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Methods For Solving Fully Fuzzy Linear Programming

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Abstract: The modeling and solving the optimization problems that come across our real life and business is one of the most important daily problems. Fully fuzzy linear programming problem is one of the most powerful tools to deal with the imprecise nature (not well-defined) of data. From studies it is noted that mathematical programming problem has a well defined objective function and set of constraints, the systematic determination of optimal solutions leads to the development of large numbers of methods and algorithms. Fully Fuzzy Linear Programming Problems are most suited way to express the real life optimization problems where the constraints are not always crisp or well defined this is why lots of research work is taking place to solve and find optimum solution fully fuzzy linear programming problems. In this paper various techniques that are emerged in a decade to solve fully fuzzy linear programming problems are discussed.

***Index Terms* - FFLP (Fully Fuzzy Linear Programming), LPP (Linear Programming Problem), Triangular Fuzzy Number, LR-Flat Fuzzy Numbers, MOLP (Multi Objective Linear Programming Problem.**

I. Introduction

Linear programming is one of the most frequently applied operation research techniques. LPP is developing new approaches to fit better with real world problems. Classical LPP have crisp or well defined constraints but in real life problems it is not the case. Constraints are not always well defined or clear or crisp. In such cases FFLP plays a very important role. Basically FFLP can be divided into two categories (a) FFLP problems with inequality constraints. (b) FFLP problems with equality constraints. These are some methods in which FFLP problem is converted into crisp linear programming problem and then obtained crisp LLP is solved to find the fuzzy optimal solution of the FFLP.

II. Basic Definitions:

Fuzzy Set: If X is a universal set then a fuzzy set A on X is defined with the help of membership function μ_A which is a function defined by $\mu_A: X \to [0,1]$ it assigns grades to each element of set A and then a fuzzy set is expressed by ordered pairs $\{(x, \mu_A(x)): x \in X\}$ here the function μ_A is called membership function of fuzzy set A.

III. Types of Fuzzy Sets:

Interval Valued Fuzzy Set: A fuzzy set whose membership function has the form $\mu_A: X \to\in ([0,1]) where \in ([0,1])$ is family of all closed intervals of real numbers in [0, 1].

Type-2 Fuzzy Set: A Fuzzy Set whose membership function has the form $\mu_A: X \to F([0,1])$ $where\ F([0,1])$ is fuzzy power set of [0, 1].

L-Fuzzy Set: A fuzzy set whose membership function has the form $\mu_A: X \to L$ where L is a lattice or Partially Ordered Set.

Level-2 Fuzzy Set: A fuzzy set whose membership function has the form $\mu_A: F(X) \to [0,1]$ where F(X) fuzzy power set of X.

Fuzzy Number: A fuzzy number is a fuzzy set of real line with normal, convex and continuous membership function of bounded support.

Triangular Fuzzy Number: A fuzzy number A is called triangular number with peak point 'a' left width α > 0 and right width β > 0 if its membership functions has the following for$A(x) = \begin{cases} 1 - \frac{a-x}{\alpha}; & a - \alpha \leq x \leq a \\ 1 - \frac{x-a}{\beta}; & a \leq x \leq a + \beta \\ 0 \quad ; & otherwise \end{cases}$

We use this notation as $A = (a, \alpha, \beta)$. This fuzzy number $A = (a, \alpha, \beta)$is called nonnegative if if and only if $a > 0, \alpha > 0, \beta > 0$

Trapezoidal Fuzzy Number: A fuzzy set A is called trapezoidal fuzzy number with tolerance interval [a, b], left width α and right width β if its membership has the following form $A(x) = \begin{cases} 1-\frac{a-x}{\alpha}; & a-\alpha \le x \le a \\ 1 & ; a \le x \le b \\ 1-\frac{x-b}{\beta}; & b \le x \le b+\beta \\ 0 & ; \quad otherwise \end{cases}$

We use this notation as $= (a, b, \alpha, \beta)$. This fuzzy number $A = (a, b, \alpha, \beta)$is called non-negative if if and only if $a > 0, b > 0, \alpha > 0, \beta > 0$.

Ranking Function: A ranking function is a function$R: F(R) \to R$, where F(R) is the set of fuzzy numbers defined on the set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $A = (a, \alpha, \beta)$ be a triangular fuzzy number then $R(A) = \frac{a+2\alpha+\beta}{4}$

LR-Type Fuzzy Number:

A fuzzy number $A \in F$ can be described as $A(x) = \begin{cases} L(\frac{a-x}{\alpha}); & a-\alpha \le x \le a \\ 1 & ; a \le x \le b \\ R(\frac{x-b}{\beta}); & b \le x \le b+\beta \\ 0 & ; \quad otherwise \end{cases}$

Where [a, b] is the peak or core of A,

$L: [0,1] \to [0,1]$ *and* $R: [0,1] \to [0,1]$ are continuous and non-increasing shape functions with $L(0) = R(0) = 1$ *and* $R(1) = L(1) = 0$ we call this fuzzy interval of LR-type and refer to it by $A = (a, b, \alpha, \beta)_{LR}$

IV. Some Arithmetic Operations on Fuzzy Numbers:

Let $A = (a, \alpha, \beta)_{LR}$ *and* $B = (b, \gamma, \delta)_{LR}$ *then* $A \oplus B = (a+b, \alpha+\gamma, \beta+\delta)_{LR}$

$-A = (-a, \alpha, \beta)_{LR}$

$A \ominus B = (a-b, \alpha+\delta, \beta+\gamma)_{LR}$

$A \otimes B = (ab, a\gamma + b\alpha, a\delta + b\beta)_{LR}$ *if A, B are positive.*

$A \otimes B = (ab, b\alpha - a\gamma, b\beta - a\delta)_{LR}$ *if A is negative and B is positive.*

$A \otimes B = (ab, -b\beta - a\delta, -b\alpha - a\gamma,)_{LR}$ *if A and B are negative.*

$\mu \otimes A = \begin{cases} (\mu a, \mu\alpha, \mu\beta)_{LR}, ; \mu > 0 \\ (\mu a, -\mu\beta, -\mu\alpha)_{LR}; \ \mu < 0 \end{cases}$

Let $A_1 = (a_1, b_1, \alpha_1, \beta_1)_{LR}$ and $A_2 = (a_2, b_2, \alpha_2, \beta_2)_{LR}$ be two LR flat fuzzy numbers then $A_1 \oplus A_2 = (a_1 + a_2, b_1 + b_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)_{LR}$

V. Fuzzy Linear Programming:

The most general type of fuzzy linear programming problem is formulated as follows:

Max $\sum_{j=1}^{n} C_j X_j$

Subject to $\sum_{j=1}^{n} A_{ij} X_j \le B_i \ (i \in N_m), X_j \ge 0 \ (j \in N_n)$,

Where A_{ij}, B_i, C_j are fuzzy numbers and X_j are variables whose states are fuzzy numbers$(i \in N_m, j \in N_n)$; the operations of addition and multiplication are operations of fuzzy arithmetic and ≤ denote the ordering of fuzzy numbers.

There are two special types of fuzzy linear programming problems.

Case-I Fuzzy linear programming problem in which only the right-hand side numbers B_i are fuzzy numbers:

Max $\sum_{j=1}^{n} C_j X_j$

Subject to $\sum_{j=1}^{n} a_{ij} X_j \le B_i \ (i \in N_m), x_j \ge 0 \ (j \in N_n)$

Case-II

Fuzzy linear programming problems in which the right-hand-side numbers B_i and the coefficients A_{ij} of the constraints matrix are fuzzy numbers:

Max $\sum_{j=1}^{n} C_j X_j$

Subject to $\sum_{j=1}^{n} A_{ij} X_j \le B_i \ (i \in N_m), X_j \ge 0 \ (j \in N_n)$

VI. Methods for Solving Fully Fuzzy Linear Systems

Real world engineering systems are too complex to be defined in precise terms; imprecision is often involved in any engineering design process. Fully fuzzy systems have an essential role in this fuzzy, modelling, which can formulate uncertainty in actual environment. Mehdi Dehghan et al. worked in LR- Fuzzy numbers which were defined and used by Dubois and Prade with some useful and easy approximation arithmetic operations on them. They employ some heuristics based methods on Dubois and Prade's approach, finding some positive vector x which satisfies Ax=y, where A and b are fuzzy matrix and a fuzzy vector respectively. They proposed some new methods to solve this system that are comparable to the well known methods such as the Cramer's rule, Gaussian elimination, LU decomposition method and its simplifications. They developed a new method to solve square and non-square fuzzy systems. [1]

Since FFLP can be divided into two groups according to fuzziness of decision parameters and decision variables, so many researchers have given solutions by assuming that decision parameters are fuzzy numbers while the decision variables are crisp numbers. Tanaka was the first to initially propose a method for solving the FFLP with fuzzy decision variables. It was observed by T. Allahviranlov et al. that most of the methods developed for fuzzy decision variables were based on application of Zadeh's Extension Principle. In these methods a FFLP is solved by some multiplication and special type of distance. T. Allahviranlov et al. proposed a new method for solving FFLP by applying the concept of comparison. They solved the FFLP by using kind of defuzzification method. The core of the nearest symmetric triangular fuzzy number is applied for an approximation of fuzzy numbers in objective function and coefficient matrix in the constraints. [2]

Any LP model representing real world situation involves a lot of parameters whose values are assigned by experts. However both experts and decision makers frequently do not precisely know the value of these parameters. There for it is useful to consider the knowledge of expert about the parameters as fuzzy data introduced by Zadeh. Amit Kumar et al. proposed Mehar's Method for solving the special type of fuzzy linear programming problems. They proved that it is easy to apply as compare to the other methods for solving the same type of fuzzy linear programming problems. [3]

Lotif et al. proposed a new method to find the fuzzy optimal solution of FFLP problems with equality constraints. This method can be applied only if the elements of the coefficient matrix are symmetric fuzzy numbers. If this method is to be applied on a FFLP in which the elements of coefficient matrix are non-symmetric triangular fuzzy numbers then firstly it is required to approximate the non- symmetric fuzzy numbers to the nearest symmetric fuzzy number, due to this conversion there were some limitations or shortcomings of these methods as the given solution of FFLP problem but these solutions do not satisfy the constraints. That is it was not possible to obtain the fuzzy number of the right hand side of the constraint by putting the obtained solution in the left hand side of the constraint. To overcome the shortcomings of this method, Amit Kumar et al. proposed a new method for finding the fuzzy optimal solution of FFLP with equality constraints. They claimed that new method can overcome the shortcomings of existing method and perform better as compared to existing methods. [4] In this method firstly linear ranking function is used to convert the fuzzy objective function to the crisp objective function by doing this fuzziness of the objective function is neglected. [13] However this method proposed by Amit Kumar at el. is revised by H. Saberi et al. They studied this method and presented a revised method by imposing conditions of non-negative to the variables. [10] Abbas Hatami et al. used the reference of method proposed by Amit Kumar et al. for solving fuzzy linear programming in which the elements of coefficient matrix of the constraints are represented by real numbers and rest of the parameters are represented by fuzzy numbers .[11]

There are some methods in which FFLP is decomposed into three crisp linear programming problems (CLPP) with bounded variable constraints, then three CLPP are solved separately and using its solution the optimal solution of FFLP is obtained. In this method fuzzy ranking function and addition of non negative variables is not used. The solution obtained by this method exactly satisfies the constraints. There for bound and decomposition method can serve managers by providing an approximate best solution to a variety of linear programming models with fuzzy numbers in simple and effective way. [5]

Amit. K. et al. proposed a general method to solve FFLP problems in which all the parameters are flat fuzzy numbers positive or negative both signs. Earlier such type of problems has solutions only if parameters have positive sing. Their method worked better than other methods. [6] In 2013 Amit Kumar et al. proposed Mehar's method to solve FFLP in which some or all the parameters are represented by unrestricted L-R flat fuzzy numbers and they proved that this method can provide solution. [7] Amit Kumar et al. also proposed Mehar's method to deal with sensitivity analysis of such fuzzy linear programming problems in which all the parameters are represented by LR flat fuzzy number. [8] Izaz Ullah Khan et al. used a modified version of well known simplex method for solving FFLP problems. They used ranking method together with Gaussian elimination method. The algorithm proposed by them is flexible, easy and reasonable. They solved FFLP problem directly without converting it into crisp LPP problem. [9]

S. Uday et al. proposed a new method for solving FFLP problems whose coefficients and decision variables are triangular fuzzy number and all the constraints are fuzzy equality or inequality. They used similarity measure and ranking function to transform FFLP into crisp nonlinear programming problem, ranking function was applied on the constraints to convert them into crisp constraints and similarity measure on objective function. [12] A new lexicographic ordering on triangular fuzzy numbers a novel algorithm is proposed by R. Ezzati et al. To solve FFLP problem by converting it to its equivalent multi objective linear programming problem and then it is solved by lexicographic method. They showed it by a theorem that lexicographic optimal solution of MOLP problem can be considered as an optimal solution of the FFLP. Earlier Lotif proposed a new method in this method the parameters of FFLP have been approximated to the nearest symmetric triangular fuzzy numbers. After that a fuzzy optimal approximate solution has been achieved by solving MOLP but shortcoming of his method was that optimal solution of FFLP was not exact that is not reliable. However by the method proposed in [4] an exact solution is achieved using linear ranking function. In this method firstly linear ranking function is used to convert the fuzzy objective function to the crisp objective function by doing this fuzziness of the objective function is neglected. S. Uday et al. revised this method. [13]

A. Hosseinzadeh et al. proposed a new method to solve the fully fuzzy linear programming problems with L-R fuzzy numbers. In this method FFLP problem is converted into an MOLP problem by using fuzzy calculus and then solved by lexicography method and linear programming method. The proposed scheme presented promising results from the aspect of computing efficiency and performance. [14]

VII. Conclusion

In this article various methods that are proposed in last decade for solution of FFLP are analyzed. It is observed that currently lot of research in the area of FFLP is taking place as the fuzziness of the real world is very difficult to exactly model and find an exact solution of true real problem.

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