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New Sets In Pythagorean Neutrosophic Refined Topological Spaces

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Abstract :

In this paper a new sets called PYTHAGOREAN NEUTROSOPHIC REFINED PRE OPEN SET& PYTHAGOREAN NEUTROSOPHIC REFINED PRE CLOSED SET are introduced and applied to Pythagorean Neutrosophic Refined Topological spaces and some of the basic definitions and theorems are introduced and explained with suitable examples.

Keywords:

Pythagorean Neutrosophic Refined Topology, Pythagorean Neutrosophic Refined Pre Open Set, Pythagorean Neutrosophic Refined Pre Closed Set.

1.INTRODUCTION :

In recent times many ideas have been introduced to deal with indeterminacy, uncertainty, vagueness. Fuzzy set theory[1], intuitionistic fuzzy sets[2],rough set theory[3] play different measures in handling inconsistent datas. However, all these above theories failed to deal with inconsistent information which exist in beliefs system.

In 1995, Smarandache [4] developed a new concept called neutrosophic set (NS) which generalizes probability set, fuzzy set and intuitionistic fuzzy set. Neutrosophic set can be described by membership degree, indeterminacy degree and non-membership degree. Smarandache[5] gave n-valued refined neutrosophic logic and its applications. Then, Ye and Ye [44] gave single valued neutrosophic sets and operations laws. R. Jhansi [6] introduced the concept of Pythagorean Neutrosophic set with T and F as dependent components. In this paper Pythogarean Nuetrosophic Refined Topology is introduced and some of the basic concepts are explained .

This paper is arranged in the following manner. In section 2, some definitions and notion about Neutrosophic set , Neutrosophic refined set and Neutrosophic Pythagorean Refined set theory are given. These definitions will help us in later section. In section 3 we study the concept of Pythagorean Neutrosophic Refined Topological Spaces their properties are explained, In section 4 Pythagorean Neutrosophic Refined Pre Open sets are introduced and their properties are explained, In section 5 Pythagorean Neutrosophic Refined Pre Closed sets are introduced and their properties are explained. Finally we conclude the paper.

2.PRELIMINARIES:

DEFINITION:2.1

Let U be a universe. A Neutrosophic set A on U can be defined as follows:

A = { $< x, T_A(x), I_A(x), F_A(x) >$: x ∈ U }

Where T_A, I_A, F_A : U → [0,1] and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminancy and $F_A(x)$ is the degree of non-membership.

DEFINITION:2.2

Let U be a Universe, a Neutrosophic refined set on can be defined as follows:

$$A = \{ <x, (T_A^1(X), T_A^2(X), T_A^3(X), \ldots T_A^P(X)), (I_A^1(X), I_A^2(X), I_A^3(X), \ldots I_A^P(X)),$$
$$(F_A^1(X), F_A^2(X), F_A^3(X), \ldots F_A^P(X)) : x \in U\}$$

Where $T_A^1(X), T_A^2(X), T_A^3(X), \ldots T_A^P(X)$: U → [0,1] , $I_A^1(X), I_A^2(X), I_A^3(X), \ldots I_A^P(X)$: U → [0,1] and

$F_A^1(X), F_A^2(X), F_A^3(X), \ldots F_A^P(X)$: U → [0,1] such that $0 \leq T_A^j(X) + I_A^j(X) + F_A^j(X) \leq 3$ for

j = 1,2,3,…p and for any x ∈ U . ($T_A^1(X), T_A^2(X), T_A^3(X), \ldots T_A^P(X)$) , ($I_A^1(X), I_A^2(X), I_A^3(X), \ldots I_A^P(X)$) ,

($F_A^1(X), F_A^2(X), F_A^3(X), \ldots F_A^P(X)$) is the Truth-membership sequence, Indeterminatemembership

sequence & Falsity-membership sequence of the element x , respectively. Also, p is called the dimension

of Neutrosophic refined set (NRS) A .

DEFINITION:2.3

Let U be a universe. A Pythagorean Neutrosophic set with T and F are dependent Neutrosophic components A on U is an object of the form

A = { $< x, T_A(x), I_A(x), F_A(x) >$: x ∈ U }

Where T_A, I_A, F_A : U → [0,1] and $0 \leq (T_A(X))^2 + (I_A(X))^2 + (F_A(X))^2 \leq 2$

$T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminancy and $F_A(x)$ is the degree of non-membership.

DEFINITION: 2.4

Let U be a Universe. A Pythagorean Neutrosophic Refined Set can be defined as follows:

$$P_{PNR} = \{ < x, (T_P^1(X), T_P^2(X), T_P^3(X), \ldots T_P^K(X)), ((I_P^1(X), I_P^2(X), I_P^3(X), \ldots I_P^k X)),$$
$$(F_P^1(X), F_P^2(X), F_P^3(X), \ldots F_P^k(X))> : x \in U\}$$

Where $T_P^1(X), T_P^2(X), T_P^3(X), \ldots T_P^k(X)$: U → [0,1] ,

$I_P^1(X), I_P^2(X), I_P^3(X), \ldots I_P^k(X)$: U → [0,1] and

$F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X)$: U → [0,1] such that

And $0 \leq (T_P^k(X))^2 + (I_P^k(X))^2 + (F_P^k(X))^2 \leq 2$

for j = 1,2,3,…p and for any x ∈ U . $T_P^k(X)$ is the degree of membership sequence, $I_P^k(X)$ is the degree of indeterminacy membership sequence and $F_P^k(X)$ is the degree of non-membership sequence.

DEFINITION:2.5

Let P_{PNR} and Q_{PNR} be Pythagorean Neutrosophic Refined sets (PNRS) in U. P_{PNR} is said to be Pythagorean Neutrosophic Refined Subset of Q_{PNR},

If $T_P^k(X) \leq T_Q^k(X)$,$I_P^k(X) \geq I_Q^k(X)$, $F_P^k(X) \geq F_Q^k(X)$ for every x ∈ U.

It is denoted by $P_{PNR} \subseteq Q_{PNR}$

DEFINITION: 2.6

Let P_{PNR} and Q_{PNR} be Pythagorean Neutrosophic Refined sets (PNRS) in U. P_{PNR} is said to Pythagorean Neutrosophic Refined equal set of Q_{PNR},

If $T_P^k(X) = T_Q^k(X)$, $I_P^k(X) = I_Q^k(X)$, $F_P^k(X) = F_Q^k(X)$ for every x ∈ U.

It is denoted by $P_{PNR} = Q_{PNR}$.

DEFINITION:2.7

Let P_{PNR} be Pythagorean Neutrosophic Refined sets (PNRS) in U. It's compliment is defined as follows:

$${P_{PNR}}^c = \{ \langle x , (F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X)),$$

$$\left(1 - I_P^1(X), 1 - I_P^2(X), 1 - I_P^3(X), \ldots\ldots\ldots\ldots 1 - I_P^k(X) \right),$$

$$T_P^1(X), T_P^2(X), T_P^3(X), \ldots\ldots T_P^k(X) \rangle : x \in U \}.$$

It is denoted as ${P_{PNR}}^c$

DEFINITION : 2.8

1. If $T_P^k(X) = 0$ and $I_P^k(X) = F_P^k(X) = 1$ for all j = 1,23,…..p ,then the set P_{PNR} is called null – Pythagorean Neutrosophic Refined Set . It is denoted as $\emptyset_{PNR}$

2. If $T_P^k(X) = 1$ and $I_P^k(X) = F_P^k(X) = 0$ for all j = 1,23,…..p ,then the set P_{PNR} is called Universal – Pythagorean Neutrosophic Refined Set . It is denoted as U_{PNR}

DEFINITION: 2.9

Let X be a non empty set in U,

$$P_{PNR} = \{ \langle x , (T_P^1(X), T_P^2(X), T_P^3(X), \ldots\ldots T_P^K(X)) , \left(I_P^1(X), I_P^2(X), I_P^3(X), \ldots\ldots I_P^k X \right) ,$$

$$(F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X)) \rangle : x \in U \}$$

$$Q_{PNR} = \{ \langle x , (T_Q^1(X), T_Q^2(X), T_Q^3(X), \ldots\ldots T_Q^K(X)) , \left(I_Q^1(X), I_Q^2(X), I_Q^3(X), \ldots\ldots I_Q^k X \right) ,$$

$$(F_Q^1(X), F_Q^2(X), F_Q^3(X), \ldots\ldots F_Q^k(X)) \rangle : x \in U \}$$

are Pythagorean Neutrosophic Refined sets (PNRS) in U.

The union of P_{PNR} and Q_{PNR} is defined as Follows :

$P_{PNR} \cup Q_{PNR}$ = { < x, s(($T_P^1(X), T_Q^1(X)), (T_P^2(X), T_Q^2(X), \ldots\ldots (T_P^k(X), T_Q^k(X))$,

t(($I_P^1(X), I_Q^2(X), (I_P^2(X), I_Q^2(X), \ldots\ldots (I_P^k(X), I_Q^k(X)$),

t(($F_P^1(X), F_Q^1(X)), (F_P^2(X), F_Q^2(X)), \ldots\ldots (F_P^k(X), F_Q^k(X))$> : x ∈ U}.

DEFINITION: 2.10

Let X be a non empty set in U,

P_{PNR} = { < x , ($T_P^1(X), T_P^2(X), T_P^3(X), \ldots\ldots T_P^K(X)$) , ($I_P^1(X), I_P^2(X), I_P^3(X), \ldots\ldots I_P^k X$)) ,

($F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X)$)>: x ∈ U}

Q_{PNR} = { < x , ($T_Q^1(X), T_Q^2(X), T_Q^3(X), \ldots\ldots T_Q^K(X)$) , ($I_Q^1(X), I_Q^2(X), I_Q^3(X), \ldots\ldots I_Q^k X$)) ,

($F_Q^1(X), F_Q^2(X), F_Q^3(X), \ldots\ldots F_Q^k(X)$)>: x ∈ U}

are Pythagorean Neutrosophic Refined sets (PNRS) in U.

The intersection of P_{PNR} and Q_{PNR} is defined as Follows:

$P_{PNR} \cap Q_{PNR}$ = { < x, t(($T_P^1(X), T_Q^1(X)), (T_P^2(X), T_Q^2(X), \ldots\ldots (T_P^k(X), T_Q^k(X))$,

s(($I_P^1(X), I_Q^2(X), (I_P^2(X), I_Q^2(X), \ldots\ldots (I_P^k(X), I_Q^k(X)$),

s(($F_P^1(X), F_Q^1(X)), (F_P^2(X), F_Q^2(X)), \ldots\ldots (F_P^k(X), F_Q^k(X))$> : x ∈ U}.

DEFINITION: 2.11

A neutrosophic topology (NT) is a non-empty set X is a family τ of a neutrosophic sets in X satisfying the following condition.

a) (NT 1) $0_N, 1_N \in \tau$
b) (NT 2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$ N
c) (NT3) $\cup G_i \in \tau$ for every { $G_i; i \in$ j } ⊆ τ

In this case (X,τ) is called a Neutrosophic topological space.

DEFINITION: 2.12

The elements of τ is neutrosophic open sets the complement of neutrosophic open set is called neutrosophic closed set.

DEFINITION:2.13

Let A be neutrosophic set of a neutrosophic topology. Then A is said to be Neutrosophic preopen[NPO]set of X if there exists a neutrosophic open set (NO) such that NO⊆A ⊆Nint(Ncl(A)

PYTHAGOREAN NEUTROSOPHIC REFINED TOPOLOGICAL SPACE

DEFINITION 3.1

A Pythagorean Neutrosophic Refined topology (PNRT) is a non-empty set X is a family τ of a Pythagorean Neutrosophic Refined sets in X satisfying the following conditions

a. (PNRT 1) $0_{PNRS}, I_{PNRS} \in \tau$
b. (PNRT 2) $P_{PNRS_1} \cap P_{PNRS_2} \in \tau$ for any $P_{PNRS_1}, P_{PNRS_2} \in \tau$
c. (PNRT 3) $\cup G_{PNRS_i} \in \tau$ for every $\{G_{PNRS_i}; i \in j\} \subseteq \tau$

In this case (X,τ) is called a Pythagorean Neutrosophic Refined topological space.

EXAMPLE: 3.2

Let X ={x},

A_{PNRS} = { < x,(0.4,0.5,0.6),(0.2,0.3,0.8),(0.4,0.5,0.6),(0.1,0.5,0.9) >}

B_{PNRS} = { < x,(0.6,0.4,0.4),(0.8,0.1,0.2),(0.6,0.5,0.4),(0.9,0.3,0.1) >}

C_{PNRS} = { < x,(0.2,0.4,0.8),(0.5,0.1,0.5),(0.6,0.1,0.4),(0.7,0.3,0.3) >}

Then the family τ = { $0_{PNRS}, I_{PNRS}, A_{PNRS}, B_{PNRS}, C_{PNRS}$ } of Pythagorean Neutrosophic Refined Sets in X is Pythagorean Neutrosophic Refined Topology in X.

DEFINITION:3.3

If (X,τ) is called Pythagorean Neutrosophic Refined topological space and

A_{PNR} ={ < x , ($T_P^1(X), T_P^2(X), T_P^3(X), \ldots\ldots T_P^K(X)$) , (($I_P^1(X), I_P^2(X), I_P^3(X), \ldots\ldots I_P^k X$)) ,

($F_P^1(X), F_P^2(X), F_P^3(X), \ldots\ldots F_P^k(X)$)>: x ∈ X}

is a Pythagorean Neutrosophic Refined Set in X then Pythagorean Neutrosophic Refined Closure and Pythagorean Neutrosophic Refined Interior are defined by

1) PNRcl(A_{PNR}) = ∩ { $A_{PNR} \subseteq P_{PNRS}$, where P_{PNRS} is a collection of Pythagorean Neutrosophic Refined closed sets in X(PNRCS)}
2) PNRint(A_{PNR}) = ∪ {$Q_{PNRS} \subseteq A_{PNR}$, where Q_{PNRS} is a collection of Pythagorean Neutrosophic Refined open sets in X(PNROS)}

We can also show that PNRcl(A) is Pythagorean Neutrosophic Refined closed set and PNRint(A_{PNR}) is Pythagorean Neutrosophic Refined open set in X

A_{PNR} is Pythagorean Neutrosophic Refined open set ⇔ A_{PNR} = PNRint(A_{PNR})

A_{PNR} is Pythagorean Neutrosophic Refined closed set ⇔ A_{PNR} = PNRcl(A_{PNR})

PROPOSITION:3.4

If (X,τ) is called Pythagorean Neutrosophic Refined topological space and if $P_{PNR} \in$ (X,τ) then we have,

1) PNRcl($P_{PNR}{}^c$)) = (PNRint(P_{PNR}))^c
2) PNRint($P_{PNR}{}^c$)) = (PNRcl(P_{PNR}))^c

PROPOSITION:3.5

If (X,τ) is called Pythagorean Neutrosophic Refined topological space and if P_{PNR} , Q_{PNR} ∈ (X,τ) are two Pythagorean Neutrosophic Refined Sets then the following properties hold true

a) PNRint(P_{PNR}) ⊆ P_{PNR}
b) P_{PNR} ⊆ PNRcl(P_{PNR})
c) P_{PNR} ⊆ Q_{PNR} then ${Q_{PNR}}^c$ ⊆ ${P_{PNR}}^c$
d) P_{PNR} ⊆ Q_{PNR} then PNRint(P_{PNR}) ⊆ PNRint(Q_{PNR})
e) P_{PNR} ⊆ Q_{PNR} then PNRcl(P_{PNR}) ⊆ PNRint(Q_{PNR})

PROPOSITION:3.6

If (X,τ) is called Pythagorean Neutrosophic Refined topological space and if P_{PNR} , Q_{PNR} ∈ (X,τ) are two Pythagorean Neutrosophic Refined Sets then the following properties hold true

a) PNRint (PNRint(P_{PNR})) = PNRint(P_{PNR})
b) PNRcl (PNRcl(P_{PNR})) = PNRcl(P_{PNR})
c) PNRint(P_{PNR} ∩ Q_{PNR}) = PNRint(P_{PNR}) ∩ PNRint(Q_{PNR})
d) PNRint(P_{PNR} ∪ Q_{PNR}) ⊇ PNRint(P_{PNR}) ∪ PNRint(Q_{PNR})
e) PNRcl(P_{PNR} ∪ Q_{PNR}) = PNRcl(P_{PNR}) ∪ PNRcl(Q_{PNR})
f) PNRcl(P_{PNR} ∩ Q_{PNR}) ⊆ PNRcl(P_{PNR}) ∪ PNRcl(Q_{PNR})
g) PNRint(0_{PNR}) = PNRcl(0_{PNR}) = 0_{PNR}
h) PNRint(I_{PNR}) = PNRcl(I_{PNR}) = I_{PNR}

4.PYTHAGOREAN NEUTROSOPHIC REFINED PRE OPEN SETS IN PNRTS

DEFINITION:4.1

Let P_{PNR} be a Pythagorean Neutrosophic Refined Set in Pythagorean Neutrosophic Refined topological space. Then P_{PNR} is called Pythagorean Neutrosophic Refined Pre-Open set if

PNRO ⊆ P_{PNR} ⊆ PNRint(PNRcl(P_{PNR}))

THEOREM :4.2

A subset P_{PNR} in Pythagorean Neutrosophic Refined topological spaces X is Pythagorean Neutrosophic Refined Pre-Open Set if and only if P_{PNR} ⊆ PNRint(PNRcl(P_{PNR}))

PROOF:

Suppose P_{PNR} ⊆ PNRint(PNRcl(P_{PNR}))

To Prove: P_{PNR} is in Pythagorean Neutrosophic Refined topological spaces

i.e) P_{PNR} is Pythagorean Neutrosophic Refined Pre-Open Set.⟹ P_{PNR} ⊆ PNRint(PNRcl(P_{PNR}))

We know that , PNRO = PNRint(P_{PNR}) ,PNRO ⊆ P_{PNR} ⊆ PNRint(PNRcl(P_{PNR}))

Therefore P_{PNR} is Pythagorean Neutrosophic Refined topological Set.

Conversely,

Let P_{PNR} is Pythagorean Neutrosophic Refined topological Set,

$\Longrightarrow$ PNRO $\subseteq P_{PNR} \subseteq$ PNRint(PNRcl(P_{PNR}))

We know that $P_{PNR} \subseteq$ PNRint (P_{PNR}) ,Therefore $P_{PNR} \subseteq$ PNRint(cl(P_{PNR})).

THEOREM :4.3

In Pythagorean Neutrosophic Refined topological the union of two Pythagorean Neutrosophic Refined Pre-Open Sets again a Pythagorean Neutrosophic Refined Pre-Open Set.

PROOF:

Let P_{PNR} and Q_{PNR} be the two Pythagorean Neutrosophic Refined Pre-Open Set in X.

$P_{PNR} \subseteq$ PNRint(PNRcl(P_{PNR}))
$Q_{PNR} \subseteq$ PNRint(PNRcl(Q_{PNR}))
$P_{PNR} \cup Q_{PNR} \subseteq$ PNRint(PNRcl(P_{PNR})) $\cup$ PNRint(PNRcl(Q_{PNR})) $\subseteq$ PNRint((PNRcl(P_{PNR}) $\cup$ PNRcl(Q_{PNR})) $\subseteq$ PNRint(PNRcl($P_{PNR} \cup Q_{PNR}$))

By the definition, $P_{PNR} \cup Q_{PNR}$ is a Pythagorean Neutrosophic Refined Pre-Open Set in X.

REMARK :4.4

The intersection of any two Pythagorean Neutrosophic Refined Pre Open sets need not be a Pythagorean Neutrosophic Refined Pre Open set in X.

Example: 4.5

Let X= {a, b}

P_{PNR} = { < x,(0.4,0.8,0.6) , (0.7,0.2,0.3) >}

Q_{PNR} = { < x,(0.5,0.8,0.5),(0.8,0.4,0.2)>}

R_{PNR} = { < x,(0.4,0.7,0.6) ,(0.6,0.4,0.4)>}

S_{PNR} = { <x,(0.5,0.7,0.5) ,(0.8,0.4,0.2) >}

Then τ = { $0_{PNRS}, I_{PNRS}, P_{PNR}, Q_{PNR}, R_{PNR}, S_{PNR}$ } is Pythagorean Neutrosophic Refined topological Set.

Consider

A_{PNRS1} = { < x, (0.5,0.4,0.5),(0.1,0.2,0.9) >}

A_{PNRS2} = { < x, (0.2,0.1,0.8),(0.2,0.5,0.8) >}

$A_{PNRS1} \cap A_{PNRS2}$ = { < x, (0.2,0.4,0.8),(0.1,0.5,0.9) >} is not a Pythagorean Neutrosophic Refined Pre - open set in X.

THEOREM:4.6

Every Pythagorean Neutrosophic Refined Open set in the Pythagorean Neutrosophic Refined Topological Space X is Pythagorean Neutrosophic Refined Pre - open set set in X.

PROOF:

Let P_{PNR} be a Pythagorean Neutrosophic Refined Open set in PNRT

P_{PNR} = PNRint(P_{PNR}) Clearly, $P_{PNR} \subseteq$ PNR(cl(P_{PNR})) ,PNRint(P_{PNR}) $\subseteq$ PNRint(PNR(cl(P_{PNR})))

$P_{PNR} \subseteq$ PNRint(PNR(cl(P_{PNR})))

Therefore P_{PNR} be a Pythagorean Neutrosophic Refined Pre Open set in PNRT.

THEOREM:4.7

Let P_{PNR} be Pythagorean Neutrosophic Refined Pre Open set in the Pythagorean Neutrosophic Refined topological space X and suppose $P_{PNR} \subseteq Q_{PNR} \subseteq$ PNRcl(P_{PNR}) then, Q_{PNR} is Pythagorean Neutrosophic Refined Pre Open set in X

PROOF:

Let P_{PNR} be Pythagorean Neutrosophic Refined Open Set in Pythagorean Neutrosophic Refined Topological Space.

Then P_{PNR} = PNRint(P_{PNR}) also, $P_{PNR} \subseteq$ PNRcl(P_{PNR}) ,PNRint(P_{PNR}) $\subseteq$ PNRint(PNRcl(P_{PNR}))

$P_{PNR} \subseteq$ PNRint(PNRcl(P_{PNR})), if $P_{PNR} \subseteq Q_{PNR}$, $Q_{PNR} \subseteq$ PNRint(PNRcl(P_{PNR}))

Hence Proved.

THEOREM:4.8

Let P_{PNR} be a Pythagorean Neutrosophic Refined Open set in X and Q_{PNR} a Pythagorean Neutrosophic Refined pre-open set in X then there exists an Pythagorean Neutrosophic Refined Open set G_{PNR} in X then $P_{PNR} \cap Q_{PNR}$ is Pythagorean Neutrosophic Refined Pre Open Set in X .

PROOF:

Let P_{PNR} be a Pythagorean Neutrosophic Refined Open set in X and Q_{PNR} a Pythagorean Neutrosophic Refined pre-open set in X then there exists an Pythagorean Neutrosophic Refined Open set G_{PNR} in X Such that, $Q_{PNR} \subseteq G_{PNR} \subseteq$ PNRcl(Q_{PNR}) $\Longrightarrow P_{PNR} \cap Q_{PNR} \subseteq P_{PNR} \cap G_{PNR} \subseteq P_{PNR} \cap$ PNRcl(Q_{PNR}) $\subseteq$ PNRcl($P_{PNR} \cap Q_{PNR}$) ,Since $P_{PNR} \cap G_{PNR}$ is Pythagorean Neutrosophic Refined Open set , from the above theorem $P_{PNR} \cap Q_{PNR}$ is Pythagorean Neutrosophic Refined Pre Open Set in X .

5.PYTHAGOREAN NEUTROSOPHIC REFINED PRE CLOSED SETS IN PNRTS

DEFINITION:5.1

Let P_{PNR} be a Pythagorean Neutrosophic Refined Set in Pythagorean Neutrosophic Refined topological space. Then P_{PNR} is called Pythagorean Neutrosophic Refined Pre-Closed set if

PNRcl(PNRint(P_{PNR})) $\subseteq P_{PNR} \subseteq$ PNRC

THEOREM :5.2

A subset P_{PNR} in Pythagorean Neutrosophic Refined topological spaces X is Pythagorean Neutrosophic Refined Pre-Closed Set if and only if PNRcl(PNRint(P_{PNR})) ⊆ P_{PNR}

PROOF:

Comsider PNRcl(PNRint(P_{PNR})) ⊆ P_{PNR}

To Prove: P_{PNR} is in Pythagorean Neutrosophic Refined Pre closed Set.

We know that , PNRC = PNRcl(P_{PNR}) , P_{PNR} ⊆ PNRint(PNRcl(P_{PNR})) ⊆ PNRC

Therefore P_{PNR} is Pythagorean Neutrosophic Refined Closed Set.

Conversely,

Let P_{PNR} is Pythagorean Neutrosophic Refined Pre Closed Set,

⟹ PNRcl(PNRint(P_{PNR})) ⊆ P_{PNR} ⊆ NPRC, PNRcl(PNRint(P_{PNR})) ⊆ NPRC, PNRcl(P_{PNR}) ⊆ NPRC

Hence proved.

THEOREM:5.3

Let (X, τ) be a Pythagorean Neutrosophic Refined topological Space and P_{PNR} be a Pythagorean Neutrosophic Refined subset of X is called Pythagorean Neutrosophic Refined Pre Closed Set if and only if ${P_{PNR}}^c$ is Pythagorean Neutrosophic Refined Pre Open in X.

PROOF:

Let P_{PNR} be a Pythagorean Neutrosophic Refined subset of X

Clearly, PNRcl(PNRint(P_{PNR})) ⊆ P_{PNR}, Taking Compliments On bothsides

${P_{PNR}}^c$ ⊆ (PNRcl(PNRint(P_{PNR})))^c , ${P_{PNR}}^c$ ⊆ PNRint(PNRcl(${P_{PNR}}^c$))

${P_{PNR}}^c$ is Pythagorean Neutrosophic Refined Pre Open in X

Conversely,

${P_{PNR}}^c$ is Pythagorean Neutrosophic Refined Pre Open in X,

${P_{PNR}}^c$ ⊆ PNRint(PNRcl(${P_{PNR}}^c$)), Taking Compliments On Bothsides, PNRcl(PNRint(P_{PNR})) ⊆ P_{PNR}

Therefore P_{PNR} is Pythagorean Neutrosophic Refined Pre Closed Set

Hence Proved.

THEOREM:5.4

In Pythagorean Neutrosophic Refined topological the intersection of two Pythagorean Neutrosophic Refined Pre-Closed Sets again a Pythagorean Neutrosophic Refined Pre-Closed Set.

PROOF:

Let P_{PNR} and Q_{PNR} be the two Pythagorean Neutrosophic Refined Pre-Closed Set in X.

PNRcl(PNRint(P_{PNR})) ⊆ P_{PNR}
PNRcl(PNRint(Q_{PNR})) ⊆ Q_{PNR}
P_{PNR} ∩ Q_{PNR} ⊇ PNRcl(PNRint(P_{PNR})) ∪ PNRcl(PNRint(Q_{PNR})) ⊇ PNRcl((PNRint(P_{PNR}) ∪ PNRint(Q_{PNR})) ⊇ PNRcl(PNRint(P_{PNR} ∪ Q_{PNR}))

By the definition, P_{PNR} ∪ Q_{PNR} is a Pythagorean Neutrosophic Refined Pre-Closed Set in X.

REMARK:5.5

The union of any two Pythagorean Neutrosophic Refined Pre Closed sets need not be a Pythagorean Neutrosophic Refined Pre Closed set in X.

EXAMPLE: 5.6

Let X= {a, b}

P_{PNR} = { < x,(0.4,0.8,0.6) , (0.7,0.2,0.3) >}

Q_{PNR} = { < x,(0.5,0.8,0.5),(0.8,0.4,0.2)>}

R_{PNR} = { < x,(0.4,0.7,0.6) ,(0.6,0.4,0.4)>}

S_{PNR} = { <x,(0.5,0.7,0.5) ,(0.8,0.4,0.2) >}

Then τ = { $0_{PNRS}, I_{PNRS}, P_{PNR}, Q_{PNR}, R_{PNR}, S_{PNR}$ } is Pythagorean Neutrosophic Refined topological Set.

Consider

P_{PNRS1} = { < x, (0.6,0.4,0.4),(0.1,0.2,0.9) >}

P_{PNRS2} = { < x, (0.2,0.1,0.8),(0.2,0.5,0.8) >}

P_{PNRS1} ∪ P_{PNRS2} = { < x, (0.6,0.1,0.4),(0.2,0.2,0.8) >} is not a Pythagorean Neutrosophic Refined Pre - Closed set in X.

THEOREM:5.7

Every Pythagorean Neutrosophic Refined Closed set in the Pythagorean Neutrosophic Refined Topological Space X is Pythagorean Neutrosophic Refined Pre -Closed set in X.

PROOF:

Let P_{PNR} be a Pythagorean Neutrosophic Refined Closed set in PNRT

P_{PNR} = PNRcl(P_{PNR}), and PNRint(P_{PNR}) ⊆ P_{PNR} ,

PNRcl(PNRint(P_{PNR})) ⊆ P_{PNR}

Therefore P_{PNR} be a Pythagorean Neutrosophic Refined Pre Closed set in PNRT.

THEOREM:5.8

Let P_{PNR} be Pythagorean Neutrosophic Refined Pre Closed set in the Pythagorean Neutrosophic Refined topological space X and suppose PNRint(P_{PNR}) $\subseteq Q_{PNR} \subseteq P_{PNR}$ then, Q_{PNR} is Pythagorean Neutrosophic Refined Pre Closed set in X

PROOF:

Let P_{PNR} be Pythagorean Neutrosophic Refined Closed Set in Pythagorean Neutrosophic Refined Topological Space.

Then P_{PNR} = PNRcl(P_{PNR}) also, PNRint(P_{PNR}) $\subseteq Q_{PNR} \subseteq P_{PNR}$,

PNRcl(PNRint(Q_{PNR}) $\subseteq Q_{PNR}$

Q_{PNR} is Pythagorean Neutrosophic Refined Pre Closed set in X

Hence Proved.

THEOREM:5.9

Let P_{PNR} be a Pythagorean Neutrosophic Refined Closed set in X and Q_{PNR} a Pythagorean Neutrosophic Refined pre-Closed set in X then there exists an Pythagorean Neutrosophic Refined Closed set G_{PNR} in X then $P_{PNR} \cap Q_{PNR}$ is Pythagorean Neutrosophic Refined Pre Closed Set in X

PROOF:

Let P_{PNR} be a Pythagorean Neutrosophic Refined Closed set in X and Q_{PNR} a Pythagorean Neutrosophic Refined pre-Closed set in X then there exists an Pythagorean Neutrosophic Refined Closed set G_{PNR} in X such that,

PNRint(Q_{PNR}) $\subseteq G_{PNR} \subseteq Q_{PNR}$

$P_{PNR} \cap Q_{PNR} \supseteq P_{PNR} \cap G_{PNR} \supseteq P_{PNR} \cap$ PNRint(Q_{PNR}) $\supseteq$ PNRint ($P_{PNR} \cap Q_{PNR}$)

Since $P_{PNR} \cap G_{PNR}$ is Pythagorean Neutrosophic Refined Closed set , from the above theorem

$P_{PNR} \cap Q_{PNR}$ is Pythagorean Neutrosophic Refined Pre Closed Set in X .

CONCLUSION:

This paper ensures the work of introducing the new sets namely Pythagorean Neutrosophic Refined Pre Open Set & Pythagorean Neutrosophic Refined Pre Closed Set by developing the concepts of Nuetrosophic Refined Topological Spaces. Some of the basic theorem and properties are illustrated with an examples.

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REFERENCES:

1. L. A. Zadeh, Fuzzy Sets, Inform. and Control, 8 (1965) 338-353.
2. K. T. Atanassov, Intuitionistic Fuzzy Set. Fuzzy Sets and Systems, 20(1), 87–86, 1986.
3. Z. Pawlak, Rough sets, Int. J. Comput. Inform. Sci., 11 (1982) 341–356
4. F. Smarandache, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, Rehoboth: American Research Press (1998).
5. F. Smarandache, n-Valued Refined Neutrosophic Logic and Its Applications in Physics, Progress in Physics, 143-146, Vol. 4, 2013.
6. R, Jhansi, K. Mohana and Florentin Smarandache, Correlation measure for pythagorean Neutrosophic sets with T and F as dependent neutrosophic components
7. Neutrosophic Refined Sets in Medical Diagnosis A.Edward Samuel and R.Narmadhagnanam, Vol. 14, No. 1, 2017, 117-123
8. On neutrosophic refined sets and their application in medical ,Irfan Deli, Said Broumi, Florentin Smarandache. ISSN: 2149-1402
9. F. Smarandache, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability and Statistics University of New Mexico, Gallup, NM 87301, USA (2002)
10. Xindong Peng, Yong Yang, Some results for pythagorean fuzzy sets, International Journal of Intelligent systems,30(2015).1133-1160
11. A.A. Salama and S.A. Al – Bloumi, Neutrosophic Set and Neutrosophic topological spaces, IOSR Journal of Math., Vol. (3) ISSUE4 (2012), 31 – 35
12. An Introduction to Pythogorean Neutrosophic Refined Set ans Some of their Basic Operations Emimanimcy.M^1, Francina Shalini.A^2 in the International Journal of Creative Research Thoughts, 7.97 Impact factor, Refereed Journal, Peer Journal and Indexed Journal (ISSN: 2320-2882).Volume 10, Issue 3.
13. Neutrosophic Pre open sets and Neutrosophic Pre Closed sets in Neutrosophic Topology
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