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# (4,2) -Fuzzy Set and Their Topological Space

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## Abstract

In this paper we study about the new concept (4,2)-fuzzy set. Then (4,2)-Fuzzy topological space and their properties. We define the (4,2) continuous maps and some properties.

**Key Words:** Fuzzy set, (4,2) Fuzzy set, continuous maps.

## 1.Introduction:

The concept of fuzzy set was defined by Zadeh [8] in the year 1965.In fuzzy set there is degree of membership and degree of membership within [0,1]. In year 1983, Atanassov [1] introduced the new concept Intuitionistic fuzzy set. Application of this fuzzy set is used in many fields such as medical diagnosis, optimization problems and multicriteria decision making. After this Yager [7] introduced a generalization of intuitionistic fuzzy set called Pythagorean fuzzy set in year 2013 and applicable in decision making.

Fermatean fuzzy set is proposed in year 2020 by Senapati and Yager [5] can handle the problem more effectively than that of intuitionistic fuzzy set. In this there are some mathematical operations over the Fermatean fuzzy set.

The concept of fuzzy topological space was introduced by Chang [2]in year 1968. He studied the topological concepts like continuity and compactness. H. Z. Ibrahim [3] defined the concept of Fermatean fuzzy topological space. H.Z.Ibrahim [4] introduce the new concept namely (3,2)-fuzzy set.

In this paper we defined the new concept (4,2) -Fuzzy set following the idea of Zadeh. Following the idea of Chang, we defined a topological structure for (4,2) Fuzzy set as an extension of fuzzy topological space. Next, we discuss the topological concept such continuity.

## 2.Perliminaries:

### Definition 2.1:

Let X be a universal set. Then the fuzzy set F is defined on X by,

$$F=\{\langle r, \alpha_F(r) \rangle : r \in X\}$$

Where the degree of membership $\alpha_F(r)$ :X ⟶ [0,1].

## Definition 2.2:

Let X be a universal set. Then the intuitionistic fuzzy set (IFS) is defined by following,

$$F=\{\langle r, \alpha_F(r), \beta_F(r)\rangle : r \epsilon X\}$$

With the condition $0 \leq (\alpha_F(r)) + (\beta_F(r)) \leq 1$,

where $\alpha_F(r)$ : X ⟶ [0,1] is the degree of membership and $\beta_F(r) : X \longrightarrow [0,1]$ is the degree of non-membership of every r $\epsilon X$ to F.

## Definition 2.3:

Let X be a universal set. Then the Pythagorean fuzzy set (PFS) is defined by following,

$$F=\{\langle r, \alpha_F(r), \beta_F(r)\rangle : r \epsilon X\}$$

With the condition $0 \leq (\alpha_F(r))^2 + (\beta_F(r))^2 \leq 1$,

where $\alpha_F(r)$ : X⟶ [0,1] is the degree of membership and $\beta_F(r) : X \longrightarrow [0,1]$ is the degree of non-membership of every r $\epsilon X$ to F.

## Definition 2.4:

Let X be a universal set. Then the Fermatean fuzzy set (FFS) is defined by following,

$$F=\{\langle r, \alpha_F(r), \beta_F(r)\rangle : r \epsilon X\}$$

With the condition $0 \leq (\alpha_F(r))^3 + (\beta_F(r))^3 \leq 1$,

where $\alpha_F(r)$ : X ⟶[0,1] is the degree of membership and $\beta_F(r) : X \longrightarrow [0,1]$ is the degree of non-membership of every r $\epsilon X$ to F.

## Definition 2.5:

Let X be a universal set. Then the (3,2)-fuzzy set is defined by,

$$F=\{\langle r, \alpha_F(r), \beta_F(r)\rangle : r \epsilon X\}$$

With the condition $0 \leq (\alpha_F(r))^3 + (\beta_F(r))^2 \leq 1$,

where $\alpha_F(r)$ : X ⟶ [0,1] is the degree of membership and $\beta_F(r) : X \longrightarrow [0,1]$ is the degree of non-membership of every r $\epsilon X$ to F.

# 3.(4,2)-Fuzzy set:

## Definition 3.1:

Let X be a universal set. Then the (4,2)-fuzzy set is defined by following,

$$F=\{\langle r, \alpha_F(r), \beta_F(r)\rangle : r \epsilon X\}$$

With the condition $0 \leq (\alpha_F(r))^4 + (\beta_F(r))^2 \leq 1$,

where $\alpha_F(r)$ : X⟶ [0,1] is the degree of membership and $\beta_F(r) : X \longrightarrow [0,1]$ is the degree of non-membership of every r $\epsilon X$ to F.

The degree of indeterminacy of r $\epsilon X$ to F is defined by,

$$\pi_F(\mathrm{r}) = \sqrt[6]{1 - \left[(\alpha_F(r))^4 + (\beta_F(r))^2\right]}$$

Where $(\alpha_F(r))^4 + (\beta_F(r))^2 + (\pi_F(r))^6 = 1$ and $\pi_F(r) = 0$ whenever $(\alpha_F(r))^4 + (\beta_F(r))^2 = 1$.

# 4.Topology with respect to (4,2)-fuzzy sets:

Here we formulate the concept of (4,2)-fuzzy topology on the family of (4,2)-fuzzy sets whose complement are (4,2)-fuzzy sets and study main properties. Then we define (4,2)-fuzzy continuous maps.

## (4,2)-fuzzy topology:

Let $\tau$ be a family of (4,2)-fuzzy subsets of a non-empty set X. If,

1. $1_X, 0_X \in \tau$ where $1_X = (1, 0)$ and $0_X = (0, 1)$,
2. $F_1 \cap F_2 \in \tau$, for any $F_1, F_2 \in \tau$,
3. $\cup_{i\in I} F_i \in \tau$, for any $\{F_i\}_{i\in I} \subset \tau$,

then $\tau$ is called a (4,2)-fuzzy topology on X and (X, $\tau$ ) is a (4,2)- fuzzy topological space.F is open if it is a member of $\tau$ .

## Remark:

If $\tau$ contains all (4,2)-fuzzy subsets, then it is discrete (4,2)- fuzzy topology, otherwise it is indiscrete.

## Example:

Let $\tau = \{1_X, 0_X, A, B, C\}$ be the family of (4,2)-fuzzy subset of $X = \{x_1, x_2\}$, where

$A = \{\langle x_1, \alpha_A(x_1) = 0.9, \beta_A(x_1) = 0.53\rangle, \langle x_2, \alpha_A(x_2) = 0.83, \beta_A(x_2) = 0.62\rangle\}$.

$B = \{\langle x_1, \alpha_B(x_1) = 0.78, \beta_B(x_1) = 0.63\rangle, \langle x_2, \alpha_B(x_2) = 0.79, \beta_B(x_2) = 0.69\rangle\}$.

$C = \{\langle x_1, \alpha_C(x_1) = 0.8, \beta_C(x_1) = 0.62\rangle, \langle x_2, \alpha_C(x_2) = 0.81, \beta_C(x_2) =0.62\rangle\}$.

∴ All the three conditions are satisfied.

Hence, is (4,2)-fuzzy topology on X.

## Definition 4.2:

Let (X, $\tau$) be a (4,2)-fuzzy topological space and $F = \{\langle x, \alpha_F(x), \beta_F(x)\rangle;\ x \in X\}$ be a (4,2)-FS in X. then, the (4,2)-fuzzy interior and (4,2)-fuzzy closure of F are defined by,

1. $cl(F)=\cap$ {A: A is closed (4,2)-fuzzy set in X and $F\subset A$}.
2. $Int (F)=\cup$ {B: B is open (4,2)-fuzzy set in X and $B\subset F$}.

## Remark:

Let (X, $\tau$) be a (4,2)-fuzzy topological space and F be any (4,2)- Fuzzy set in X. Then,

1. int (F) is an open (4,2)-FS.
2. cl (F) is a closed (4,2)-FS.
3. $int (1_x) = cl(1_x) = 1_x$ and $int (0_x) = cl(0_x) = 0_x$.

## Theorem 1:

Let (X, $\tau$) be a (4,2)-fuzzy topological space and $F_1$ ,$F_2$ be (4,2)- fuzzy set in X. Then the following properties holds:

1. $int (F_1) \subset F_1$ and $F_1 \subset cl(F_1)$.
2. If $F_1\subset F_2$, then $int (F_1) \subset int (F_2)$ and $cl (F_1) \subset cl (F_2)$.
3. $F_1$ is an open (4,2)-fuzzy set iff $F_1=int (F_1)$.
4. $F_1$ is an closed (4,2)-fuzzy set iff $F_1=cl (F_1)$.

**Proof:** Let (X, $\tau$) be a (4,2)-fuzzy topological space. $F_1$, $F_2$ be (4,2)- fuzzy set in X,

1. By definition, $int (F_1) \subset F_1$ and also $F_1 \subset cl(F_1)$.
2. It is obvious that,if $F_1\subset F_2$, then $int (F_1) \subset int (F_2)$ and $cl (F_1) \subset cl (F_2)$.
3. If we say $F_1$ is open then, $F_1=int (F_1)$.
4. Similarly, $F_1$ is closed then, $F_1=cl (F_1)$.

## Corollary:

Let (X, $\tau$) be a (4,2)-fuzzy topological space and $F_1$, $F_2$ be (4,2) fuzzy set in X. Then the following properties holds:

1. $int(F_1) \cup int(F_2) \subset int(F_1 \cup F_2)$.
2. $cl(F_1 \cap F_2) \subset cl(F_1) \cap cl(F_2)$.
3. $int(F_1 \cap F_2) = int(F_1) \cap int(F_2)$.
4. $cl(F_1) \cup cl(F_2) = cl(F_1 \cup F_2)$.

## Proof:

1. If $int(F_1) \subset F_1$ and $int(F_2) \subset F_2$ then,

$$int(F_1) \cup int(F_2) \subset int(F_1 \cup F_2).$$

2. If $F_1 \subset cl(F_1)$ and $F_2 \subset cl(F_2)$ then,

$$cl(F_1 \cap F_2) \subset cl(F_1) \cap cl(F_2).$$

3. Since $int(F_1 \cap F_2) \subset int(F_1)$ and $int(F_1 \cap F_2) \subset int(F_2)$ then,

$int(F_1 \cap F_2) \subset int(F_1) \cap int(F_2)$

by (1), $(int(F_1) \cup int(F_2) \subset int(F_1 \cup F_2))$

$\therefore int(F_1 \cap F_2) = int(F_1) \cap int(F_2)$.

4. Since $cl(F_1 \cup F_2) \subset cl(F_1)$ and $cl(F_1 \cup F_2) \subset cl(F_2)$then,

$cl(F_1 \cup F_2) \subset cl(F_1) \cup cl(F_2)$

by $F_1 \subset cl(F_1)$, $cl(F_1) \cup cl(F_2) \subset cl(F_1 \cup F_2))$

$\therefore cl(F_1) \cup cl(F_2) = cl(F_1 \cup F_2)$.

## Theorem 2:

Let (X, $\tau$) be a (4,2)-fuzzy topological space and F be (4,2)-fuzzy set in X. Then the following properties holds:

1. $cl(F^c) = int(F)^c$.
2. $int(F^c) = cl(F)^c$.
3. $cl(F^c)^c = int(F)$.
4. $int(F^c)^c = cl(F)$.

## Proof:

1.Let $F=\{\langle x, \alpha_F(x), \beta_F(x)\rangle : x \in X\}$,suppose the family of (4,2) fuzzy set contained in F indexed by,

$$\{\langle x, \alpha_{Ui}, \beta_{Ui}\rangle : i \in J\}$$

$int(F) = \{\langle x, \vee\alpha_{Ui}(x), \wedge\beta_{Ui}(x)\rangle\}$

$int(F)^c = \{\langle x, \wedge\alpha_{Ui}(x), \vee\beta_{Ui}(x)\rangle\}$

Now, $F^c = \{\langle x, \beta_F(x), \alpha_F(x)\rangle\}$ is family of all closed (4,2)-fuzzy set containing $F^c$ .

$cl(F^c) = int(F)^c$ .

2.Similarly, we prove $int(F^c) = cl(F)^c$ .

3.If $cl(F^c) = int(F)^c$ then, $cl(F^c)^c = int((F)^c)^c = int(F)$

$$\therefore cl(F^c)^c = int(F).$$

4.Similarly, if int $(F^c) = cl(F)^c$ then, $int(F^c)^c = cl((F)^c)^c = cl(F)$.Therefore, $int(F^c)^c = cl(F)$.

## 5.(4,2)- Fuzzy Continuous Maps:

### Definition:

Let f be a function f:X ⟶ Y and U and V be the (4,2) fuzzy subsets of X and Y respectively. The functions of membership and non-membership of the image of U denoted by $f[U]$ ,are defined by,

$$\alpha_{f[U]}(y) = \begin{cases} \sup_{z\, \epsilon f^{-1}(y)} \alpha_U(z) & if\ f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

$$\beta_{f[U]}(y) = \begin{cases} \inf_{z\, \epsilon f^{-1}(y)} \beta_U(z) & if\ f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

The function of membership and non-membership preimage of V is denoted by $f^{-1}(V)$,

$$\alpha_{f^{-1}[V]}(x) = \alpha_V(f(x))$$

$$\beta_{f^{-1}[V]}(x) = \beta_V(f(x))$$

**Theorem 5.1:**

Let f : X ⟶ Y such that U and V are (4,2)-fuzzy subset of X and Y. Then ,

1) $f^{-1}[V^c] = f^{-1}[V]^c$.

2) $f[U]^c \subseteq f[U^c]$.

3) $f[f^{-1}[V]] \subseteq V$.

4) $U \subseteq f[f^{-1}[U]]$.

## Proof:

**1**. Let x ϵ X and V be a (4,2)-fuzzy subsets of Y, then

$$\alpha_{f^{-1}[V^c]}(x) = \alpha_{V^c}(f(x)$$
$$= \beta_V(f(x)) = \beta_{f^{-1}[V]}(x)$$
$$= \alpha_{f^{-1}[V]^c}(x).$$

Similarly , $\beta_{f^{-1}[V^c]}(x) = \beta_{f^{-1}[V]^c}(x)$ .Therefore , $f^{-1}[V^c] = f^{-1}[V]^c$

**2**. Let y ϵ Y and for any (4,2)-fuzzy subset U of X , then

$$\alpha_{f[U^c]}(y) = \sup\ \alpha_{U^c}(z) \quad \text{where } z \in [f^{-1}[y]].$$
$$= \sup\ \beta_U(z) \quad \text{where } z \in [f^{-1}[y]].$$
$$= \sup \sqrt{(\gamma_U(z))^6 - (\alpha_U(z))^4} \quad \text{,where } z \in [f^{-1}[y]].$$
$$\geq \sqrt{sup(\gamma_U(z))^6 - sup(\alpha_U(z))^4} \quad \text{,where } z \in [f^{-1}[y]].$$
$$\geq \sqrt{(\gamma_{f[U]}(y))^6 - (\alpha_{f[U]}(y))^4}$$
$$= \beta_{f[U]}(y)$$

$= \alpha_{f[U]^c}(y).$

Similarly, we prove it for $\beta_{f[U^c]}(y) \leq \beta_{f[U]^c}(y)$,therefore $f[U]^c \subseteq f[U^c]$.

**3.** For any y ϵ Y and f(y)≠ ∅, $\alpha_{f[f^{-1}[V]]}(y) = \sup \alpha_{f^{-1}[V]}(z) = \sup \alpha_V(f(x)) \leq \alpha_V(y)$.

Therefore, $f[f^{-1}[V]] \subseteq V$.

**4.**For any x$\epsilon X$ , $\alpha_{f[f^{-1}[U]]}(x) = \alpha_{f[U]}(f(x)) = \sup \alpha_U(z) \geq \alpha_U(x)$.

Therefore, U$\subseteq f[f^{-1}[U]]$.

## Conclusion:

In this paper, we defined new fuzzy concept namely (4,2)-fuzzy set and discuss interior, closure of this fuzzy set. In addition, (4,2)-fuzzy topology and their continuous mapping.

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