propto^2} + \propto T(\propto) = P \cdot T(\propto)$$

Since, $t_\circ = 0, N = N_\circ$

$$\propto T(\propto) - P[T(\propto)] = \frac{N}{\propto^2}$$

$$\therefore T(\propto)(\propto - P) = \frac{N_\circ}{\propto^2}$$

$$\therefore T(\propto) = \frac{N_\circ}{\alpha^2(\alpha - P)}$$

Applying inverse Emad Sara Transform on above equation,

$$(ES)^{-1}[T(\propto)] = (ES)^{-1}\left[\frac{N_\circ}{(\alpha - P)\alpha^2}\right]$$

$$\Rightarrow N(t) = N_{\circ}(ES)^{-1}\left[\frac{1}{(\alpha-P)\alpha^2}\right]$$
$$\Rightarrow N(t) = N_{\circ}e^{Pt}$$

which is the required amount of population at time $t$.

**3.2) Emad Sara Transform for Decay Problem:**

In this section, we use Emad-Sara Transform for Decay problem which is given as follows:

The Decay problem of the substance is governed by the first order linear ordinary differential equation.

$\frac{dN}{dt} = -P \cdot N$ (3)

With initial condition, as

$N(t_{\circ}) = N_{\circ}$ (4)

Where $N$ is the amount of substance at time, $t$; $P$ is positive real number and $N_{\circ}$ is the initial amount of the substance at time, $t_{\circ}$.In equation(3), the negative sign to R.H.S is taken becaus of the mass of the substance is decreasing with time and so the derivative $\frac{dN}{dt}$ must be negative.

Applying Emad-Sara Transform on both side of equation (3)

$$ES\left\{\frac{dN}{dt}\right\} = -P \cdot (ES)\{N(t)\}$$

Now, applying the property, Emad-Sara Transform of derivative of function on above equation,

$$-\frac{N}{\propto^2} + \propto T(\propto) = -P \cdot T(\propto)$$

Since, $t_{\circ} = 0$ , $N = N_{\circ}$ ,

$$\propto T(\propto) + P \cdot T(\propto) = \frac{N_{\circ}}{\propto^2}$$

$$T(\propto) = \frac{N_{\circ}}{(\propto + P)\propto^2}$$

Applying inverse Emad-Sara Transform on above equation ,

$$(ES)^{-1}T(\propto) = N_{\circ}(ES)^{-1}\left[\frac{1}{(\propto + P)\propto^2}\right]$$
$$\Rightarrow N(t) = N_{\circ}e^{-Pt}$$

Which is required amount of substance at time, $t$.

**4. Applications:**

In this section, we solve some problems on population growth and decay.

**Application(1):** The population of the city grows at the rate proportional to the number of people presently living in the city. If after two years, the population has doubled and after three years the population is20000, Estimate the number of people initially in the city.

**Solution:**

This problem can be written in mathematical form as:

$\frac{dN}{dt} = PN$ (5)

Where $N$ denote the number of people living in the city at any time $t$ and $P$ is the constant of proportionality.

Consider, $N_{\circ}$ is the number of people initially living in the city at time, $t = 0$ .

Applying Emad- Sara Transform on both sides of equation(5),

$$ES\left\{\frac{dN}{dt}\right\} = P.ES\{N(t)\}$$

Now apply the property of Emad-Sara Transform of derivative of function, in above equation,

$$\frac{N}{\propto^2} + \propto T(\propto) = P \cdot T(\propto)$$

Since, $N = N_{\circ}$ , $t = 0$ ,

$$T(\propto)(\propto - P) = \frac{N_{\circ}}{\propto^2}$$

$$T(\propto) = \frac{N_{\circ}}{\propto^2(\propto - P)}$$

Applying inverse Emad-Sara Transform on above equation,

$$(ES)^{-1}(T(\propto)) = N_{\circ}(ES)^{-1}\left[\frac{1}{\propto^2(\propto - P)}\right]$$

$\Rightarrow N(t) = N_{\circ}e^{Pt}$ (6)

Now, at $t = 2$ , $N = 2N_{\circ}$

$\therefore 2N_{\circ} = N_{\circ}e^{2P}$

$$\therefore 2 = e^{2P}$$
$$\therefore P = \frac{1}{2}\log_e 2$$

$\therefore P = 0.3466$

Now, put $t = 3, N = 20000$

Put this value in equation $(6)$

$$\therefore 20000 = N_{\circ}e^{3P}$$

$$\therefore 20000 = N_{\circ}e^{3(0.3466)}$$

$$\therefore 20000 = N_{\circ}(2.8287)$$

$$\therefore N_{\circ} = 7070.3857 \approx 7070$$

which is the required number of people living in the city, initially.

**Application$(2)$ :** A radioactive substance is known to decay at a rate proportional to the amount present . It initially there is 100 miligrams of the radioactive substance present and after two hours it is observed that the radioactive substance has lost 10 percent of it's original mass, Find half life of the radioactive substance.

**Solution:** This problem can be written in the form as:

$$\frac{dN}{dt} = -PN \qquad (7)$$

Where $N$ denote the amount of radioactive substance at time $t$ and $P$ is the proportionality constant.

Consider $N_{\circ}$ is the initial amount of radio-active substance at time $t = 0$

Applying Emad Sara transform on both side of equation $(7)$,

$$ES\left\{\frac{dN}{dt}\right\} = -P \cdot ES\{N(t)\}$$

Now, applying the property, Emad-Sara Transform of derivative of function , on above equation $\frac{-N}{\propto^2} + \propto T(\propto) = -\ P \cdot T(\propto)$

Since, $N = N_{\circ}, t = t_{\circ}$

$$\propto T(\propto) + P \cdot T(\propto) = \frac{N_{\circ}}{\propto^2}$$

$$T(\propto)(\propto + P) = \frac{N_{\circ}}{\propto^2}$$

$$\Rightarrow T(\propto) = \frac{N_{\circ}}{\propto^2\ (\propto + P)}$$

Since, $t = 0\ , N_{\circ} = 100$

$$T(\propto) = \frac{100}{\propto^2 + P}$$

Now, applying inverse Emad-Sara Transform on above equation,

$$(ES)^{-1}[T(\propto)] = (ES)^{-1}\left[\frac{100}{\propto^2 + P}\right]$$

$$= 100(ES)^{-1}\left[\frac{1}{\propto^2 + P}\right]$$

$$\Rightarrow N(t) = 100e^{-Pt} \qquad (8)$$

Now, by the given condition in the problem at $t = 2$ , the radio-active substance has lost 10 percent of it's original mass 100 miligrams.

So, $N = 100 - 10 = 90$

$$\therefore 90 = 100e^{-2P}$$

$$\therefore e^{-2P} = 0.9$$

$$\therefore -2P = \log_e 0.9$$

$$\therefore P = \frac{-1}{2}\log_e 0.9$$

$$\therefore P = 0.0527$$

We required half time of radioactive substance $(t)$ when $N = \frac{N_{\circ}}{2} = \frac{100}{2} = 50$

Substitute this value in equation $(8)$ ,

$$\therefore 50 = 100e^{-Pt}$$

$$\therefore 50 = 100e^{-0.0527t}$$

$$\therefore 0.5 = e^{-0.0527t}$$

$$\therefore\ -0.0527t = \log_e(0.5)$$

$$\therefore\ -0.0527t = -0.6932$$

$$\therefore t = 13.1537\ hours$$

This is required half life time of radioactive substance.

**References:**


(1) S. R. Kushare, D. P. Patil and A. M. Takate, The new integral transform “Kushare transform”, International Journal of Advanced in Engineering and Management, Vol. 3,Issue 9,September 2021,pp.1589-1592 .

(2) D. P. Patil and S. S. Khakale, The new integral transform "Soham transform", International Journal of Advances in Engineering and Management, Vol. 3,Issue 10,October 2021,pp.126-132.

(3) R. S. Sanap and D. P. Patil, Kushare integral transform for Newton's Law of cooling, International Journal of Advances in Engineering and Management, Vol. 4, Issue 1, January 2022 pp. 166-170.

(4) D. P. Patil, P. S. Nikam, S. D. Shirsath and A. T. Aher, Kushare transform for solving the problems on growth and decay, Journal of Engineering Technologies and Innovative Research, Vol. 9,Issue 4,April 2022,pp.h317-h323.

(5) D. P. Patil, Applications of Sawi transform in Bessel Functions, Aayushi International Interdisciplinary Research Journal ,Special issue No. 86,pp.171-175.

(6) D. P. Patil, Application of Sawi transform of error function for evaluating improper integrals, Journal of Research and Development, Vol. 11,Issue 20,June 2021,pp.41-45.

(7) D. P. Patil, Application of integral transform (Laplace and Shehu) in chemical sciences, Aayushi International Interdisciplinary Research Journal, Special issue No. 88, pp. 437-441.

(8) D. P. Patil, Sawi transform and convolution theorem for initial Boundary Value Problems(wave equation), Journal of Research and Development, Vol.11, Issue 14, June 2021, pp. 133-136.

(9) D. P. Patil, Application of Mahgoub transform in parabolic boundary value problems, International Journal of Current Advanced Research, Vol.9 Issue 4(c), April 2020,pp.21949-21957.

(10) D. P. Patil, Solution of wave equation by double Laplace and double Sumudu transform, Vidyabharti International Interdisciplinary Research Journal, Special Issue IV CIMs 2021, August 2021, pp.135-138.

(11) D. P. Patil, Dualities between double integral transforms, International Advanced Journal in Science, Engineering and Technology, Vol.7, Issue 6, June 2020, pp. 74-82.

(12) S. R. Kushare, D. P. Patil, A. M. Takale, Comparison between Laplace, Elzaki and Mabgoub transforms for solving System of First order and First degree differential equations, Vidyabharti International Interdisciplinary Research Journal, Special Issue IVCIMS 2021, August 2021, pp. 139-144.

(13) D. P. Patil, Aboodh and Mahgoub transform in boundary Value problems of System of ordinary differential equations, International Journal of Advanced Research in Science, communication and Technology, Vol.6, Issue 1, June 2021, pp. 67-75.

(14) D. P. Patil, Comparative study of Laplace, Sumudu, Aboodh, Elzaki and Mahgoub transforms and Applications in boundary value problems, International Journal of Research and Analytical Reviews, Vol. 5,Issue 4,December 2018.

(15) D. P. Patil, Double Mahgoub transform for the solution of parabolic boundary value Problems, Journal of Engineering mathematics and statistics, Vol. 4, Issue 2,May 2020,pp.28-36..

(16) D. P. Patil, Shweta Rathi and Shrutika Rathi, The new integral transform Soham thransform for system of differential equations, International Journal of Advances in Engineering and Management, Vol. 4, Issue 5 , May 2022, PP. 1675- 1678.

(17) D. P. Patil, Y. S. Suryawanshi, M. D. Nehete, Application of Soham transform for solving Volterra integral equations of first kind, International Advanced Research Journal in Science , Engineering and Technology, Vol. 9, Issue 4, April 2022

(18) Sara Falih Maktoof, Emad Kuffi and Elaf Sabah Abbas, “Emad-Sara Transform” a new integral transform, Journal of Interdisciplinary Mathematics, Nov. 2021.

(19) D. P. Patil, P. D. Shinde and G. K. Tile, Volterra integral equations of first kind by using Anuj transform, International Journal of Advences in Engineering and Management, Vol. 4, Issue 5 , May 2022, pp. 917-920.

(20) D. P. Patil, Sonal Borse and Darshana Kapadi, Applications of Emad-Falih transform for general solution of telegraph equation, International Journal of Advanced Research in Science, Engineering and Technology, Vol. 9, Issue 6, June 2022, pp. 19450-19454.

(21) D. P. Patil, K. S. Kandekar and T. V. Zankar, Application of general integral transform of error function for evaluating improper integrals, International Journal of Advances in Engineering and Management, Vol. 4, Issue 6, June 2022.

(22) Dinkar Patil, Prerana Thakare and Prajakta Patil, A double general integral transform for the solution of parabolic boundary value problems, International Advanced Research in Science, Engineering and Technology, Vol. 9, Issue 6, June 2022, pp. 82-90.

(23) D. P. Patil, Shweta Vispute and Gauri Jadhav, Applications of Emad Sara transform for general solution of telegraph equation, International Advanced Research Journal in Science, Engineering and Management, Vol. 9, Issue 6, June 2022, pp. 127-132.

(24) Dinkar Patil, Areen Fatema Shaikh, Neha More and Jaweria Shaikh, The HY integral transform for handling growth and Decay problems, Journal of Emerging Technology and Innovative Research, Vol. 9, Issue 6, June 2022, pp. f334-f 343.

(25) D. P. Patil, S. A. Patil and K. J. Patil, Newton’s law of cooling by, “ Emad- Falih Transform” , International Journal of Advances in Engineering and Management, Vol. 4, Issue 6, June 2022, pp. 1515- 1519.

(26) D. P. Patil, J. P. Gangurde, S. N. Wagh, T. P. Bachhav, Applications of the HY transform for Newton’s law of cooling, International Journal of Research and Analytical Reviews, Vol. 9, Issue 2, June 2022, pp.740-745.