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CHROMATIC BURNING OF CERTAIN GRAPHS

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Abstract: Consider n≥2 of an undirected simple graph G(m,n), where 'm' represents vertices and 'n' represents lines. A graph with a proper coloring in which each vertex of the adjacent have distinct color. Chromatic & burning number is a minimum number of colors to burn a graph this is called a Chromatic burning. The Chromatic burning number is $\chi_b(G)$. In this paper we are finding a Chromatic burning number for the Path, Cycle and Star graphs.

Introduction

A graph G= (V,E), is a finite undirected connected graph without loops or multiple edges such that V(G) is the vertex set, contains a vertices and E(G) is the edge set contains a edges and there exists an order pair of (v_i,v_j) relationship. The vertices v_i, v_j associated with edge e_k are called the end vertices of e_k.[1]

Coloring Problem has origin from the Four-color Conjecture almost in the year 1852. In 1840 August Mobius(1700-1868) posed a problem , i.e., 'The problem of the five Princes'. [2] "There once was a king who had five sons. In his will he stated that on his death his kingdom should be divided into five regions for his sons in such a way that each region should have a common boundary with the other four. How can this be done? [2]

According to the king's desire his Kingdom has to be divided in to five regions, and it requires 5 colors to color the regions. However, there was no such configuration of five regions. In Any Map it does not contains the five regions, every two of which are neighboring.

Let us make a few observations about Coloring Graphs and the chromatic number. Even for one edge at least two Colors are required to Color the Graph. That is, $\chi(G)=1$. Proper coloring of a graph G, it means that, the assignment of colors to each of the vertices with the distinct colors, such that the adjacent vertices are colored differently. The smallest number of colors in any coloring of a graph G is called the Chromatic Number.[3]

Burning a number is an information diffusion process to spread information widely in a social networks like Face book, twitter….etc.,[4] starts with one burnt vertex called the information sources. It is proposed to model the spread of social contagion, as in the communication of disease from one person to another. 'Burning a Graphs as a model of social contagion' by Antony Bonato et al., in the year 2014, Burning a number concept was introduced by Antony Bonato.[4]

The burning process involves the discrete steps; each vertex is either burned or unburned. To burn a graph at least we need one edge. Initially the process begins with any vertex. In the first round choose any vertex to burn, but, if you choose any unburned vertex then only the chosen vertex will get burned. Continue the steps till unburned vertex to burn. Once the vertex starts get burning and its adjacent vertex also it will get burn only by selecting the unburned vertex. The burning number of a graph G, denoted by b(G), is the minimum number of rounds needed for the process to end.

2. Chromatic Number

2.1. Definition: The chromatic number of a graph, is the smallest number of colors with which it can be colored. We shall denote the chromatic number of a graph by "χ"

Burning a number:

2.2. Definition: Burning a number of a graph G is the minimum number of burning steps required to burn a graph.

Chromatic Burning:

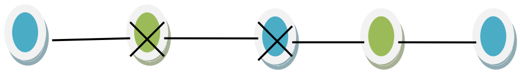
2.3. Definition: A graph $\chi_b(G)$ is said to be Chromatic burning only if Chromatic and burning numbers must be equal, otherwise it is non Chromatic burning.

Star Graph

2.4. Definition: A Star is a tree as well as Complete bipartite graph and it is denoted by S_k.

2.5. Chromatic Burning of a Path graph

Example: Let us discuss the P_5 path graph



Path graph in Figure 1

Crossed vertices are burned and uncrossed vertices are source vertices or burning number of a path graph. The chromatic and burning number of P_5 graph is 2 & 3.

Some observations noted from Burning Number.[5][6]

Note: √ n is a burning number of a path P_n graph as well as cycle C_n graph too
Hypothesis: For any graph on n vertices a burning number is at most √ n

Table 2.5.1

(Pn)	χ (P)	b(P)
1	1	-
2	2	2
3	2	2
4	2	2
5	2	3
6	2	3
7	2	3
8	2	3
9	2	3
10	2	4

2.5.2. Theorem: For a Path graph P_n is a chromatic burning, only if $n \geq 2$, otherwise it is $b(P_n) \geq \chi(P_n)$.

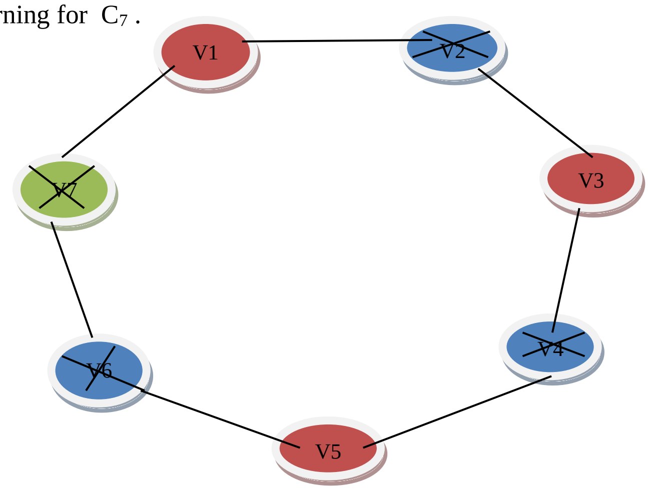
Proof: We know that an even or an odd vertices of a Path P_n graph, n>1, then $\chi(P_n)$ is 2.

Let us consider $2 \leq n \leq 4$ then only the Chromatic burning number is trivially true. Otherwise it is $b(P) \geq \chi(P)$

Clearly it shows that the chromatic burning of a path graph $\chi_b(P)$ is $b(P_n) \geq \chi(P_n)$

2.6. Chromatic Burning of a Cycle graph

Example: Let us discuss the chromatic burning for C_7 .



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Cycle graph in Figure 2

Crossed vertices are burned vertices and uncrossed vertices are source vertices or burning number of a Cycle graph. The chromatic and burning number of C_7 graph is 3 & 3. Where v_1,v_3 and v_5 are sources vertices or burning number of a graph 2. Therefore the chromatic burning of $\chi_b(C_7)$ is 3.

Table 2.6.1

(Cn)	χ (C)	b(C)
3	3	2
4	2	2
5	3	3
6	2	3
7	3	3
8	2	3
9	3	3
10	3	4

Theorem 2.6.1: For a Cycle graph C_n , is a chromatic burning if, $n > 3$, n = 4, 5, 7 & 9 otherwise it is $b(C_n) \geq \chi(C_n)$.

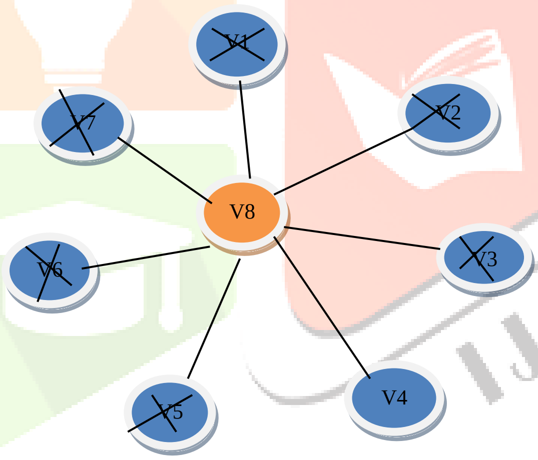
Proof: the burning number of a cycle graph C_n is $\sqrt{}$ n, as we know and from the table

$$\chi(C_n) = \begin{cases} 3, & \text{if n is odd} \\ 2, & \text{if n is even} \end{cases}$$

the chromatic number of odd cycle is 3 and for an even cycle is 2. For a cycle graph, chromatic burning attempts only when n = 4, 5, 7 and 9. Otherwise it is $b(C_n) \geq \chi(C_n)$.

2.7. Chromatic Burning of a Star graph

Example: Let us discuss the Star graph S_8



Star graph in figure 3

Crossed vertices are burned and uncrossed vertices V_4 and V_8 are source vertices or burning number of a star graph. Chromatic and Burning number of S_8 graph is 2.

Table 2.7.1

(Sn)	X(S)	b(S)
1	1	-
2	2	2
3	2	2
4	2	2
5	2	2
6	2	2
7	2	2
8	2	2
9	2	2
10	2	2

Theorem 2.7.1: For a Star graph $S_{n,}$, n>1 is the chromatic burning $\chi_b(S_n) = 2$.

Proof: for n>1, chromatic number and burning number of star graph $\chi(S_n)$ is 2 and $b(S_n)$ is 2, $\chi(S_n) = b(S_n) = 2$.

Therefore the star graph is a chromatic burning graph $\chi_b(S_n)$.

3. Conclusion: In this paper, thus after discussing the chromatic burning χ_b of Path, Cycle and Star graphs, it is observed that the burning number is greater than or equal to chromatic number. This paper can further be extended by identifying the graph families for which these burning and chromatic numbers are equal.

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