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A New class of Binary Open Sets in Binary Topological Space

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Abstract: In this paper, we define and study $^b\alpha gs$- closed sets in binary topological spaces. Also we study some of its properties. Furthermore, we examine its relationship with other existing sets in binary topological space. Also, we defined binary αgs-closure and defined its properties.

Keywords: *$^b\alpha gs$- closed set, $^b\alpha gs$-open set, $^b\alpha gs$-closure*

I. Introduction

In 2011, S.Nithyanantha Jothi and P.Thangavelu [2] introduced topology between two sets and also studied some of their properties. Topology between two sets is the binary structure from X to Y which is defined to be the ordered pairs (A,B) where A⊆X and B⊆Y. In 2004, Rajamani.M and Vishwanathan.K [8], introduced αgs- closed sets in topological spaces. In 2014, S.N.Jothi and P.Thangavelu [5] introduced generalized binary closed sets in binary topological spaces. Also, S.N.Jothi and P.Thangavelu [3] introduced binary semiopen open sets and discussed some of their properties in binary topological spaces. In continuation, we have found $^b\alpha gs$ -closed sets in binary topological spaces and analyzed some of their properties and also explored its relationship with other existing sets. Furthermore, we introduced binary αgs-closure and discussed its properties.

II. Preliminaries

Definition 2.1[2]:

Let X and Y be any two nonempty sets. A binary topology isa binary structure $M \subseteq P(X) \times P(Y)$ from X to Y which satisfies the following axioms:

(i) $(\emptyset, \emptyset) \in M$; $(X, Y) \in M$.

(ii) $(A_1 \cap A_2, B_1 \cap B_2) \in M$ where $A_1, A_2, B_1, B_2 \in M$

(iii) If $(A_\alpha, B_\alpha : \alpha \in A)$ is a family of members of M, then $(\cup_{\alpha\in A} A_\alpha, \cup_{\alpha\in A} B_\alpha) \in M$.

If M is a binary topology from X to Y, then the triplet (X, Y, M) is called binary topological space and the members of M are called the binary open sets of the binary topological space (X, Y, M).

Definition 2.2.[2] Let X and Y be any two nonempty sets and let (A, B) and $(C, D) \in P(X) \times P(Y)$. If $A \subseteq C$ and $B \subseteq D$, then $(A, B) \subseteq (C, D)$.

Definition 2.3.[2] Let (X, Y, M) be a binary topological space and $(A, B) \subseteq (X, Y, M)$. The ordered pair $((A, B)^{1^\circ}, (A, B)^{2^\circ})$ is called the binary interior of (A, B) where

$(A, B)^{1^\circ} = \cup\{A_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$ and

$(A, B)^{2^\circ} = \cup\{B_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$.

The binary interior of (A, B) is denoted by $b-int(A, B)$.

Definition 2.4.[2] Let (X, Y, M) be a binary topological space and (A, B) ⊆ (X, Y ,M). The ordered pair $((A, B)^{1^\star}, (A, B)^{2^\star})$ is called the binary closure of (A, B) where

$(A, B)^{1^\star} = \cap\{A_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A_\alpha, B_\alpha) \supseteq (A, B)\}$ and

$(A, B)^{2^\star} = \cap\{B_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A_\alpha, B_\alpha) \supseteq (A, B)\}$.

The binary closure of (A, B) is denoted by b − cl(A, B).

Definition 2.5.[3] A subset (A, B) of a binary topological space (X, Y, M) is called

(i) binary regular open if (A, B) = b -int(b -cl(A, B)) and binary regular closedif (A, B) = b -cl(b-int(A, B)).

(ii) binary semi open set if (A, B) ⊆ b- int(b- cl(A, B)). The compliment of binary semiopen set is binary semi closed set.

Definition 2.6[3]. A subset (A, B) of a binary topological space (X, Y, M) is called

(i) binary pre closed if b − cl(b − int(A, B)) ⊆ (A, B)

(ii) binary semi pre closed (or binary β closed if b−cl(b−int(b−cl(A, B))) ⊆ (A, B)

(iii) binary α closed if b − int(b − cl(b − int(A, B))) ⊆ (A, B).

Definition 2.7.[5] Let (X, Y, M) be a binary topological space. Let (A, B) ∈ P (X) ×P (Y). Then the subset (A, B) is called generalised binary closed if b- cl(A, B) ⊆ (U, V) whenever (A, B) ⊆ (U, V) and (U, V) is binary open in (X, Y, M)

Definition 2.8.[9] Let (X, Y, M) be a binary topological space. Then (A, B) ⊆ (X, Y) is called generalized binary semiclosed if b-cl(A, B) ⊆ (U, V) whenever(A, B) ⊆ (U, V) and (U, V) is binary semi open.

Definition 2.9.[8] A subset A of a topological space (X, τ) is said to be αgs-closed if α -cl(A) ⊆ U whenever A ⊆ U and U is semiopen in (X, τ).

III. ^{b}αgs -CLOSED SET

Definition 3.1.

Let (X, Y, M) be a binary topological space. Let (A, B) ⊆ (X, Y). Then (A, B) is called binary α generalized semi-closed (shortly ^{b}αgs -closed) set if ^{b}α-cl(A, B) ⊆ (U, V) whenever (A, B) ⊆ (U, V) and (U, V) is binary semiopen in (X, Y, M).

Theorem 3.2. In a binary topological space (X, Y, M), arbitrary union of ^{b}αgs - closed set is ^{b}αgs -closed set but the intersection of two ^{b}αgs - closed set need not be ^{b}αgs -closed set.

Remark 3.3. Let (A, B) be a ^{b}αgs -closed set in (X, Y, M), then A need not be a αgs-closed set in X and B need not be a ^{b}αgs-closed set in Y.

Proposition 3.4 Let (X, Y, M) be a binary topological space. Then,

(i) Every ^{b}α-closed set is ^{b}αgs -closed set.

(ii) Every ^{b}αgs -closed set is binary semiclosed set.

(iii) Every generalised binary semi closed set is ^{b}αgs -closed set.

(iv) Every generalised binary closed set is ^{b}αgs -closed set.

Theorem 3.5. Suppose (X, ρ) and (Y, σ) are two topological spaces. If A is open in X and B is open in Y. Then (A, B) is ^{b}αgs -open set in binary topological space (X, Y, ρ × σ).

Proof. Since A is open in X and B is open in Y. By proposition 2.15 [2], (A, B)is binary open in ρ × σ. Since every binary open is ^{b}αgs -open, we get (A, B) is ^{b}αgs -open set in (X, Y, ρ × σ).

Theorem 3.6. Let $(X, Y, \mathcal{M})$ be a binary topological space and $A \subseteq X, B \subseteq Y$. If (A, B) is binary open in $(X, Y, \mathcal{M})$, then A^c is ${}^b\alpha gs$ -closed set in X and B^c is ${}^b\alpha gs$ -closed set in Y.

Proof. By proposition 2.14[2], $\mathcal{M}_x = \{A \subseteq X; (A, B) \in \mathcal{M}$ for some $B \subseteq Y\}$ is a topology on X and $\mathcal{M}_y = \{B \subseteq Y; (A, B) \in \mathcal{M}$ for some $A \subseteq X\}$ is a topology on Y. Since (A, B) is a binary open in $(X, Y, \mathcal{M}) A \in \mathcal{M}_x$ and $B \in \mathcal{M}_y$. That is A is open in $(X, \mathcal{M}_x)$ which implies A^c is closed in $(X, \mathcal{M}_x)$. Similarly, B^c is closed in $(Y, \mathcal{M}_y)$. Since, every binary closed set is ${}^b\alpha gs$ -closed set, A^c and B^c are ${}^b\alpha gs$ -closed set in $(X, \mathcal{M}_x)$ and $(Y, \mathcal{M}_y)$.

Remark 3.7. The converse of the above theorem need not be true from the fact that every ${}^b\alpha gs$ -closed sets need not be binary closed sets.

Theorem 3.8. Let (A, B) be a ${}^b\alpha gs$ -closed set in a binary topological space $(X, Y, \mathcal{M})$ and suppose $(A, B) \subseteq (C, D) \subseteq^b \alpha cl(A, B)$. Then (C, D) is a ${}^b\alpha gs$ -closed set.

Proof. Since (A, B) is ${}^b\alpha gs$ -closed set, there exists a binary semiopen set (U, V) such that ${}^b\alpha cl(A, B) \subseteq (U, V)$. Since $(C, D) \subseteq^b \alpha cl(A, B) {}^b\alpha cl(C, D) \subseteq^b \alpha cl({}^b\alpha cl(A, B)) {}^b\alpha cl(C, D) \subseteq^b \alpha cl(A, B) \subseteq (U, V) {}^b\alpha cl(C, D) \subseteq (U, V)$ whenever (U, V) is binary semiopen. Hence (C, D) is ${}^b\alpha gs$ -closed set.

Theorem 3.9. Let (A, B) be binary semiopen and ${}^b\alpha gs$ -closed set. Then (A, B) is binary α-closed set.

Proof. Let (A, B) be ${}^b\alpha gs$ -closed set. Then by definition 2.13, ${}^b\alpha cl(A, B) \subseteq (U, V)$ whenever $(A, B) \subseteq (U, V)$ and (U, V) is binary semiopen. Since (A, B) is binary semiopen and ${}^b\alpha gs$ -closed, we have ${}^b\alpha cl(A, B) \subseteq (A, B)$. Also, $(A, B) \subseteq^b \alpha cl(A, B)$. Hence $(A, B) =^b \alpha cl(A, B)$. Therefore, (A, B) is binary α-closed set.

IV. ${}^b\alpha gs$ -closure

Definition 4.1

Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A, B) \subseteq (X, Y)$. Let

1. $(A, B)^{\alpha gs-1^*} = \cap \{A_\alpha : (A_\alpha, B_\alpha)$ is ${}^b\alpha gs$ -closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$
2. $(A, B)^{\alpha gs-2^*} = \cap \{B_\alpha : (A_\alpha, B_\alpha)$ is ${}^b\alpha gs$ -closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$

The ordered pair $((A, B)^{\alpha gs-1^*}, (A, B)^{\alpha gs-2^*})$ is called the binary αgs-closure of (A,B) and is denoted by ${}^b\alpha gs$ -cl(A, B) in the binary topological space $(X, Y, \mathcal{M})$ where $(A, B) \subseteq (X, Y)$.

Proposition 4.2

Let $(A, B) \subseteq (X, Y)$. If (A,B) is binary αgs-closed in $(X, Y, \mathcal{M})$, then $(A, B) = {}^b\alpha gs$ -cl(A, B)

Proof. Suppose (A, B) is binary αgs-closed set, we have $(A, B) \subseteq {}^b\alpha gs$ -cl(A, B). Therefore, $A \subseteq (A, B)^{\alpha gs-1^*}$ and $B \subseteq (A, B)^{\alpha gs-2^*}$. Since (A, B) is ${}^b\alpha gs$ -closed set containing (A, B), we have $((A, B)^{\alpha gs-1^*}, (A, B)^{\alpha gs-2^*}) \subseteq (A, B)$. Hence, $(A, B) = {}^b\alpha gs$ -cl(A, B).

Proposition 4.3

Suppose $(A, B) \subseteq (C, D) \subseteq (X, Y)$ and $(X, Y, \mathcal{M})$ is a binary topological space. Then,

1. ${}^b\alpha gs$ -cl$(\emptyset, \emptyset) = (\emptyset, \emptyset)$
   ${}^b\alpha gs$ - cl$(X, Y) = (X, Y)$
2. $(A, B) \subseteq {}^b\alpha gs$ -cl(A, B)
3. $(A, B)^{\alpha gs-1^*} \subseteq (C, D)^{\alpha gs-1^*}$
4. $(A, B)^{\alpha gs-2^*} \subseteq (C, D)^{\alpha gs-2^*}$
5. ${}^b\alpha gs$ - cl$(A, B) \subseteq {}^b\alpha gs$ -cl(C, D)
6. ${}^b\alpha gs$ –cl $({}^b\alpha gs$ -cl$(A, B)) = {}^b\alpha gs$ -cl(A, B)

Proof. (i) and (ii) are obvious.

(iii) $(A, B)^{\alpha gs-1^*} = \cap \{A_\alpha : (A_\alpha, B_\alpha)$ is ${}^b\alpha gs -$ closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$
$\subseteq \cap \{A_\alpha : (A_\alpha, B_\alpha)$ is ${}^b\alpha gs -$ closed and $(C, D) \subseteq (A_\alpha, B_\alpha)\}$
$= (C, D)^{\alpha gs-1\star}$

Similarly, (iv) also holds.

(v) ${}^b\alpha gs - cl(A, B) = ((A, B)^{\alpha gs-1^*}, (A, B)^{\alpha gs-2^*})$
$\subseteq ((C, D)^{\alpha gs-1^*}, (C, D)^{\alpha gs-2^*})$
$=^b \alpha gs - cl(C, D)$.

(vi)It follows from the definition

Theorem 4.4

Let (A, B) and (C, D) be contained in (X, Y) where $(X, Y, \mathcal{M})$ be a binary topological space. Then

1. $(A, B)^{\alpha gs-1^\star} \cup (C, D)^{\alpha gs-1^\star} \subseteq (A \cup C, B \cup D)^{\alpha gs-1^\star}$.
2. $(A, B)^{\alpha gs-2^\star} \cup (C, D)^{\alpha gs-2^\star} \subseteq (A \cup C, B \cup D)^{\alpha gs-2^\star}$.

Proof. Since (A, B) and (C, D) are contained in $(A \cup C, B \cup D)$, by the proposition 4.3 [(iii),(iv)], we have $(A, B)^{\alpha gs-1^\star} \subseteq (C, D)^{\alpha gs-1^\star}$ and $(A, B)^{\alpha gs-2^\star} \subseteq (C, D)^{\alpha gs-2^\star}$. Therefore, $(A, B)^{\alpha gs-1^\star} \cup (C, D)^{\alpha gs-1^\star} \subseteq (A \cup C, B \cup D)^{\alpha gs-1^\star}$ and $(A, B)^{\alpha gs-2^\star} \cup (C, D)^{\alpha gs-2^\star} \subseteq (A \cup C, B \cup D)^{\alpha gs-2^\star}$

Theorem 4.5

Let (A, B) and (C, D) be contained in (X, Y) where $(X, Y, \mathcal{M})$ is a binary space. Then,

1. $(A \cap C, B \cap D)^{\alpha gs-1^\star} \subseteq (A, B)^{\alpha gs-1^\star} \cap (C, D)^{\alpha gs-1^\star}$
2. $(A \cap C, B \cap D)^{\alpha gs-2^\star} \subseteq (A, B)^{\alpha gs-2^\star} \cap (C, D)^{\alpha gs-2^\star}$

Proof. Same as before.

Remark 4.6 Examples can be framed to show that the converse of above two theorems need not be true.

V. References

[1] S. Jayalakshmi and A. Manonmani, Binary Pre Generalized Regular Beta Closed Sets in Binary Topological spaces, International Journal of Mathematics Trends and Technology (IJMTT), Volume 66 Issue 7, July 2020.

[2] S. Nithyanantha Jothi and P. Thangavelu, Topology between two sets, Journal of Mathematical Sciences and Computer Applications,1 (3): 95–107, 2011.

[3] S. Nithyanantha Jothi, Binary semiopen sets in binary topological spaces, International Journal of Mathematical Archive-7(9), 2016.

[4] S. Nithyanantha Jothi, P. Thangavelu, Generalized Binary Regular Closed Sets, IRA-International Journal of Applied Sciences, Vol.04, Issue 02 (2016).

[5] Nithyanantha Jothi.S and Thangavelu.P, Generalized binary closed sets in binary topological spaces, Ultra Scientist Vol.26 (1) A, (2014)25-30.

[6] S. Nithyanantha Jothi and P.Thangavelu, On binary topological spaces, Pacific-Asian Journal of Mathematics, 5(2)(2011), 133-138.

[7] S. Nithyanantha Jothi and P.Thangavelu, On binary continuity and binary separation axioms, Ultra Scientist Of Physical Sciences, Vol. 24(1)A, (2012), 121 -126.

[8] Rajamani M. and Viswanathan K. : On αgs-closed sets in topological spaces, Acta Cienicia Indica, Vol No. 3 (2004), 521 -526.

[9] C.Santhini and T. Dhivya, New notion of generalised binary closed sets in binary topological space, International Journal of Mathematical Archive-9(10), 2018.

[10] D. Savithiri and C. Janaki, Binary Regular Generalized Closed Sets In Binary Topological Spaces, IJSRSET, Volume 6, Issue 3, 2019.