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GENERALISED CLASSES OF MIXED ESTIMATOR IN REGRESSION MODEL WITH PRIOR INFORMATION

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ABSTRACT

Generalised classes of estimator in regression model with prior information deals with developing generalised classes mixed regression type estimator of the classes than existing ones under different criterion of choosing better estimators expected loss or concentration probabilities. Generalised mixed regression model, estimator is more efficient than other estimators.

Key Words : Steyn rule , mixed regression , concentration, m n d, linear restrictions,

INTRODUCTION

In social, chemical, physical, biological and medical sciences engineering, business, planning and another fields of practical importance, we require the analysis of causes and effects involved in the mechanism of various intricate phenomena under our study. On the basis of best experience or some theoretical justification, the relationship between causes and effects are specified. This specified relationship between causes and effects together with certain other assumptions constitutes the theoretical framework in the form of providing the data generating mechanism of the phenomena under our study. The simple well known theoretical frame-work representing the complex mechanism underlying various intricate phenomena is the linear regression model which is linear in parameters in its basic nature. Linear regression model under certain assumptions contains some unknown parameters which are to be estimated on the basis of apriori information.

In the context of linear restricted regression model, Srivastava and Srivastava (1984), proposed two families of shrinkage estimators by synthesizing Stein rule estimator with the restricted regression estimator in an appropriate manner, discussed their properties with respect to bias vectors, mean squared error matrices and the risks under quadratic loss function when the disturbances are assumed to be small and 1ater on, Srivastava and Chandra (1991) studied these two families of estimators when disturbances are not necessarily normal. we consider the following generalised families of estimators There are several methods of obtaining estimators in the literature but the question now arises whether some of many possible estimators are better in some sense, than the others. To solve this problem, we present some criteria of judging the performance of an estimator that will help us in deciding whether one estimator is better than another.

Let the classical linear regression model be

$$\underset{T\times 1}{y} = \underset{T\times p}{x}\ \underset{p\times 1}{\beta} + \underset{T\times 1}{u} \qquad (1)$$

where u follows a multivariate normal distribution with mean vector

$E(u) = 0$ and dispersion matrix $E(u\,u') = \sigma^2 I_T$, σ^2 being unknown variance of disturbances.

Let the available prior stochastic information on coefficient vector β be in the form

$$q = Q\beta + \upsilon \qquad (2)$$

where $E(q) = Q\beta$, q is a $J\times 1$ vector, Q is known $(J\times p)$ matrix of full row rank and υ is a $(J\times 1)$ vector of disturbance, *u and v are uncorrelated* where u follows a multivariate normal distribution with mean vector

$$E(u) = 0$$

and dispersion matrix

$$E(u\,u') = \sigma^2 I_T$$

σ^2 being unknown variance of disturbances.

For u following the multivariate normal distribution with mean vector zero and dispersion matrix $\sigma^2 I_T$, we know that the ordinary least square (OLS) estimator

$$b = (x'x)^{-1}x'y \tag{3}$$

is the best linear unbiased estimator of β and dispersion matrix $\sigma^2 (x'x)^{-1}$.

Mixed Regression Model

Theil and Goldberger (1961) developed the mixed regression model using the linear restrictions RB r stochastically. Thus, we have the linear restrictions on the parameter vector β as

$$r = R + \beta v \tag{4}$$

where r is a Jx1 vector of j (s p) random variables, R is a Jxp matrix with known elements and v is a Jx1 vector of stochastic elements having mean vector null dispersion matrix and the elements of v are un correlated with those of u , that is

E (v) = 0 And E (u v') = 0.

Noting E (v) = 0 , we see that the restrictions may be written as which shows that r contains the unbiased estimator of J linear parametric functions. Incorporating the available a priori stochastic information Theil and Goldberger (1961) introduced the mixed regression estimator given by

$$b_{TG} = [(x'x) + \sigma^2 Q'\Psi^{-1}Q]^{-1}(x'y + \sigma^2 Q'\Psi^{-1}q) \tag{5}$$

which is an unbiased estimator of β and has the dispersion matrix

$$V(b_{TG}) = \sigma^2[(x'x) + \sigma^2 Q'\Psi^{-1}Q]^{-1} \tag{6}$$

When the disturbance variance σ^2 is known, it may be replaced by its unbiased estimator

$$s^2 = \frac{1}{T-p}(y - xb)'(y - xb) \tag{7}$$

and thus, the adaptive version of the mixed regression estimator b_{TG} is

$$b_T = (x'x + s^2 Q'\Psi^{-1}Q)^{-1}(x'y + s^2 Q'\Psi^{-1}q) \tag{8}$$

Srivastava and Srivastava (1983) considered the following two families of estimators

$$b_{TS} = \left[1 - \frac{k(y-xb_T)'(y-xb_T)}{b'_T C b_T}\right] b_T \tag{9}$$

and

$$b_{ST} = \left[x'x + \frac{(y=xb_S)'(y-xb_S)Q'\Psi^{-1}Q}{T-p}\right]^{-1} \times \left[x'y + \frac{(y=xb_S)'(y-xb_S)}{T-p}\, Q'\Psi^{-1}q\right] \tag{10}$$

$$b_{gT} = g(z^*)b_T \tag{11}$$

$$b_{hT} = [x'x + (y - xh(z)b)'(y - xh(z)b)Q'\Psi^{-1}Q]^{-1} \times \left[x'y + \frac{(y-xh(z)b)'(y-xh(z)b)Q'\Psi^{-1}q}{T-p}\right] \tag{12}$$

$$z^* = \frac{(y-xb_T)'(y-xb_T)}{b'_T x C b_T}, \tag{13}$$

and

$$z = \frac{(y-xb)'(y-xb)}{b'Cb}, \tag{14}$$

g(z)* and *h(z)* satisfying the validity conditions of Taylor's series expansion with appropriate finite expectations and having first two derivatives bounded, are the functions of z^* and z such that

$$g(z^*=0) = 1,\ h(z=0) = 0$$

and

$$g(z^*) = O(1),\ h(z) = O(1)$$

as $\sigma \to 0$ respectively and z^*, z have least first $k\ (\geq 4)$ moments finite. Here an attempt has been made to analyse the properties of the estimator b_{gT} by Singh et al. (1995) and b_{hT} with respect to the criterion of the concentration probability around the true parameter.

Concentration Probabilities of Estimators, we derive the small σ asymptotic expression for the sampling distributions of the estimators

$$y = x\beta + \sigma\omega \tag{15}$$

so that ω follows multivariate normal distribution with mean vector zero and dispersion matrix I_T.

$$\left.\begin{aligned} Z &= (x'x)^{-\frac{1}{2}}x'\omega \\ \bar{P}_x &= [I - x(x'x)^{-1}x'] \end{aligned}\right\} \tag{16}$$

we observe that (i)Z follows multivariate p normal distribution with mean vector zero and dispersion matrix I_p, (ii) $\omega'\bar{P}_x\omega$ follows a chi square distribution with

$$n = T - p \qquad \text{degrees of freedom} \tag{17}$$

.

The estimation error of b_{gT} {see Singh et al. (1995)} may be written a

$$(b_{gT} - \beta) = \sigma\upsilon_1 + \sigma^2\upsilon_2 + \sigma^3\upsilon_3 + \sigma^4\upsilon_4 + O(\sigma^5) \tag{18}$$

Standardizing the estimation error $(b_{gT} - \beta)$, we have

$$r_g = \frac{1}{\sigma}(x'x)^{-\frac{1}{2}}(b_{gT} - \beta) \tag{19}$$

$$= A_0 + \sigma A_1 + \sigma A_2 + \sigma^2 A_2 + \sigma^3 A_3 + O(\sigma^4) \tag{20}$$

The characteristic function of r_g is given by

$$\begin{aligned}\phi_g(h) &= E\left(e^{ih'r_g}\right) \\ &= E\left(e^{ih'A_0}e^{(\sigma ih'A_1 + \sigma^2 ih'A_2 + \sigma^3 ih'A_3 + O(\sigma^4))}\right)\end{aligned} \tag{21}$$

By using the results in (17) , the characteristic function of the vector r_g to order $O(\sigma^3)$, is

$$\emptyset_g(h) = (1 + \sigma\emptyset_1 + \sigma^2\emptyset_2 + \sigma^3\emptyset_3)e^{-\frac{1}{2}h'h} \tag{22}$$

$$\emptyset_1 = -\frac{ig'(0)n}{\emptyset}(\alpha'_1 h)$$

$$\emptyset_2 = -n(h'Gh) - \frac{(n+2)}{2n}(h'Ah) - \{g'(0)\}^2\frac{n(n+2)}{2\emptyset^2}(\alpha'_1 h)^2$$

for in similar manor other terms for above equation, by the inversion theorem, the joint probability density function of the elements of r_g is given by

$$g(r_g) = \frac{1}{(2\pi)^p}\int_{-\infty}^{\infty} e^{ih'r_g}\emptyset_g(h)dh. \tag{23}$$

Substituting ϕ_g (h) from (18) in (19) and utilizing the following results for a fixed vector *a* and a fixed matrix A:

Noting the concentration probability of the estimator b_{gT} around β for the region bounded by the constants $\overline{m}_1, \overline{m}_2 \ldots\ldots \overline{m}_p$ in the *p*-dimensional Euclidean space to be

$$CP(b_{gT}) = \int_{-m_p}^{m_p} \ldots \int_{-m_1}^{m_1} g(r_g)dr_{g1} \ldots dr_{gp} \tag{24}$$

Proceeding on the same lines for the generalized estimator b_{hT} as in case of generalized estimator b_{gT}, we have

$$\text{with } e_j = \frac{m_j e^{-\frac{1}{2}m_j^2}}{\int_0^{m_j} e^{-\frac{1}{2}r_{hj}^2} dr_{hj}}; j = 1, 2, \ldots..p$$

$$\emptyset\,(m) = \int_{-m_p}^{m_p} \ldots \int_{-m_1}^{m_1} \xi(r_h) dr_{h1} dr_{h2} \ldots. dr_{hp.}$$

Generalised classes of estimator in regression model, mixed regression estimator is better estimator from Shukla, also all the estimator is particular case of mixed regression estimator. Extending the results of Srivastava and Srivastava (1984), Srivastava and Chandra (1991) (and and many others, Singh (1994) proposed a generalized class of restricted regression estimators which includes some improved efficient estimators than the existing ones. Further Singh et al. (1995) considered a generalized class of mixed regression estimators containing an improved subclass of estimators over the existing mixed mixed or mixed type regression estimators with respect to a general quadratic loss function in linear regression model under constraints.

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