



# $P_X$ – POWER SET OF EVEN AND ODD NUMBERS OF A NUMBER ‘ X ’

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**Abstract :** The **power set** is defined as the set of all subsets of the set including the set itself and the null or empty set . In this connection we have introduced the newly concept the **power set of number X** is called X-power set of X .To understand concept unit digit, we must know the concept is mainly about the unit digit of a numbers and it's repetitive and non repetitive pattern on being divided by a certain number and the X-power set does not contain null set or empty set . In this paper we have newly presented on X-Power set , Even Power numbers , Odd Power number and Prime Power numbers .

**Keyword:** Numbers , sets , power set , power set of number X ,X-power set , algorithm.

## I. Introduction

Set theory is the branch of mathematical logic that studies sets , which can be informally described as collections of objects . Although objects of any kind can be collected into a set , set theory , as a branch of mathematics .The modern study of set theory was initiated by the German mathematicians Richard Dedekind and Georg Cantor in the 1970s . In particular , Georg Cantor is commonly considered the founder of set theory .Ferreiros , Jose (1996) “ On the Relation between Georg Cantor and Richard Dedekind ” [12] .The positive integers are actually part of a large group of numbers called integers .Boyer.“ Fundamental Steps in the Development of Numeration ”[ 1 ] . Integers are all the whole numbers , both positive and negative .By whole numbers we mean numbers without fractions or decimals. 1,2,3,4..... , are positive integers and-1,-2,-3..... ,are negative integers . Here 0 is an integer which is neither positive nor negative . On an integer number line , all numbers to the right of 0 are positive integers and all numbers to the left of 0 are negative integers . Also the new concept two digit , three digit and so on the natural numbers equal to two focused by Dr.A.Sudhakaraiyah , A. Madhan kumar , 2021 [ 11 ] .

Set theory is the mathematical theory of well determined collections called sets , Jech , Thomas , set theory , Springer Monographs in Mathematics 2003 [ 9 ] of objects that are called members or elements ,of the set .Pure set theory deals exclusively with set , so the only sets under consideration are those whose members are also sets . Set theory , as a separate mathematical discipline , which begins in the work of Georg Cantor . One might say that set theory was born in late 1873 , when he made the amazing discovery that the linear continuum , that is , the real line , is not countable , meaning that its points can not be counted using the natural numbers . Jonson , Philip (1972) , A History of the set Theory [ 8 ] . Set theory is one of the greatest achievements of modern mathematics .

## II . Preliminaries

In this paper we have discussed sum preliminary definitions like digits , numbers , whole numbers , even numbers , even digits and odd numbers , odd digits , prime numbers , prime digits , factorial , permutations , combinations , sets , subset , power set .all these we have use our newly developed concept the X – Power set .

## 1. Power set :

In the set theory , the power set of a set A is define as the set of all subsets of the set A including the set itself and the null or empty set . It is denoted by  $P(A)$  . Basically , this set is the combination of all subset including null set and itself of the given set .

**Formula :**  $P(A) = 2^n$ .

## 2. X - Power number

Let X be a number then the X – power set contains either even or odd numbers and itself not contained empty set then the X is called X – power number or let X be any whole number , if we can define an X-Power set then X is called Power number.

For example ,  $X = 45$  ,  $P_X = \{ 4 , 5 , 45 , 54 \}$  .

## 3. X – Power set

Recently we have defined the new definition of X-Power set by Dr.A.Sudhakaraiah and A.Madhan kumar (2022), Department of Mathematics , Sri Venkateswara University Tirupati , Andhra Pradesh , India . In set theory the X-Power set of a number X is denoted by  $P_X$  which can be defined as the set  $P_X$  which consist of the numbers which can be formed by using the digits of the number X , then that set  $P_X$  is called “ the number power set number X ” or “ X-Power set . X-Power set is also defined as  $X = x_1x_2, x_1x_2x_3, x_1x_2x_3x_4, \dots, x_1x_2x_3 \dots x_n$  be a set of whole numbers such that  $X \in W$  or  $X \in I^+$  and  $P_X$  is X-Power set consisting of all possible numbers which can be formed by using digits of a given number X is called X-Power set and repeated each digit of X the number of times while forming the numbers in  $P_X$  as the number of times that digit required in X .

For example  $X = 42$  ,  $P_X = \{ 4 , 2 , 24 , 42 \}$  or  $\{ 2 , 4 , 24 , 42 \}$  . Another example of  $X = 40$  ,  $P_X = \{ 0 , 4 , 40 , 04 = \{ 0 , 4 , 40 \}$  , since  $4 = 04$  . Also  $X = 44$  ,  $P_X = \{ 4 , 4 , 44 , 44 \} = \{ 4 , 44 \}$  . Here  $4 = 4$  and  $44 = 44$  .

## 4. X is a Even power number

Let X be a positive even number then the X - Power set formed by the digits of the number X . If Such a X – Power set contains all the numbers are even numbers then X is called even power number .

For example ,  $X = 48$  is even number then the X – Power set  $P_X = \{ 4 , 8 , 48 , 84 \}$  here all numbers of X – power set is even numbers , therefore  $X = 48$  is a even power number . Suppose  $X = 34$  is a even number then the X – Power set  $P_X = \{ 3 , 4 , 34 , 43 \}$  .

Here all numbers of  $P_X$  are not even numbers .

Since 3 and 43 are not even so that  $X = 34$  is not an even power number .

## 5. X is an Odd Power number

let X be a positive odd number then the X - Power set formed by the digits of the number X . If Such a X – Power set contains all the numbers are odd numbers then X is called an odd power number .

For example ,  $X = 17$  is an odd number then the X – Power set  $P_X = \{ 1 , 7 , 17 , 71 \}$  here all numbers of X – power set is an Odd numbers .Therefore  $X = 17$  is an odd power number .

Suppose  $X = 21$  is an odd number then the X – Power set  $P_X = \{ 2 , 1 , 12 , 21 \}$  .Here all numbers of  $P_X$  are not odd numbers .

Since 2 and 12 are not odd numbers . So that  $X = 21$  is not an odd power number .

## 6. X is a Prime power number

Let X be a positive prime number then the X – Power set formed by the digits of the number X . If such a X – Power set contains all the numbers are prime numbers then X is called prime power number.

For example ,  $X = 37$  is a prime number then the X – Power set  $P_X = \{ 3 , 7 , 37 , 73 \}$  .

Here all numbers of  $X$  – power set is prime numbers and therefore  $X = 37$  is a prime power number . Suppose  $X = 23$  is a prime number then the  $X$  – Power set  $P_X = \{ 2 , 3 , 23 , 32 \}$  .

Here all numbers of  $P_X$  is not prime numbers .

Since 32 is not a prime so that  $X = 23$  is not a prime power number

### III. Main theorems

**1.Theorem :** Let  $X = x_1x_2$  or  $y_1y_2$  or  $z_1z_2$  ..... , be a positive even numbers and all the digits of  $X$  are even if and only if the numbers of the  $X$ -power set  $P_X$  are even numbers only .

**Proof :** Let us consider a number of  $X = x_1x_2$ , be a positive even number and suppose  $x_1x_2$  is a two digit even number or  $y_1y_2$  is a two digit even number and also  $z_1z_2$ . Now we will prove that  $P_X$  be the  $X$ -power set of  $X$  and is the all digits of  $X$  is even digits if and only if in the numbers in the  $X$  - Power set are even numbers only .

In this it will arise two conditions namely , Necessary condition and Sufficient condition .

**Necessary condition :** We consider the digits of the number  $X = x_1x_2$  or  $y_1y_2$  or  $z_1z_2$  ..... , is an even number . Then we want to show that , the numbers in the  $X$ -Power set  $P_X$  of  $X$  is even numbers only . Let  $x_1x_2 \in X$  be a positive even number . By the definition of the  $X$ -power set such that ,  $x_1 \in P_X$  and  $x_2 \in P_X$  both are even digits . Since  $P_X$  is the  $X$ -Power set of  $X = x_1x_2$  which means  $P_X$  is formed by the digits of the given number  $X = x_1x_2$  . In this connection we have already considered  $X = x_1x_2$  is a positive even number and also considered the digits of the number  $X = x_1x_2$  is even number . So , All the numbers formed by the digits  $x_1 , x_2 , x_1x_2 , x_2x_1 \in P_X$  of the number  $X = x_1x_2$  are even numbers only . Therefore , by definition the number formed by even digits are even numbers only .

**Sufficient condition :** Let  $x_1 , x_2 , x_1x_2 , x_2x_1 \in P_X$  be a  $X$ -Power set of  $X$  . Our aim to show that the digits of  $X = x_1x_2$  is an even number and also  $x_1$  and  $x_2$  are even digits . Let the numbers in the  $P_X$  are even such that  $x_1 , x_2 , x_1x_2 , x_2x_1$  are even numbers . Which implies that , we get  $P_X$  is  $X$ - Power set . Since  $X \in P_X$  and therefore  $X$  is an even number .

Suppose , if all the digits of the number  $X = x_1x_2$  are not even then it not possible to get all even number in  $P_X$  . Since  $X = x_1x_2$  even number ,  $X = x_1x_2$  and  $x_1 , x_2$  are even .

#### 1.1.Illustration :

Let us consider  $X = x_1x_2 = 42$  where  $x_1 = 4$  and  $x_2 = 2$  are both even  $P_{42}$  be the 42-Power set .  $P_{42} = \{ 2 , 4 , 24 , 42 \}$  . Here 2 , 4 , 24 , 42 are even numbers .

An another example  $X = x_1x_2 = 54$  even number but here  $x_1 = 5$  and  $x_2 = 4$  are not both even . Since  $P_{54}$  be the 54-Power set .  $P_{54} = \{ 5 , 4 , 54 , 45 \}$  . Here 5 , 4 , 54 , 45 all numbers are not even .

#### 1.2.To find $P_X$ is even $X$ - power set with algorithm :

Let us consider any even number  $X = x_1x_2$  or  $y_1y_2$  or  $z_1z_2$  ..... , .

Input :  $X = x_1x_2$  an even number

Out put :  $P_X$  is an even  $X$ - power set

Step 1 :  $X = x_1x_2$  an even number

Step 2 : Store  $x_1 , x_2$

Step 3 : check  $x_1 , x_2$  are even . go to step 4 , otherwise go to step 8

Step 4 : store  $x_1 , x_2 , x_1x_2 , x_2x_1$  in  $P_X$  set which are formed by  $x_1 , x_2$

Step 5 : check  $P_X$  has distinct numbers go to step 7 , otherwise step 6

Step 6 : Remove same or repeated numbers in  $P_X$  and we should take the numbers only one time in  $P_X$  or  $X$ -power .

Step 7 : check all numbers are even in  $P_X$  go to step 9 , otherwise go to step 8

Step 8 : Check the number  $X$  is an even or not

Step 9 :  $P_X$  is an even  $X$ - power set .

### 1.3.Algorithm with illustration :

Input :  $X = x_1x_2 = 48$  is an even number

Output :  $P_X$  is an even  $X$ -power set = { 4 , 8 , 48 , 84 }

Step 1 :  $X = x_1x_2 = 48$  an even number

Step 2 : Store  $x_1 = 4$  ,  $x_2 = 8$

Step 3 : 4 , 8 are even numbers

Step 4 :  $P_X = \{ 4 , 8 , 48 , 84 \}$

Step 5 :  $P_X$  has distinct numbers

Step 6 : 4 , 8 , 48 , 84 are even numbers

Step 7 :  $P_X$  is an even  $X$ -power set.

**2.Theorem :** Let  $X = x_1x_2$  or  $x_1x_2x_3x_2$  or  $x_1x_2x_3x_4$  or  $x_1x_2x_3x_4x_5 \dots \dots \dots x_n$ , be a positive even numbers and all the digits of  $X$  are even if and only if the numbers of the  $X$ -power set  $P_X$  are even numbers only .

**Proof :** The proof as follows theorem 3.1 .

#### 2.1.Illustration :

We consider  $X = 464$  or  $X = 2486$  or  $X = 2222$  and so on all are even numbers as well as even digits . Let us take  $X = 484$  be a positive even number . Here 4,8 are even digits .

$P_{484} = \{ 4 , 8 , 48 , 44 , 84 , 448 , 484 , 844 \}$  . Here all 4 , 8 , 48 , 44 , 84 , 448 , 484 , 844 numbers in  $P_{484}$  are even .

#### 2.2.Conversely with illustration :

Let us take the number  $X = 2322$  is even number but 3 is not even digit of  $X$  .  $P_{2322} = \{ 2 , 3 , 22 , 23 , 32 , 223 , 232 , 322 , 2223 , 2232 , 2322 , 3222 \}$  . Here 3 , 223 , 2223 are not even numbers so that  $P_{2322}$  is not even  $X$ -power set.

**3.Theorem :** Let  $X = x_1x_2$  or  $y_1y_2$  or  $z_1z_2 \dots \dots \dots$ , be the positive odd numbers and all the digits of  $X$  are odd if and only if the numbers of the  $X$  - Power set  $P_X$  are odd numbers only .

**Proof :** We consider for any two digit odd number  $X$  where  $X = x_1x_2$  or  $y_1y_2$  or  $z_1z_2 \dots \dots \dots$ , be the positive odd numbers such that  $x_1x_2 \in \mathbb{I}^+$  and also  $x_1 , x_2 \in \mathbb{I}^+$  .

We will prove that  $P_X$  be the  $X$ -power set of  $X$  and all digits of  $X$  is odd digits if and only if in the numbers in the  $X$  - Power set  $P_X$  are odd numbers only .

We will prove the theorem by the Necessary and Sufficient conditions .

**Necessary condition :** If all the digits of the number  $X = x_1x_2$  is odd digits .Then ,We want to show that , the numbers in the  $X$ -Power set  $P_X$  of  $X$  is odd numbers only .

Since ,  $x_1 \in P_X$  and  $x_2 \in P_X$  both are odd digits .  $P_X$  is the  $X$ -Power set of  $X = x_1x_2$  which means  $P_X$  is formed by the digits of the given number  $X = x_1x_2$ . In this connection we have already considered  $X = x_1x_2$  is a positive odd number and also considered the digits of the number  $X = x_1x_2$  is odd number. So ,All the numbers formed by the digits  $x_1 , x_2 , x_1x_2 , x_2x_1 \in P_X$  of the number  $X = x_1x_2$  are odd numbers only . Therefore , the number formed by odd digits are odd numbers only .

**Sufficient condition :** Let  $x_1 , x_2 , x_1x_2 , x_2x_1 \in P_X$  be a  $X$ -Power set of  $X$  .

Our aim to show that , the digits of  $X = x_1x_2$  is odd number and also  $x_1$  and  $x_2$  are odd digits .Let the numbers in the  $P_X$  are odd this means that  $x_1, x_2, x_1x_2, x_2x_1$  are odd numbers .Which implies that , we get  $P_X$  is  $X$ - Power set ,  $X \in P_X$  therefore  $X$  is an odd number.

Suppose ,If all the digits of the number  $X = x_1x_2$  are not odd then it not possible to get all odd number in  $P_X$  . Since  $X = x_1x_2$  odd number ,  $X = x_1x_2$  and therefore  $x_1, x_2$  are odd .

### 3.1. Illustration :

Let us consider  $X = x_1x_2 = 53$  , Where  $x_1 = 5$  and  $x_2 = 3$  are both odd

$P_{53}$  be the 53-Power set that is  $P_{53} = \{ 5, 3, 35, 53 \}$  . Here 5, 3, 35, 53 are odd numbers .

An another example  $X = x_1x_2 = 45$  odd number . But here  $x_1 = 4$  and  $x_2 = 5$  , both are not odd.

$P_{45}$  be the 45-Power set  $P_{45} = \{ 4, 5, 45, 54 \}$  .Here 4, 5, 45, 54 all are not odd numbers.

### 3.2.To find $P_X$ is an odd power set with Algorithm :

Input :  $X = x_1x_2$  an odd number

Out put :  $P_X$  is an odd power set

Step 1 :  $X = x_1x_2$  an odd number

Step 2 : Store  $x_1, x_2$

Step 3 : check  $x_1, x_2$  are odd . go to step 4 , otherwise go to step 8

Step 4 : store  $x_1, x_2, x_1x_2, x_2x_1$  in  $P_X$  set which are formed by  $x_1, x_2$

Step 5 : check  $P_X$  has distinct numbers go to step 7 , otherwise step 6

Step 6 : Remove same or repeated numbers and take one time only in  $P_X$

Step 7 : check all numbers are odd number go to step 9 , otherwise go to step 8

Step 8 : Check the number  $X$  is an odd or not

Step 9 :  $P_X$  is odd power set .

### 3.3.Algorithm with example

Input :  $X = x_1x_2 = 35$  odd number

Out put :  $P_X$  is odd power set

Step 1 :  $X = x_1x_2 = 35$  odd number

Step 2 : Store  $x_1 = 3, x_2 = 5$

Step 3 : 3, 5 are odd numbers

Step 4 :  $P_X = \{ 3, 5, 35, 53 \}$

Step 5 :  $P_X$  has distinct numbers

Step 6 : 3, 5, 35, 53 are both odd numbers

Step 8 :  $P_X$  is Odd power set .

**4.Theorem :** Let  $X = x_1x_2$  or  $x_1x_2x_3x_2$  or  $x_1x_2x_3x_4$  or  $x_1x_2x_3x_4x_5 \dots x_n$ , be a positive odd numbers and all the digits of  $X$  are odd if and only if the numbers of the  $X$ -power set  $P_X$  are odd numbers only .

**Proof :** The proof as follows theorem 3.3.

#### 4.1.Illustration :

We consider  $X = 357$  or  $X = 7537$  or  $X = 77557$  and so on all are odd numbers as will as odd digits . Let us  $X = 335$  be a positive odd number . Here 3 , 5 are odd digits .

$$P_{335} = \{ 3 , 5 , 33 , 35 , 53 , 335 , 353 , 533 \} .$$

Here all 3 , 5 , 33 , 35 , 53 , 335 , 353 , 533 numbers in  $P_{335}$  are odd .

#### Conversely with illustration :

Let us take the number  $X = 3233$  is odd number but 2 is not odd digit of  $X$  .

$$P_{3233} = \{ 2 , 3 , 33 , 23 , 32 , 233 , 332 , 323 , 333 , 2333 , 3323 , 3233 , 2333 , 3332 \} .$$

In this we have observed that 2 , 32 are not odd numbers so that  $P_{3233}$  is not odd  $X$ -power set.

### III.Conclusion

In this paper we discussed the  $P_x$  – power set of even and odd numbers of a number  $X$  and using different numbers of  $X$  which are even and odd numbers. In this paper we have newly focused about the  $X$ -power set . And also we are going to find newly concept about the order of  $X$ -power set of  $X$  with all positive integers .

### IV.Acknowledgment

The authors are thankful to the referees for their valuable comments which have lead to implements in the presentation of the paper .This research was supported in part by the Sri Venkateswara University , Tirupati. Andhra pradesh , India .

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