



Static, Dynamic And Buckling Analysis Of FGM (SS/Al₂O₃) Coated Stainless Steel Cylinder

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Abstract:

This article focusses on finite element analysis of FGM coated stainless steel cylinder. The present investigation is having two parts. The first part is the development of a finite element (FE) model for FGM coated cylinder by using ANSYS® 15.0. The FE model is used to study the static (circumferential stress & buckling) behaviour of FGM coated cylinder. The later one is dynamic (natural frequency) behaviour of FGM coated cylinder. In this analysis, grading of constituents are SS and Alumina is considered.

Keywords: FGM coated cylinder, compression test, power-law index, critical buckling load

1. Introduction

Functionally graded materials (FGM) are denoted as a new class of advanced composite materials made of two dissimilar phase material viz. metal and ceramic. In FGM, the material properties are varying gradually along the thickness direction and so is often applied in many engineering application to avoid reliability and durability problems which are elevated in the laminated composite material. Hence the FGM has drawn more attention from the researchers and a wide variety of applications has been demonstrated in [1] & [2]. Huang [3] et al exposed the buckling behaviours of FGM cylindrical shell subjected to pure bending load based on Donnell shell theory neglecting pre-buckling deformation and the results were closer with analysis using ABAQUS code. The buckling behaviour of FGM structure (Plate and cylindrical shell) is investigated by Hajlaoui [4] et al using the Enhanced Assumed Strain (EAS) solid-shell element which is based on the first-order shear deformation concept. Sofiyev [5] investigates the dynamic instability of FG sandwich cylindrical shells under static and time-dependent periodic axial loadings using the Shear Deformation Theory (SDT). Rao et al [6] deal that the analysis of FGM shell under thermal and mechanical

load by finite element method and also the free vibration analysis of FGM spherical shell is presented. Finite element formulation is derived using the newly designed cylindrical element for the structural analysis of FG hollow cylinders by Taghvaeipour et al [7]. The Sanders–Koiter theory is applied by Strozzi et al [8] to model the nonlinear dynamic system in the case of FG circular cylindrical shell. Rahimi et al [9] studied the vibrational behaviour of FG cylindrical shell with intermediate ring supports by Sanders' thin shell theory. The hybrid finite element method for predicting the flutter boundaries of FGM circular cylindrical shell is presented by Sabri et al [10] and the results are reliable and efficient. Free vibration analysis of FGM shell structure was done based on discrete double director shell element by Wali et al [11]. The vibration and buckling of cylindrical sandwich shells covered by different types of coatings, such as FGM coating (metal and ceramic) under uniform hydrostatic pressure using First-Order Shear Deformation Theory (FOSDT) are discussed by Sofiyev [12]. Vasiraja et al. [13] deals with the static and dynamic response of layered functionally graded material plate using finite element method. In this study, shell481 element is used to analysis FGM plate in ANSYS®15.0.

In the recent literature, the mathematical modelling of the basic 2D structure of FGM has been established very well while for 3D part profile of FGM is very limited. Since, in the case of complex engineering structure like an aircraft fuselage, it is mandatory to carry out finite element analysis of 2D and 3D structure of FGM cylinder and an attempt has been made to develop FE model of FGM cylinder, based on the commercial FEA package ANSYS®15.0.

2. Evaluation of volume fraction of constituents and material properties of FGM coating

FGM coating which is exhibited as a mixture of ceramic and metal with thickness 'h' is shown in Figure 1. The material on the bottom surface ($z = -h/2$) is ceramic rich, whereas the top surface ($z = h/2$) material is metal-rich. Here 'z' is the distance measured from the middle of the plate.

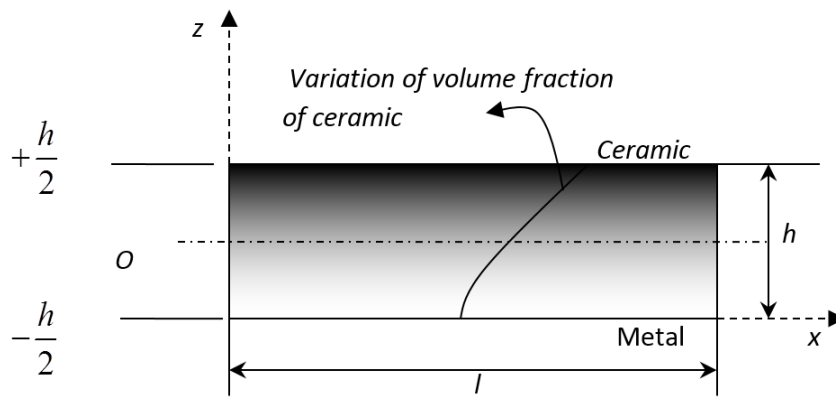


Figure 1. Schematic diagram of FGM

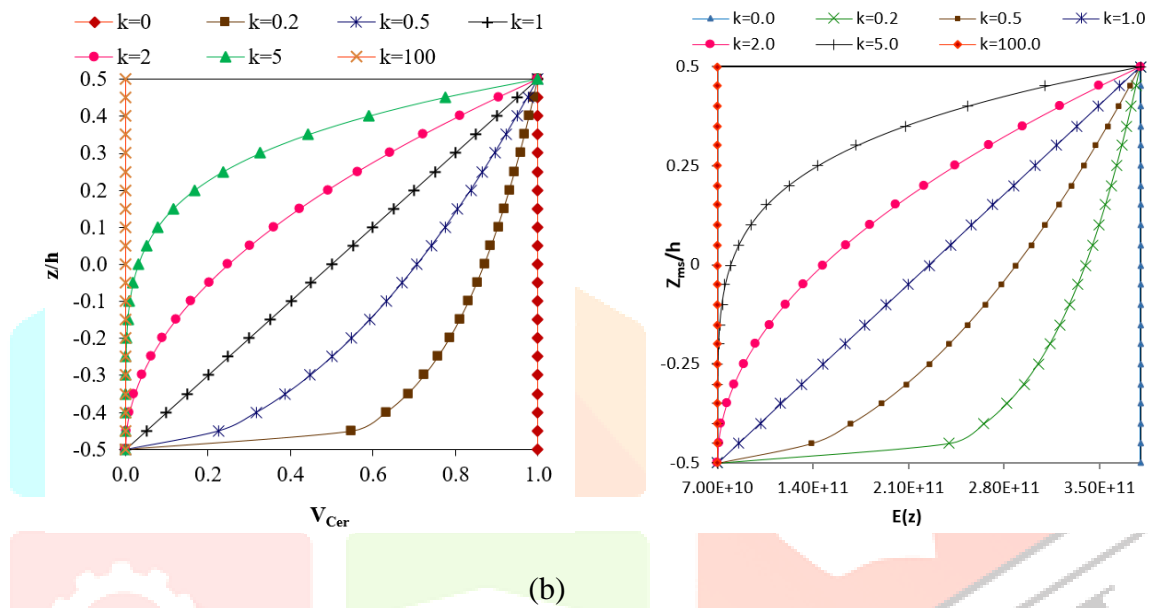


Figure 2. (a) Variation of volume fraction of ceramic (b) Variation of Young Modulus of Alumina/Aluminium FGM through the Thickness

In the present FGM model, grading of materials along the thickness has been done by using Simple power-law with considering the stepwise (layered) variation of the volume fractions of each constituent and the effective properties of modulus of elasticity, the mass density, the Poisson’s ratio for each layer are evaluated by the simple rule of mixtures techniques as represented in equations (1) to (3). The volume fractions of constituents of ceramic (V_{Cer}) at each layer is calculated using equation (4) and variation is shown in Figure 2. (a) and variation of corresponding young’s modulus values is shown in Figure 2.(b).

$$E(z) = E_{Cer}V_{Cer} + E_{Met}V_{Met} \tag{1}$$

$$\nu(z) = \nu_{Cer}V_{Cer} + \nu_{Met}V_{Met} \tag{2}$$

$$\rho(z) = \rho_{Cer}V_{Cer} + \rho_{Met}V_{Met} \tag{3}$$

$$V_{Cer} = \left[\frac{z}{h} + \frac{1}{2} \right]^k \tag{4}$$

$$V_{Cer} + V_{Met} = 1 \quad (5)$$

where the parameter power-law index, 'k' commands the material distribution through the thickness of the plate that takes on values greater than or equal to zero. The V_{Met} is determined based on the simple rule of mixture shown in equation (5).

3. Finite element modelling of FGM Cylinder

The finite element modelling for FGM cylinder has been done by using ANSYS® 15.0 software. In this FE analysis, Shell181 element is used to model the FGM cylinder with layer (stepwise) concept. The details of the FE analysis (analysis type, element type, materials, boundary and loading conditions) are given in Table 1. The finite portion of the coating thickness is considered as each layer and treated as isotropic materials. Material properties have been evaluated in the middle of each layer by using power-law index grading. A twenty layers shell element has been used for FGM cylinder.

Table 1. Finite Element model and analysis description for FGM cylinder

Parameter	Description					
Structure	Cylinder					
Software package Used	ANSYS® 15.0					
Analysis type	Structural- Static, Modal, Buckling and Thermo- mechanical (coupled field)					
Element Name	Shell181 (Structural analysis)					
Material used	Stainless Steel (SS) & Alumina (Al_2O_3)					
Material properties	Material	E (GN/m ²)	ν	ρ (kg/m ³)	K (W/m K)	α (m/K)
	Stainless steel	215	0.3	8000	16.2	17.3×10^{-6}
	Alumina	380	0.3	3800	10.4	7.4×10^{-6}
Number of layers	20					
Layer material properties	Varying from the inner surface to outer surface by simple exponential power-law Index (k) and each layer is considered as isotropic.					
Loading condition	For Static analysis: Uniform pressure on the inner surface For Buckling analysis: Axial load on top of the cylinder.					
Boundary condition	For Static analysis: $1/4^{\text{th}}$ portion of the cylinder is considered due to symmetric Two long edges are having symmetric. For Buckling and vibrational analysis: all degrees of freedom are arrested in the bottom edge.					

3.1 Finite Element Modelling and Convergence Study for Isotropic Cylinder

Initially, convergence study for isotropic cylinder has been done while finding circumferential stress and longitudinal stress using ANSYS®15.0. In this work, the quarter portion of the cylinder is considered and symmetric conditions are applied at two long edge sides. The FE model with applied internal pressure is shown in figure 3 (a) and figure (b). Figure 3 (c) shows the Circumferential stress plot for FGM cylinder when $k = 0$ or ∞ (isotropic material).

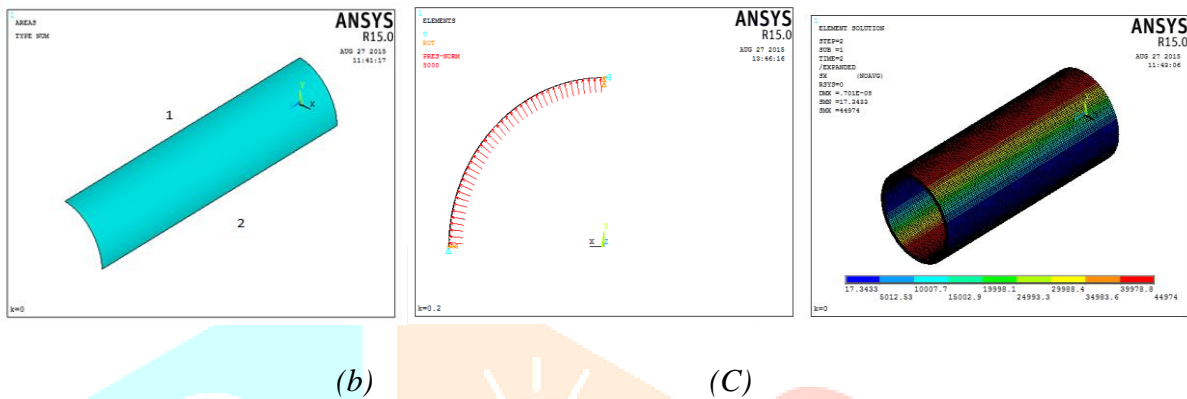


Figure 3. (a) FE model of 1/4th of cylinder (b) FE model of 1/4th of cylinder with inside pressure (c) Circumferential stress plot for cylinder when $k = 0$ or 100

The theoretical formulation for isotropic cylinder are as follow

$$\text{Circumferential stress} = \sigma_c = \frac{P \times d}{2t}$$

where, P = internal pressure in N/m^2 , d = diameter in m, t = thickness in m. The present FE model results (Table 2) are validated with theoretical results and showing the percentage of error is lesser than 0.1%. In this case, $P = 5000 N/m^2$, $d = 0.0054 m$, $t = 0.003 m$.

Table 2. Convergence study for circumferential stress σ_c

Stress (N/m^2)	Theoretical	Present FE Model (Number element used)						Error in %
		20	40	60	100	150	160	
σ_c	45000	44488.2	44856	44938	44977	44989	44989	0.057

3.2 FE Analysis of FGM Cylindrical Shell

The $\frac{1}{4}$ th model of FGM cylinder is modelled & meshed using shell181 element in the ANSYS® 15.0 and the boundary conditions are applied. The FE analysis is carried out for the various 'k' values to find out the circumferential stress variation from the ceramic to metallic phase and tabulated in table 3 and shown in figure 4. It is found that stress is low at the ceramic and metallic phase. The stress is high at the intermediate phase.

Table 3. Circumferential stress σ_c for the various power-law index of FGM cylinder

Stress (N/m ²)	Power-law index (k)								
	0	0.2	1	2	10	20	50	80	∞
σ_c	44989	48461	57334	62767	68269	64440	54590	49426	44989

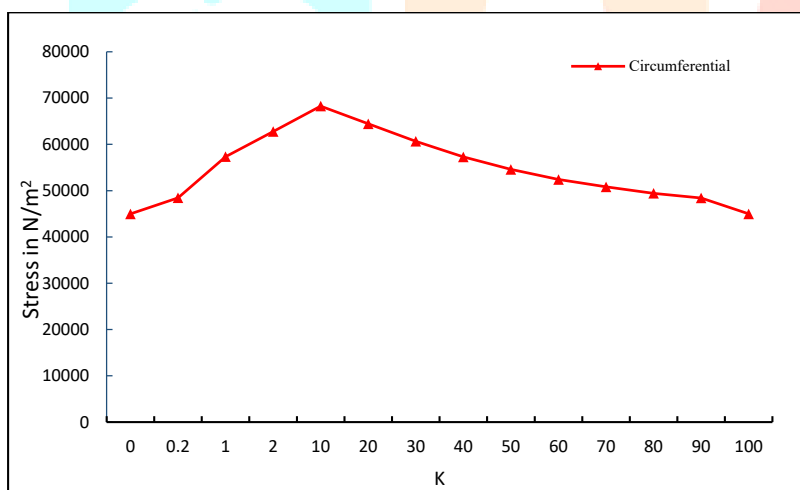


Figure 4. Variation of Circumferential stress for different 'k' value

3.3 Critical Buckling Load Analysis of FGM Cylinder

Buckling is one of the critical phenomena in structural failure due to compression load. This buckling strength of structures depends on many parameters like supports, material properties, etc. The quadratic model is considered for this FE analysis, the long edges are assumed as symmetric and the bottom end is fully constrained for the translation and rotational as shown in figure 5. Since buckling analysis is an eigenvalue problem, a unit load is assumed to act at the top edge of the FGM cylinder. The results of this analysis (Table 4) reveals the critical buckling load is high when $k=0$ and low when $k=100$. This is due to load carrying capacity is high for the ceramic and gradually decreases towards the metallic phase means.

It is concluded that the critical buckling load directly depends on the material composition and position of the measured variable.

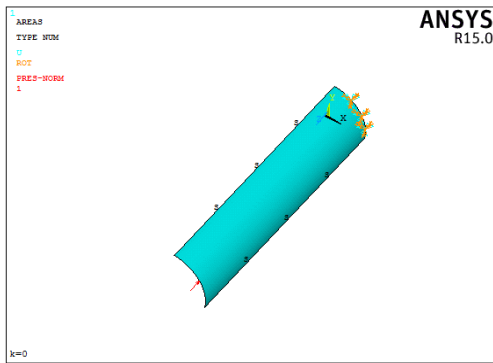


Figure 5. FE model of FGM cylinder for buckling analysis

Table 4. Critical Buckling Load of FGM cylinder for different power-law index

K values	Critical Buckling load (N)
K=0	0.21170E+8
K=0.2	0.13094E+8
K=10	0.13094E+8
K=100	0.11699E+8

3.3 Natural frequency of FGM Cylinder

Vibration is also a mechanical phenomenon whereby failure occurs due to resonance. Therefore it is necessary to find the natural frequency (Modal analysis) of the FGM cylinder to avoid the resonance conditions. The FGM cylinder is designed as per the following specification. Outside Diameter = 76 mm, Inside Diameter = 70 mm, Height of cylinder = 150 mm, Number of Layers = 20, Thickness = 3 mm (total), Each Layer Thickness = 0.15 mm. The quadratic model is considered for the FE analysis. The length sides are symmetric, bottom end is fixed and top end is free as shown in Figure 6 (a).

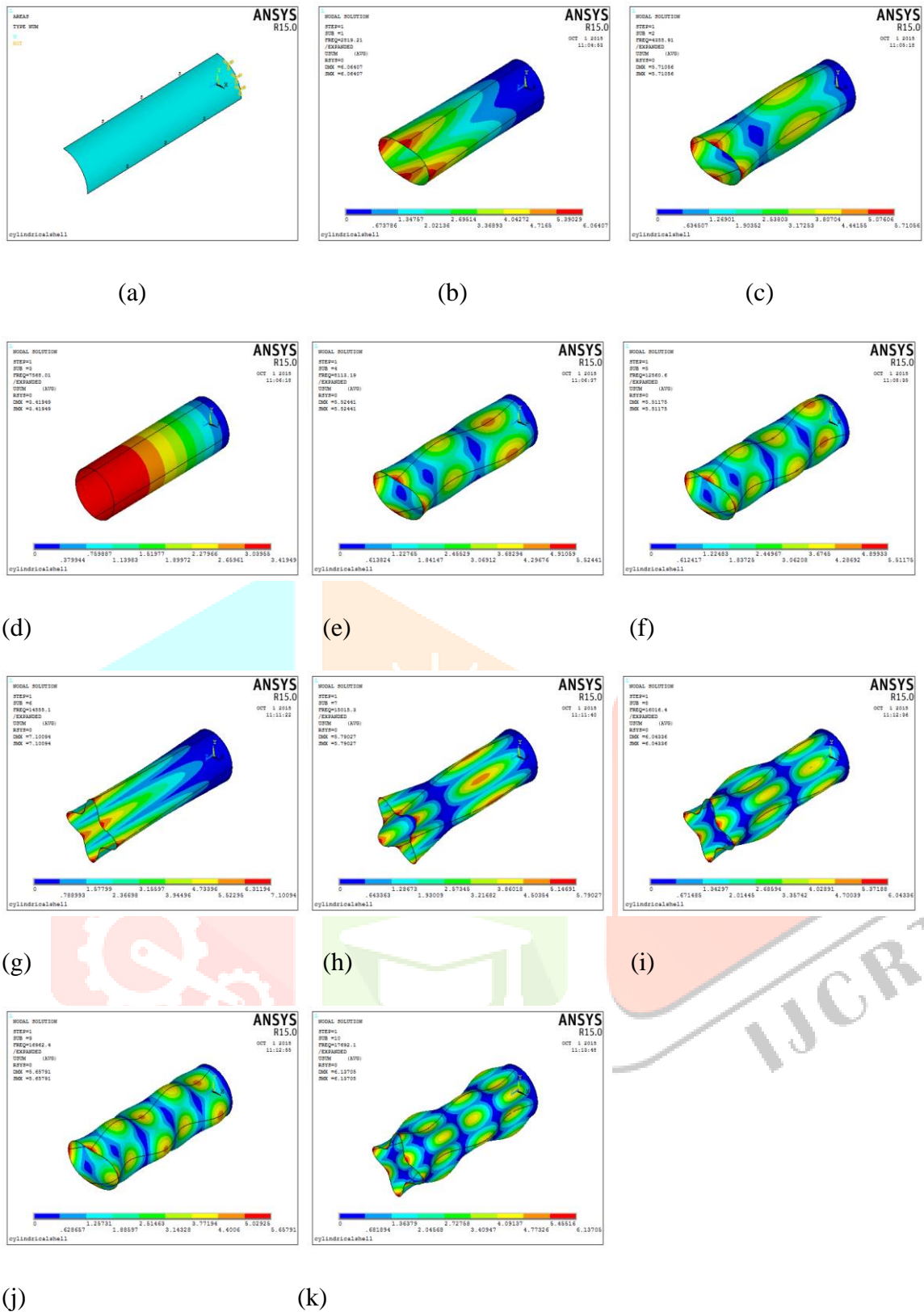


Figure 6. (a) FE model (b) to (k) Mode shape 1 to 10 of FGM cylinder for $k=1.0$

The various mode shapes from 1 to 10 are shown in figure 6 (b) to figure 6 (k). In the modal analysis of FGM cylindrical shell, the natural frequency of the FGM cylinder for the first ten modes are evaluated by varying the power-law index and tabulated in Table 6. The results implies that frequency is decreasing with

increasing power-law index. This is due to the stiffness of the materials is reduced from ceramic to metal. Figure

7 indicates the variation of frequency with different power-law index.

Table 6. Frequency of FGM cylinder for various power-law index

Mode shape	Frequency						
	k=0	k=0.2	k=0.5	k=1	k=1.5	k=2	k=100
1	3284.6	2924.2	2623.8	2375.2	2250.0	2174.2	1784.0
2	9323.8	8318.2	7504.0	6775.6	6381.7	6133.5	5029.4
3	13990	12515	11228	10197	9697.3	9399.1	7631.3
4	15427	13814	12406	11269	10711	10376	8402.1
5	16330	14546	13133	11847	11145	10700	8870.3
6	18781	16768	15136	13665	12866	12361	10152
7	18906	16954	15254	13850	13148	12719	10280
8	24312	21830	19666	17845	16920	16350	13210
9	26977	24129	21794	19689	18551	17830	14617
10	30918	27797	25055	22726	21535	20797	16799

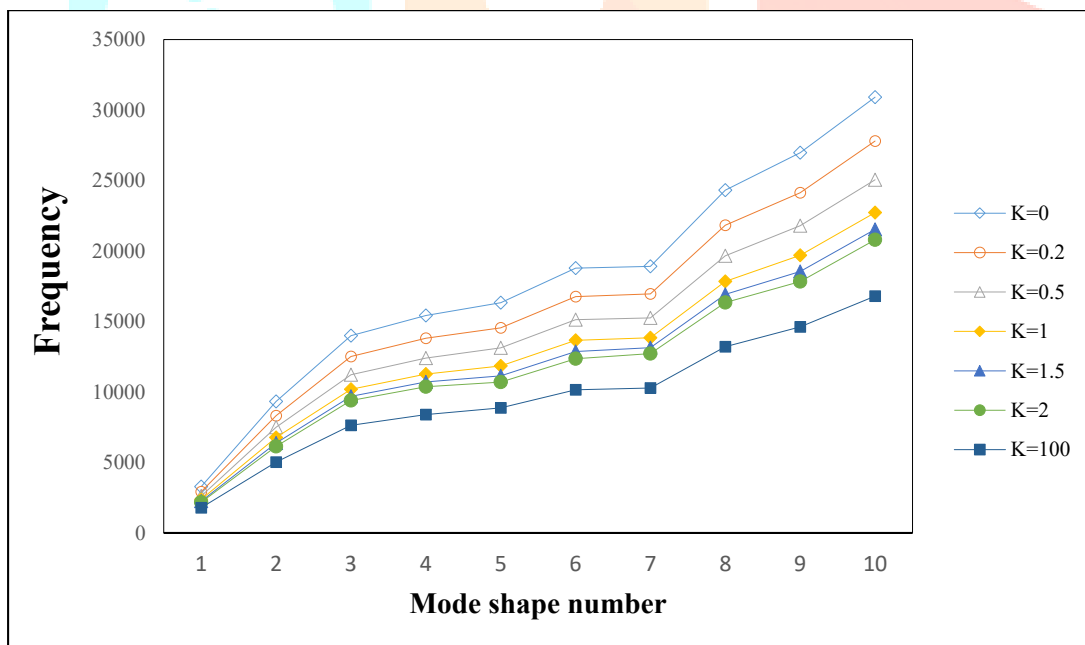


Figure 7. Variation of the frequency with different power-law index

4. Conclusions

In this investigations, the static and dynamic behaviour of FGM coated cylinder is examined by using ANSYS® 15.0 FEA software package. The following conclusions are drawn from present numerical studies,

- 1) The results for circumferential stress is increased with 'k' value.
- 2) Critical buckling load is reducing with increasing 'k' value.

Nomenclature of symbols used

FGM	Functionally Graded Material
CVD	Chemical Vapor Deposition
PVD	Physical Vapor Deposition
SS	Stainless Steel
E_{Cer}, E_{Met}	Young's modulus of ceramic and metal respectively
V_{Cer}, V_{Met}	Volume of ceramic and metal respectively
ν_{Cer}, ν_{Met}	Poisson's ratio of ceramic and metal respectively
ρ_{Cer}, ρ_{Met}	Density of ceramic and metal respectively
k	Power law Index

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