



L-C-R Series Resonance Using wxMaxima

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Abstract:

In the field of electrical and magnetism, the LCR circuit is an important element. Complex numbers and trigonometric identities are widely used in the theory of electric circuits. The entire set of equations is tough to remember and simplify. There are several equations and principles to remember. They're all working for LCR, and the whole situation is more intricate than it was before. As a result, comprehending resonance phenomena in LCR circuits is a difficult task. The purpose of this paper is to demonstrate how to use wxMaxima software to make solving mathematical problems simple easy and interesting.

Keywords: wxMaxima, LCR circuit, resonance.

1. INTRODUCTION

The resistor(R), inductor(L), and capacitor (C) are the three essential constituents of electrical circuits in the electronic world. When these three components are combined in an electrical circuit, they show chaotic instability[1]. Inherent nonlinearities cause this undesirable form of behaviour, which is also very useful in engineering and technology applications, as well as other disciplines of science, for identifying novel phenomena that linear systems are unable to reveal [2]. Nonlinearities can exist in electrical systems due to resistive, inductive, or capacitive circuit components, and they play an important role. Other essential phenomena such as subharmonic and super harmonic resonances, which are especially useful in electrical engineering applications and other fields of engineering and physical sciences, also arise in forced nonlinear systems [3]. Therefore, the analysis of nonlinear resonance phenomena in a series RLC circuit subjected to a dynamic voltage source becomes a necessity in electrical engineering. Thus, this problem requires building a nonlinear mathematical model of the RLC series circuit and carrying out its numerical simulation for a better understanding of their nonlinear electrical properties before all design processes.

A wide range of software is available to solve complex mathematical problems. One of the challenges to using technology in the teaching and learning of mathematics has been recognized as the financial expense. One solution is to introduce open-source software into the classroom, which allows mathematics teachers to integrate it into their teaching and learning. wxMaxima is a wxWidgets-based cross-platform graphical front-end for the computer algebra system. It's a symbol-based mathematical software with a variety of functions for algebraic manipulation, calculus operations, matrix, linear algebra, and other computations. It has appealing displays of mathematical results as well as menus and dialogues that offer quick access to Maxima operations [4].

2. THEORETICAL CONSIDERATION

Resonance is a phenomenon in which the output voltage or current of an electric circuit is at its maximum at a specific frequency. Resonance arises in LCR series circuits when the inductive and capacitive reactances have identical magnitude and phase of a 180° . Due to the large current at resonance, the voltage across the inductor and capacitor is quite high. High-frequency selective filters can be built using series resonance circuits[5]. As a result, studying the LCR circuit is essential. The phenomenon is exemplified by the case when wxMaxima is used to solve differential equations of the circuit.

The series resonant or series tuned circuit is formed when the resistor R, inductor L, and capacitor C are connected in series with a source of e.m.f 'E₀ sin ωt'. (figure-1).

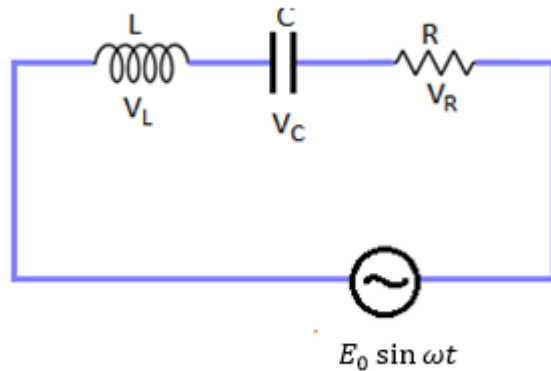


Figure 1. LCR Series Circuit

In an LCR circuit, the inductive reactance X_L is given by 2πfL. This indicates that inductive reactance increases in unison with frequency. Inductive reactance reaches infinity at infinite frequency, and the circuit behaves as an open circuit. The capacitive reactance X_c is given by 1/2πfC and capacitive reactance diminishes as frequency increases. If we plot inductive reactance vs frequency and capacitive reactance vs frequency on the same graph, the two graphs must meet at some point. At that point of junction, the inductive and capacitive reactance becomes equal, and the circuit behaves as a pure resistor (even though the circuit consists of L, C, and R). This phenomenon is called resonance[6]. This occurs at a single frequency known as the resonant frequency. At this frequency, the capacitive reactance (X_c = 1/2πfC) and the inductive reactance (X_L = 2πfL) are equal and opposite in direction. So they get to cancel each other and the circuit is purely resistive.

At resonant frequency, X_L = X_c

$$X_L = 2\pi fL \text{ and } X_C = \frac{1}{2\pi fC}$$

$$2\pi fL = \frac{1}{2\pi fC}$$

At resonance f = f₀ and on solving above equation we get,

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

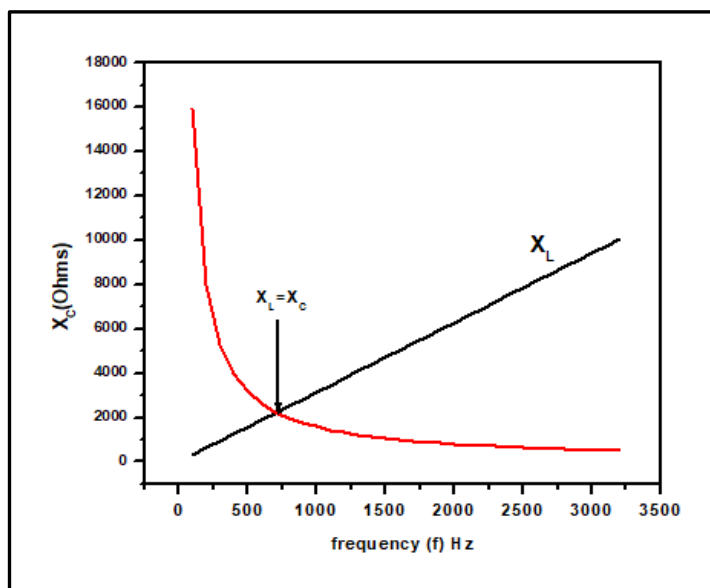


Figure 2. Variation of Capacitor Reactance and Inductor Reactance with frequency

Many researchers have worked on LCR resonance in a variety of ways, including contact measurement, resistivity, metal detection, eddy current, and more. Electrical resonance in an LCR series circuit was illustrated using an XY plotter by G. Chadha et al [7]. Falah R. Awwad & V. Ramachandran et al. rephrased the mathematical analysis of the LCR circuit[8]. The internal resonance relations in the pause states of a nonlinear LCR circuit were studied by Emanuel Gluskin. Yaakov Kraftmakher worked on metal detection using LCR as well as L.E.Helseth studied Eddy currents and contactless electrical resistivity testing[9,10]. and B Fecko and coworkers created a Matlab model and PI controller for a DC Permanent Magnet Motor that uses LCR[11]. wxMaxima, on the other hand, solves differential equations and simplifies trigonometry with a single click. The simplicity of LCR resonance has never been done using Wxmaxima. The objective of this paper is to demonstrate how to quickly solve differential equations, analyze them, and visualize them using the wxMaxima command.

3. LCR CIRCUIT MATHEMATICAL FORMULATION

Developing differential equation:

Kirchhoff's law may be applied to the LCR circuit shown in Figure 1. The total emf is calculated by adding the voltage drop across all of the elements. The voltage across the inductor is $L \frac{di}{dt}$. The voltage across the capacitor is $\frac{q}{C}$. The voltage across the resistor is IR . The total applied sinusoidal voltage is $E_0 \sin \omega t$. As a result, we would write,

$$L \frac{di}{dt} + \frac{q}{C} + IR = E_0 \sin \omega t \quad (1)$$

$$L \frac{d}{dt} \left(\frac{dq}{dt} \right) + \left(\frac{1}{C} \right) q + \left(\frac{dq}{dt} \right) R = E_0 \sin \omega t$$

$$\frac{d^2q}{dt^2} + \left(\frac{1}{LC} \right) q + \left(\frac{dq}{dt} \right) \frac{R}{L} = \frac{E_0 \sin \omega t}{L} \quad (2)$$

This is the LCR circuit differential equation[12]. The solution of a differential equation is obtained by using the commands of wxMaxima.

ode2 is used to find a solution to a differential equation, whereas diff is used to find the derivative. The result is in the format shown below.

Use of wxMaxima :

The indefinite integral code integrates (expr, x) is used to integrate the equation. Substitution is performed with the command subst, which has the following syntax: subst (what, for what, where).

Rational simplification with function ratsimp (rational simplification) is used to simplify rational expressions such as fractions, polynomials, and so on.

trigreduce rewrites a trigonometric expression as a sum of terms, each of which contains only a single sin or cos[13].

```
wxMaxima 16.04.2 [ LCR3.wxmx* ]
File Edit View Cell Maxima Equations Algebra Calculus Simplify Plot Numeric Help

--> E*sin(omega*t)=L*'diff(i,t)+R*i+q/c;
(%o3) E sin(t omega) = q/c + L (d/dt i) + R i

--> i=I*sin(omega*t-phi);
(%o6) i = I sin(t omega - phi)

--> integrate(I*sin(t*omega-phi), t);
(%o10) - I cos(t omega - phi) / omega

--> subst (-(I*cos(t*omega-phi))/omega, q, E*sin(t*omega)=q/c+L*'diff(i,t,1)+R*i);
(%o14) E sin(t omega) = - I cos(t omega - phi) / (c omega) + L (d/dt i) + R i
```

The equation $i = I \sin(\omega t - \phi)$ is the equation of instantaneous current. The equation of the charge is obtained by $q = \int i dt$.

The substitution is made with wxMaxima and the equation for $E \sin(\omega t)$ is obtained.

```

--> diff(I*sin(t*omega-phi),t);
(%o17) I*omega*cos(t*omega-phi)

--> subst(I*omega*cos(t*omega-phi),('diff(i,t,1)),E*sin(t*omega)--(I*cos(t*omega-phi))/(c*omega)+L*('diff(i,t,1))+R*i);
(%o18) E*sin(t*omega)=I*L*omega*cos(t*omega-phi)-I*cos(t*omega-phi)/c*omega+R*i

--> subst(I*sin(t*omega-phi),i,E*sin(t*omega)=I*L*omega*cos(t*omega-phi)-(I*cos(t*omega-phi))/(c*omega)+R*i);
(%o19) E*sin(t*omega)=I*R*sin(t*omega-phi)+I*L*omega*cos(t*omega-phi)-I*cos(t*omega-phi)/c*omega

```

```

--> ratsimp(E*sin(t*omega)=(I*R*c*omega*sin(t*omega-phi)+(I*L*c*omega^2-I)*cos(t*omega-phi))/(c*omega));
(%o26) E*sin(t*omega)=I*R*c*omega*sin(t*omega-phi)+(I*L*c*omega^2-I)*cos(t*omega-phi)/c*omega

--> expand(E*sin(t*omega)=(I*R*c*omega*sin(t*omega-phi)+(I*L*c*omega^2-I)*cos(t*omega-phi))/(c*omega));
(%o2) E*sin(t*omega)=I*R*sin(t*omega-phi)+I*L*omega*cos(t*omega-phi)-I*cos(t*omega-phi)/c*omega

--> ratsimp(I*L*omega*cos(t*omega-phi)-(I*cos(t*omega-phi))/(c*omega));
(%o3) (I*L*c*omega^2-I)*cos(t*omega-phi)/c*omega

```

To

simplify the equation further, the substitution $R = a \cos \phi$ and $(L\omega - \frac{1}{c\omega}) = a \sin \phi$ are made and the value of $a = \sqrt{R^2 + (L\omega - \frac{1}{c\omega})^2}$ is easily obtained using wxMaxima[14].

```

--> trigreduce((I*a*c*cos(phi)*sin(t*omega-phi)+I*a*c*sin(phi)*cos(t*omega-phi))/(c*omega));
(%o14) I*a*cos(phi)*sin(t*omega-phi)+I*a*sin(phi)*cos(t*omega-phi)

--> trigsimp(I*a*cos(phi)*sin(t*omega-phi)+I*a*sin(phi)*cos(t*omega-phi));
(%o16) I*a*cos(phi)*sin(t*omega-phi)+I*a*sin(phi)*cos(t*omega-phi)

--> trigreduce(cos(phi)*sin(t*omega-phi)+sin(phi)*cos(t*omega-phi));
(%o18) cos(phi)*sin(t*omega-phi)+sin(phi)*cos(t*omega-phi)

--> trigreduce(I*a*cos(phi)*sin(t*omega-phi)+I*a*sin(phi)*cos(t*omega-phi));
(%o23) I*a*sin(t*omega)

-->
R=a*cos(phi);
(%o9) R=a*cos(phi)

-->
(I*omega-1/(c*omega))=a*sin(phi);
(%o10) L*omega-1/c*omega=a*sin(phi)

```

```

wxMaxima 16.04.2 [LCR3.wxmx*]
File Edit View Cell Maxima Equations Algebra Calculus Simplify Plot Numeric Help
-->
R=a*cos(phi);
(%o9) R = a cos(phi)
-->
(L*omega-1/(C*omega))=a*sin(phi);
(%o10) L*omega - 1/(C*omega) = a sin(phi)
-->
trigreduce((%o10)^2+((%o9)^2));
(%o14) (L*omega - 1/(C*omega))^2 + R^2 = a^2
(%i1) i=I/sqrt((L*omega-1/(C*omega))^2+R^2);
(%o1) i = I / sqrt((L*omega - 1/(C*omega))^2 + R^2)
    
```

Further trigonometric equations are solved by using the command `trigreduce` and the final equation of the current is obtained.

$$i = \frac{I}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} \tag{3}$$

The quantity $\sqrt{\left(L\omega - \frac{1}{C\omega}\right)^2 + R^2}$ in equation (3) is called impedance or effective resistance of the LCR circuit. When, $L\omega = \frac{1}{C\omega}$, the effective impedance of the circuit is resistive and the circuit behaves as a pure resistor[15].

Further, the plot of the current i against the frequency f is plotted using the command `wxplot2d`. To plot the graph, we have to take certain values of L , C , and R . Here, $L = 0.235\text{H}$, and $C = 0.1\mu\text{F}$ are taken and the cut of frequency $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ is obtained at 1000 Hz. The maximum value of the current is 0.05mA and the range of the frequency is 100-2000Hz. It is shown in figure-3.

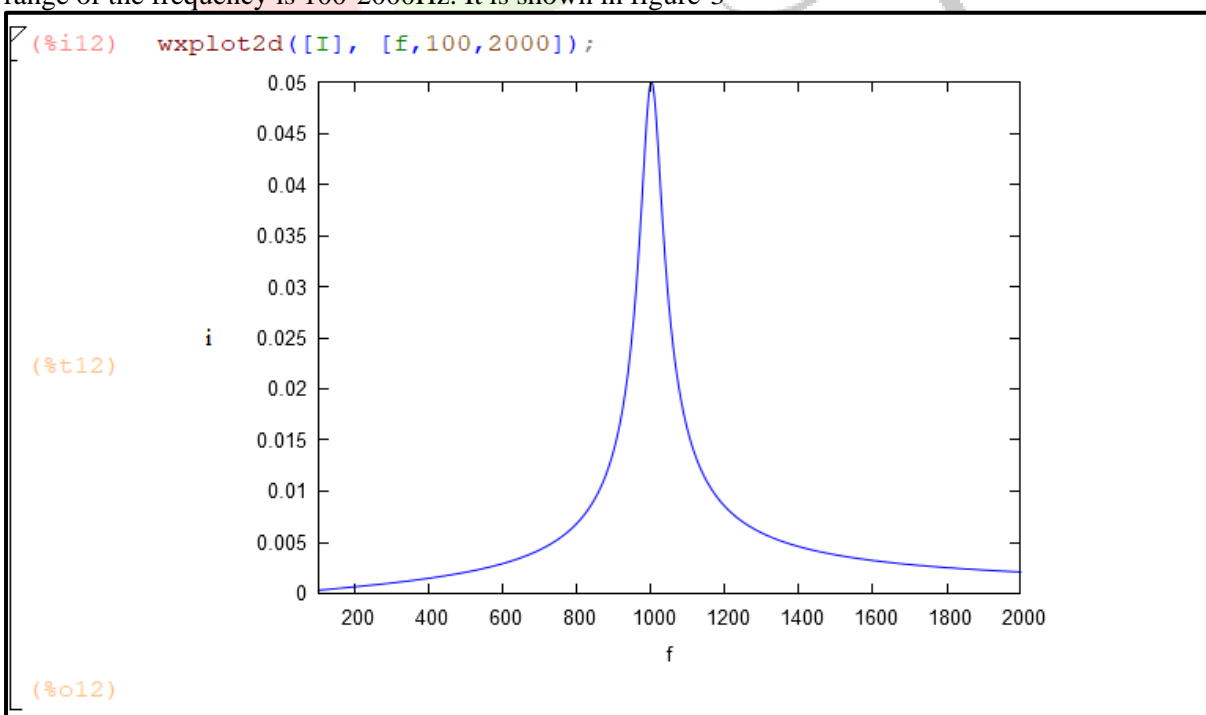


Figure 3. Plot of the equation (3) using wxMaxima

4. Result and Discussion:

The phasor diagram displayed in the figure-4 is one approach to visualize the behavior of the RLC series circuit. The inductive reactance is larger than the capacitive reactance at the frequency depicted in the phasor diagram. This would happen at a frequency that is higher than the resonant frequency.

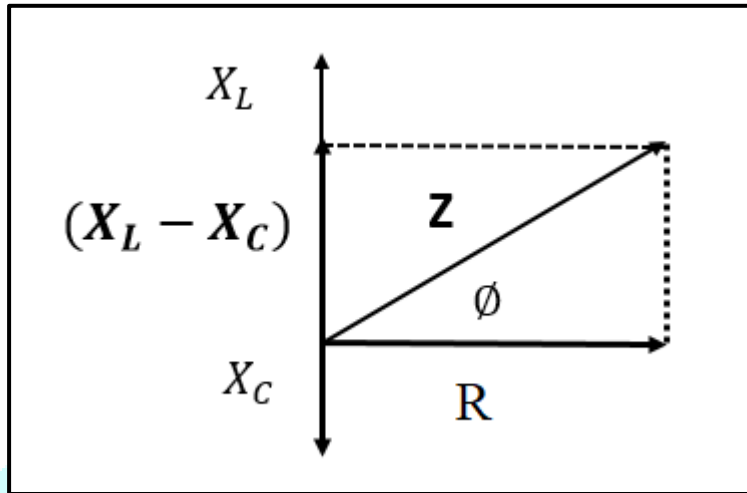


Figure 4. Phasor diagram of LCR Circuit

The phenomenon at which current I reach a maximum is called a resonance, and the frequency in the equation $\omega=2\pi f_0$ is called the resonant frequency. At resonance, the impedance becomes $Z = R$, and the amplitude of the current is maximum. [16]

$$i = \frac{I}{\sqrt{\left(L\omega - \frac{1}{C\omega}\right)^2 + R^2}} = \frac{I}{R}$$

Since $\left(L\omega = \frac{1}{C\omega}\right) = \omega = \frac{1}{\sqrt{LC}}, \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$

The graph is also verified with the data points generated in MS-Excel and the plot is drawn.

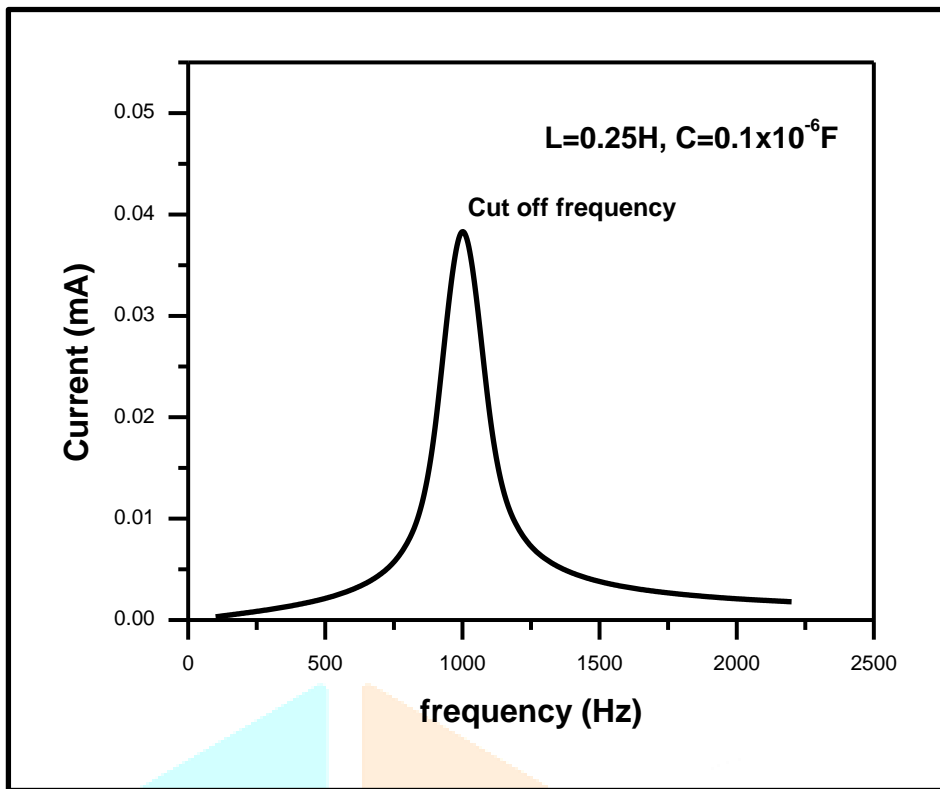


Figure 5. Variation of current against frequency

The most prominent feature of the frequency response of a resonant circuit is a sharp resonant peak in its amplitude characteristics as shown in figure-5. It is plotted for 0.25H and $0.1 \times 10^{-6}\text{F}$

Because impedance is minimum and current is maximum, series resonance circuits are also called Acceptor Circuits. The current in the circuit decreases as we increase the value of R as shown in figure-5

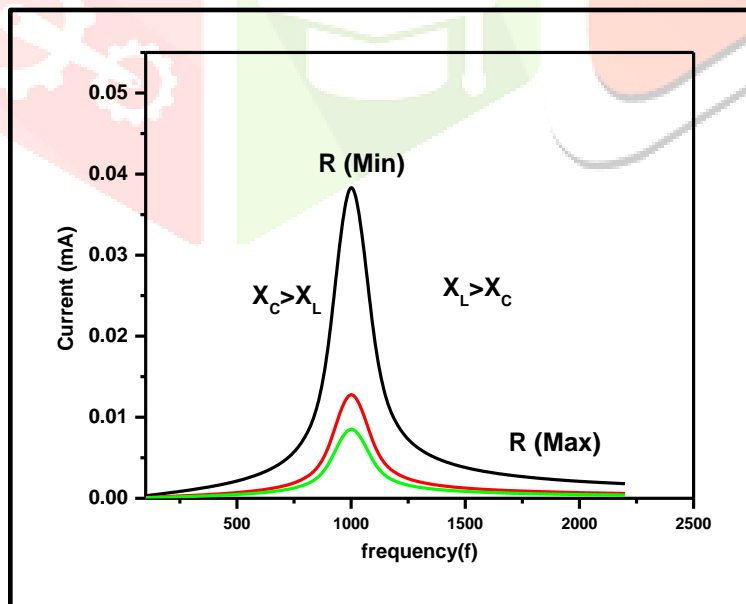


Figure 6. Variation of current against frequency for various values of R

5. CONCLUSION

The LCR series circuit with alternating current input is more difficult to solve manually. Learning and programming with wxMaxima, on the other hand, is easy, quick, and entertaining. It not only simplifies circuit equations, but also allows us to analyze circuits, solve circuit equations, and display the results in graphical form.

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