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Intuitionistic Fuzzy Strongly Alpha Generalized Star Star Closed Sets in Intutionistic Fuzzy Topological Spaces

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Abstract

In this paper, We define and study a new concepts of Intuitionistic Fuzzy sets called as Intuitionistic Fuzzy Strongly α Generalized Star Star Closed Sets in Intuitionistic Fuzzy Topological spaces. We also have analyzed their properties and some interesting theorems.

Keywords: IFSαG**C, IFSαG*O, IFCS.

1. Introduction

The concepts of fuzzy sets was introduced by Zadeh [11] in the year 1965. He introduced the concepts of Fuzzy sets. It was C.L.Chang [3] who established a generalization of Fuzzy sets in the topological space as Fuzzy topological space in the year 1968. After that Attanasov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. In the course of time, Fuzzy Topology proved to be very beneficial in fixing many realistic problems. Several mathematicians have tried almost all the pivotal concepts of General Topology for extension to the fuzzy settings. In 1981, Azad [2] gave Fuzzy version of the concepts given by Levine and thus initiated the study of weak forms of several notions in Fuzzy Topological spaces. Later, Coker [4] developed the notion of intuitionistc fuzzy topological spaces which are also generalizations to fuzzy sets. In this paper, we introduce the concepts of Intuitionistic Fuzzy Strongly α Generalized Star Star Closed Sets in Intuitionistic Fuzzy Topological spaces

2. Preliminaries

Definition 2.1 [3]

An Intuitionistic Fuzzy Topology (IFT) on X is a collection of IFSs in X satisfying the following axioms,

1. 0,1 $\epsilon\ \tau$
2. A ∪ B $\epsilon\ \tau$ for any A,B $\epsilon\ \tau$
3. $\cap A_i \ \epsilon\ \tau$ for any { A_i / i ϵ J } $\subseteq \tau$

Here, (X,τ) is called an intuitionistic fuzzy topological space (IFTS) and any IFS in τ1 is known as intuitionistic fuzzy open set (IFOS) in X.The complement (A^c) of an IFOS A in an IFTS (X,τ) is called an intutionistic fuzzy closed set (IFCS) in X.In this paper, Intuitionistic fuzzy interior is denoted by ***int*** an Intuitionistic fuzzy closure is denoted by ***cl***.

Definition 2.2 [4]

An intutionistic fuzzy set W of an intuitionistic fuzzy topological space (X,τ) is called as,

1. an intutionistic fuzzy semi open (IFSO) if W⊆cl(int(W)) and intutionistic fuzzy semi closed set (IFSC) if int(cl(W)) ⊆ W.
2. an intutionistic fuzzy pre-open set (IFPO) if W⊆int(cl(W)) and intutionistic fuzzy pre-closed set (IFPC) if cl(int(W)) ⊆W.
3. an intutionistic fuzzy semi pre-open set (IFSPO) if W⊆cl(int(cl(W))) and intutionistic fuzzy semi generalized preclosed set (IFSPC) if int(cl(int(W))) ⊆W.
4. an intutionistic fuzzy semi generalized closed set (IFGSC) if Scl(W) ⊆ O, whenever W⊆O and O is IFO in X.
5. an intutionistic fuzzy generalized star closed set (IFG*C) if cl(W) ⊆O whenever W⊆O and O is IFGO in X.
6. an intutionistic fuzzy strongly generalized star closed set (IFSG*C) if cl(int(W)) ⊆O whenever W⊆O and O is IFGO in X.
7. an intutionistic fuzzy regular weekly generalized closed set (IFRWGC) if cl(int(W))⊆O, whenever W⊆O and O is IFO in (X,τ_1)
8. an intutionistc fuzzy generalized pre closed (IFGPC) if Pcl(W)⊆O whenever W⊆O and O is IFO in (X,τ_1).
9. an intutionistc fuzzy weekly generalized closed set (IFWGC) if cl(int(W)) ⊆O, whenever W⊆O and O is IFSO in X.
10. an intutionistic fuzzy generalized star pre closed set (IFG*PC) if Pcl(W) ⊆O whenever W⊆O and O is IFGO in X.
11. an intutionistic fuzzy generalized pre semi closed set (IFGPSC) if Pcl(W)⊆O whenever W⊆O and O is IFSO in X.
12. an intutionistic fuzzy semi weekly generalized closed set (IFSWGC) if cl(int(W))⊆O whenever W⊆O and O is IFSO in X.
13. an intutionistic fuzzy semi regular closed set (IFSGRC) if Scl(W) ⊆O whenever W⊆O and O is IFRO in X.
14. an intutionistic fuzzy generalized pre regular closed set (IFGPRC) if Pcl(W)⊆O whenever W⊆O and O is IFRO in X.

3. Intutionistic Fuzzy Strongly Alpha Generalized Star Star Closed Sets (IFSαG**C)

Definition 3.1

An IFS W of an intutionistic fuzzy topological space (X,τ) is called an Intutionistic Fuzzy Strongly Alpha Generalized Star Star closed set [IFSαG**C] if αcl(W)⊆O,whenever W⊆O and O is IFSαG*O.

Definition 3.2

An IFS W of an intutionistic fuzzy topological space (X,τ) is called an Intutionistic Fuzzy Strongly Alpha Generalized Star Star open set [IFSαG**O] if the complement of W is IFSαG*C.

Theorem 3.3

Every IFCS is IFSαG**C set in X.

Proof: Suppose W is IFCS. Now, cl(int(cl(W)))⊆cl(W)=W. Let O be any IFSαG*O set containing W. Then αcl(W) ⊆ O.Then W is IFSαG**C.

Remark 3.4

The converse of the above theorem is not true and it is shown by the following example,

Let X = $\{O_1, O_2\}$, τ = $\{0, 1, O_1, O_2\}$ and τ^c = $\{0, 1, {O_1}^c, {O_2}^c\}$ where O_1 = {<0.7, 0.5>, <0.4,0.6>}, O_2 = {<0.6,0.7>,<0.3,0.8>} and ${O_1}^c$ = {<0.5,0.7>,<0.6,0.4>}, ${O_2}^c$ = {<0.7,0.6>,<0.8,0.3>}. Then (X,τ) is an IFTS. Consider the IFS, W = {<0.2,0.8>,<0.1,0.7>}. Let O be any IFSαG*O set such that W⊆O. Then αcl(W)⊆O. Hence W is IFSαG**C set. But W is not IFCS. (Since cl(W)≠W).

Theorem 3.5

Every IFSαG**C is IFGSC set but not conversely.

Proof: Let W be any IFSαG**C set and O be any IFOS such that W⊆O. Since every IFOS is IFSαG*O, O is an IFSαG*O set. Therefore, αcl(W) ⊆ O which implies scl(W) ⊆ O. Hence W is IFGSC.

Example 3.6

Let X = {a,b}, τ = {0, 1, a, b} and τ^c = {0, 1, a^c, b^c} where a = {<0.3,0.4>,<0.2,0.8>}, b = {<0.7,0.9>,<0.3,0.6>} and a^c = {<0.4,0.3>,<0.8,0.2>}, b^c = {<0.9,0.7>,<0.6,0.3>}. Then (X,τ) is an IFTS. Consider the IFS, W = {<0.3,0.5>,<0.7,0.5>}. Now scl(W) ⊆ U whenever W⊆U U is IFO. Hence W is IFGSC. Let O be any IFSαG*O and W⊆O. But αcl(W) ⊈ O. Hence W is not IFSαG**C.

Theorem 3.7

Every IFSαG**C is IFGPC set but not conversely.

Proof: Let W be any IFSαG**C set and O be any IFOS such that W⊆O. Since every IFOS is IFSαG*O, O is an IFSαG*O set. Therefore, αcl(W) ⊆ O. Now, pcl(W) ⊆ αcl(W) implies pcl(W) ⊆ O. Thus W is IFGPC.

Example 3.8

Let X = {c,d}, τ = {0, 1, c, d} and τ^c = {0, 1, c^c, d^c} where c = {<0.6,0.3>,<0.5,0.1>}, d = {<0.8,0.6>,<0.7,0.3>} and c^c = {<0.3,0.6>,0.6,0.8>,<0.3,0.7>}. Then (X,τ) is an IFTS. Consider the IFS, W = {<0.4,0.7>,<0.7,0.6>}. Now, pcl(W) ⊆ U whenever W ⊆ U and U is IFO. Hence W is IFGPC. Let O be any IFSαG*O and (W) ⊆ O. But αcl(W) ⊈ O. Hence W is not IFSαG**C.

Theorem 3.9

Every IFSαG**C is IFWGC set but not conversely.

Proof: Let W be any IFSαG**C set and O be any IFOS such that W⊆O. Since every IFOS is IFSαG*O, O is an IFSαG*O set. Therefore, αcl(W) ⊆ O ⟹ cl(int(W)) ⊆ O. Hence W is IFWGC.

Example 3.10

Let X = {a,b}, τ = {0, 1, a, b} and τ^c = {0, 1, a^c, b^c} where a = {<0.3,0.6>,<0.3,0.5>}, b = {<0.5,0.3>,<0.5,0.3>} and a^c = {<0.6,0.3>,<0.5,0.3>}, b^c = {<0.3,0.5>,<0.3,0.5>}. Then (X,τ) is an IFTS. Consider the IFS, W = {<0.6,0.4>,<0.5,0.4>}. Now cl(int(W)) ⊆ U whenever W ⊆ U and U is IFO. Hence W is IFWGC. Let O be any IFSαG*O and (W) ⊆ O. But αcl(W) ⊈ O. Hence W is not IFSαG**C.

Theorem 3.11

Every IFSαG**C is IFGPRC set.

Proof: Let W be any IFSαG**C set and O be any IFRO set such that W⊆O. Since every IFRO is IFOS and every IFOS is IFSαG*O set. Therefore αcl(W) ⊆ O. Since pcl(W) ⊆ αcl(W), we have pcl(W) ⊆ O. Hence W is IFGPRC.

Theorem 3.12

Every IFSαG**C is IFGSRC set.

Proof: Let W be any IFSαG**C set and O be any IFRO set such that W⊆O. Since every IFRO is IFOS and every IFOS is IFSαG*O set. Therefore αcl(W) ⊆ O ⟹ scl(W) ⊆ O. Hence W is IFGSRC.

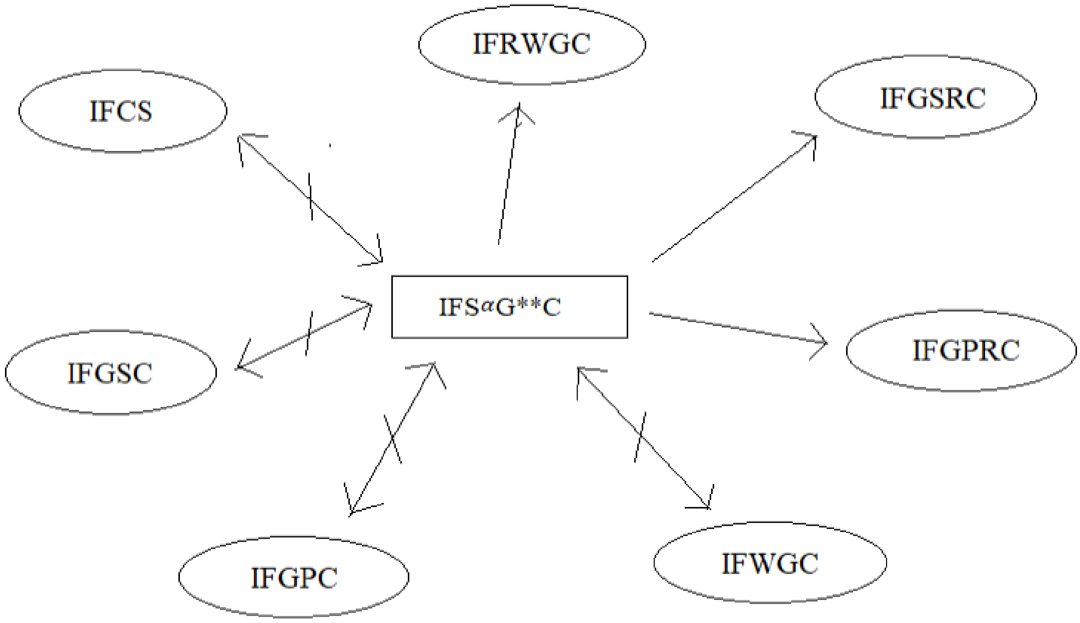
Theorem 3.13

Every IFSαG**C is IFRWGC set.

Proof: Let W be any IFSαG**C set and O be any IFRO set such that W⊆O. Since every IFRO is IFOS and every IFOS is IFSG*O set. O is an IFSαG*O set. Therefore, αcl(W) ⊆ O ⟹ cl(int(W)) ⊆ O. Hence W is IFRWGC.

Remark 3.14

From the above theorems and examples we have the following diagrammatic representation.



Characterizations of Intutionistic Fuzzy Strongly Alpha Generalized Star Star Closed Sets

Theorem 3.15

An IFS A is IFSαG**C iff αcl(A) – A containing no non zero IFSαG*C set.

Proof: Suppose that F is a non-zero IFSαG*C set such that F ⊆ αcl(A) – A. Then F ⊆ αcl(A) ∩ A^c i.e., F ⊆ αcl(A) and F ⊆ A^c which implies A ⊆ F^c. Here F^c is IFSαG*O and A is IFSαG*C. We have αcl(A) ⊆ F^c, F ⊆ αcl(A) ∩ (αcl(A)^c) = $\tilde{0}$ which implies αcl(A) – A containing no non zero IFSαG*C set.

Theorem 3.16

If B is IFSαG**C set and B ⊆ A ⊆ αcl(B) then A is IFSαG**C.

Proof: Let B be IFSαG**C and O be any IFSαG*O set such that A ⊆ O. Then B ⊆ O which implies αcl(A) ⊆ αcl(B) ⊆ O. Hence A is IFSαG**C.

Theorem 3.17

A is any IFSαG**O iff B ⊆ αint(A) where B is IFSαG*C and B ⊆ A.

Proof: Let A be any IFSαG**O set. Let B be IFSαG*C and B ⊆ A. Then A^c ⊆ B^c, Since A^c IFSαG**C and B^c is IFSαG*O. Therefore, we have B ⊆ αint(A). Conversely, Assume that B ⊆ αint(A) whenever B is IFSαG*C and B ⊆ A. Let O be any IFSαG*O. Then O^c is IFSαG*C. Therefore, by assumption O^c ⊆ αint(A) which implies αcl(A^c) ⊆ O. Then A^c is IFSαG*C. Hence, A is IFSαG*O.

Theorem 3.18

If αint(A) ⊆ B ⊆ A and A is IFSαG**O, then B is IFSαG**O.

Proof: αint(A) ⊆ B ⊆ A implies $A^c \subseteq B^c \subseteq \alpha$cl($A^c$). Since A is IFS$\alpha$G**O, A^c is IFSαG**C. Therefore by theorem 3.16, B^c is IFSαG**C. Hence B is IFSαG**O.

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