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# Common Fixed Point Theorem In Intuitionistic Menger Spaces 

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#### Abstract

The main purpose of this paper is to proof some common fixed point theorem in Instuitionistic Menger Space.

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Key words: Fixed point, Common Fixed point, Menger Space, instuitionistic Menger Space, Continuous mapping, Compatible and weakly compatible Self mapping

## Introduction

There have been a number of generalization of metric space, one such generalization is Menger Space introduced in 1942 by Menger, who was used distribution function instead of non negative real numbers as values of the metric. The concept of Fuzzy sets was introduced initially by Zadeh [1] in 1965. Since then, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and applications. In metric fixed point theory, various mathematicians weakened the notation of compatibility by introducing the notation of weak commutatively, $\overline{\text { compatibility }}$ and weak compatibility and produced the number of fixed point theorems using these notations. Recently, many mathematicians formulated the definition of weakly computing, compatible and weakly compatible maps in instuitionistic fuzzy metric spaces and prove a number of fixed point theorem in instuitionistic fuzzy metric spaces.

In this paper, we prove the common fixed point theorem for six maps under the condition of weak compatibility and compatibility in instuitionistic Menger spaces.

## Preliminaries

Definition 2.1 A binary operation $\star:[0,1] \times[0,1] \rightarrow[0,1]$ is continuous $t$-norm if $\star$ is satisfying the following condition:
(1) $\star$ is commutative and associative,
(2) $\star$ is continuous,

$$
\begin{equation*}
a \star 1=a \text { for all } a \in[0,1] \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
a \star b \leq c \star d \text { whenever } a \leq c \text { and } b \leq d, \text { for all } a, b, c, d \in[0,1] \tag{4}
\end{equation*}
$$

Examples of $t-$ norm are $a \star b=\min \{a, b\}$ and $a \star b=a b$.
Definition 2.2 A binary operation $\nabla:[0,1] \times[0,1] \rightarrow[0,1]$ is continuous $t$-conorm if $\nabla$ is satisfying the following condition:
(1) $\nabla$ is commutative and associative,
(2) $\nabla$ is continuous,
$a \nabla 1=a$ for all $a \in[0,1]$
$a \nabla b \leq c \nabla d$ whenever $a \leq c$ and $b \leq d$,for all $a, b, c, d \in[0,1]$
Examples of $t$ - conorm are $a \nabla b=\max \{a, b\}$ and $a \nabla b=\min \{1, a+b\}$.
Definition 2.3 A distribution function is a function $F:[-\infty, \infty] \rightarrow[0,1]$ which is left continuous on $\mathcal{R}$, non decreasing and $F(-\infty)=0, f(\infty)=1$.

We will denote by $\Delta$ the family of all distribution function on $[-\infty, \infty], \mathcal{H}$ is a special element of $\Delta$ defined by,

$$
\mathcal{H}(t)= \begin{cases}0 & \text { if } t \leq 0 \\ 1 & \text { if } t \geq 0\end{cases}
$$

If $X$ is a nonempty set, $F: X \times X \rightarrow \Delta$ is called a probabilistic distance on $X$ and $F(x, y)$ is usually denoted by $F_{x y}$.

Definition 2.3 The ordered pair $(X, F)$ is called the probabilistic metric space (shortly PM-space) if $X$ is a nonempty set and $F$ is a probabilistic distance satisfy the following conditions:- for all $x, y, z \in X$ and $t, s>0$,
(1) $F_{x y}(t)=1 \Leftrightarrow x=y$;
(2) $F_{x y}(0)=0$
(4) $\quad F_{x z}(t)=1, F_{z y}(s)=1 \Rightarrow F_{x y}(s+t)=1$

The order triple $(X, F, \star)$ is called Menger space if $(X, F)$ is a PM-space, $\star$ is a $t$-norm and the following condition is also satisfies: for all $x, y, z \in X$ and $t, s>0$,

$$
\begin{equation*}
F_{x y}(t+s) \geq F_{x z}(t) \star F_{z y}(s) . \tag{5}
\end{equation*}
$$

Definition 2.4 Let $(X, F, \star)$ be a Menger space and $\star$ be a continuous $t$ - norm, then
(a) A sequence $\left\{x_{n}\right\}$ in $X$ is said to be converges to a point $x$ in $X$, iff for every $\epsilon>0$ and $\lambda \in(0,1)$ there exists an integer $n_{0}=n_{0}(\epsilon, \lambda)$ such that $F_{x_{n} x}(\epsilon)>1-\lambda$ for all $n \geq n_{0}$.
(b) A sequence $\left\{x_{n}\right\}$ in $X$ is said to be Cauchy sequence converges to a point $x$ in $X$, iff for every
$\epsilon>0$ and $\lambda \in(0,1)$ there exists an integer $n_{0}=n_{0}(\epsilon, \lambda)$ such that $F_{x_{n} x_{n+p}}(\epsilon)>1-\lambda$ for all $n \geq n_{0}$ and $p>0$.
(c) A Menger space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.5 Self map $A$ and $B$ of a Menger space ( $X, F, \star$ ) are said to be weakly compatible if they commute at their coincidence points that is if $A x=B x$ for some $x \in X$ then $A B x=B A x$.

Definition 2.6 Self map $A$ and B of a Menger space $(X, F, \star)$ are said to be compatible if $F_{A B x_{n} B A x_{n}}(t) \rightarrow 1$ for all $t>0$, whenever $\left\{x_{n}\right\}$ is sequence in $X$ such that $A x_{n}, B x_{n} \rightarrow x$ for some $x \in X$ as $n \rightarrow \infty$.

Definition 2.7 A 5- tuple ( $X, M, N, \star, \nabla$ ) is said to be intuitionistic Menger space, if $X$ is arbitrary set, $\star$ is a continuous $t$-norm, $\nabla$ is a continuous $t$-conorm and $M, N$ are $P M$-space on $X \times X \times[0, \infty)$ satisfying the following conditions:

```
\(M_{x, y}(t)+N_{x, y}(t) \leq 1\). For all \(x, y \in X\) and \(t>0\),
\(M_{x, y}(o)=0\) for all \(x, y \in X\)
\(M_{x, y}(t)=1\), for all \(x, y \in X\), and \(t>0\) if and only if \(x=y\).
\(M_{x, y}(t)=M_{y, x}(t)\) For all \(x, y \in X\) and \(t>0\)
\(M_{x, y}(t) \star M_{y, z}(s) \leq M_{x, z}(t+s)\) For all \(x, y \in X\) and \(s, t>0\)
For all \(x, y \in X, M_{x, y}(\cdot):[0, \infty) \rightarrow[0,1]\) is left continuous.
\(\lim _{t \rightarrow \infty} M_{x, y}(t)=1\), for all \(x, y \in X\) and \(t>0\),
\(N_{x, y}(0)=1\), for all \(x, y \in X\),
\(N_{x, y}(t)=0\), for all \(x, y \in X\), for all \(x, y \in X\), and \(t>0\) if and only if \(x=y\).
\(N_{x, y}(t)=N_{y, x}(t)\) For all \(x, y \in X\) and \(t>0\)
    \(N_{x, y}(t) \nabla N_{y, z}(s) \geq N_{x, z}(t+s)\) for all \(x, y \in X\) and \(s, t>0\)
    For all \(x, y \in X, N_{x, y}(\cdot):[0, \infty) \rightarrow[0,1]\) is right continuous.
    \(\lim _{t \rightarrow \infty} N_{x, y}(t)=0\), for all \(x, y \in X\) and \(t>0\),
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Then $(M, N)$ is called intuionistic Menger space on $X$. The function $M_{x, y}(t)$ and $N_{x, y}(t)$ denote the degree of nearness and degree of non nearness between $x$ and $y$ with respect to $t$.

Lemma 2.8 Let $\left\{x_{n}\right\}$ be a sequence in a Menger space $(X, F, \star)$ with continuous $t$ - norm $\star$ and $t \star t \geq t$. If there exists a constant $k \in(0,1)$ such that,

$$
F_{x_{n}, x_{n+1}}(k t) \geq F_{x_{n-1}, x_{n}}(t)
$$

for all $t>0$ and $n=1,2, \ldots .$. then $\left\{x_{n}\right\}$ is a Cauchy sequence in $X$.

## Main Result,

Theorem:3.1 Let $A, B, S, T, P$ and $Q$ be self mapping of an intuitionistic Menger space, $X$ into itself, with continuous, $t$-norm, * and continuous $t$-conorm $\nabla$ defined by $t \star t \geq t$ and $(1-t) \nabla(1-t) \leq(1-$ t) for all $t \in[0,1]$, satisfying the following condition:

$$
\begin{equation*}
P(X) \subset S T(X), Q(X) \subset A B(X) \tag{1}
\end{equation*}
$$

There exists a constant $k \in(0,1)$ such that,

$$
M_{P x, Q y}^{2}(k t) \star\left[M_{A B x, P x}(k t) \cdot M_{S T y, Q y}(k t)\right] \geq\left[\alpha M_{A B x, P x}(t)+\beta M_{A B x, S T y}(t)\right] M_{A B x, Q y}(2 k t)
$$

for all $x, y \in X$ and $t>0$, where $0<\alpha, \beta<1$ such that $(\alpha+\beta)=1$ and

$$
N_{P x, Q y}^{2}(k t) \nabla\left[N_{A B x, P x}(k t) \cdot N_{S T y, Q y}(k t)\right] \leq\left[\alpha N_{A B x, P x}(t)+\beta N_{A B x, S T y}(t)\right] N_{A B x, Q y}(2 k t)
$$

for all $x, y \in X$ and $t>0$, where $0<\alpha, \beta<1$ such that $(\alpha+\beta)=1$.
(3) If one of $P(X), S T(X), A B(X), Q(X)$ is a complete subspace of $X$ then:
(a) $P$ and $A B$ have a coincidence point and
(b) $Q$ and $S T$ have a coincidence point.
(4) $A B=B A, S T=T S, P B=B P, Q T=T Q$,
(5) The pair $\{P, A B\}$ and $\{Q, S T\}$ are weakly compatible,

Then $A, B, S, T, P$, and $T$ have unique common fixed point in $X$.
Proof: Let $x_{0}$ be an arbitrary point in $X$, then by (1) there exists $x_{1}, x_{2} \in X$ such that,

$$
P x_{0}=S T x_{1}=y_{0} \text { and } Q x_{1}=A B x_{2}=y_{1} .
$$

Inductively, we can construct sequence $\left\{x_{n}\right\}$ and $\left\{y_{n}\right\}$ in $X$ such that, $P x_{2 n}=S T x_{2 n+1}=y_{2 n}$ and $Q x_{2 n+1}=$ $A B x_{2 n+2}=y_{2 n+1}$ for $n=0,1,2, \ldots \ldots$

On taking $x=x_{2 n}$ and $y=x_{2 n+1}$ in (2), we have

$$
\begin{align*}
M_{P x_{2 n}, Q x_{2 n+1}}^{2}(k t) \star\left[M_{A B x_{2 n}, P x_{2 n}}(k t)\right. & \left.. M_{S T x_{2 n+1}, Q x_{2 n+1}}(k t)\right] \\
& \geq\left[\alpha M_{A B x_{2 n}, P x_{2 n}}(t)+\beta M_{A B x_{2 n}, S T x_{2 n+1}}(t)\right] M_{A B x_{2 n}, Q x_{2 n+1}} \tag{2kt}
\end{align*}
$$

And

$$
M_{y_{2 n}, y_{2 n+1}}^{2}(k t) \star\left[M_{y_{2 n-1}, y_{2 n}}(k t) \cdot M_{y_{2 n}, y_{2 n+1}}(k t)\right]
$$

$$
\geq\left[\alpha M_{y_{2 n-1}, y_{2 n}}(t)+\beta M_{y_{2 n}, y_{2 n-1}}(t)\right] M_{y_{2 n-1}, y_{2 n+1}}(2 k t)
$$

$$
N_{y_{2 n}, y_{2 n+1}}^{2}(k t) \nabla\left[N_{y_{2 n-1}, y_{2 n}}(k t) \cdot N_{y_{2 n}, y_{2 n+1}}(k t)\right]
$$

$$
\leq\left[\alpha N_{y_{2 n-1}, y_{2 n}}(t)+\beta N_{y_{2 n}, y_{2 n-1}}(t)\right] N_{y_{2 n-1}, y_{2 n+1}}(2 k t)
$$

$$
M_{y_{2 n}, y_{2 n+1}}^{2}(k t) \star\left[M_{y_{2 n-1}, y_{2 n}}(k t) \cdot M_{y_{2 n}, y_{2 n+1}}(k t)\right] \geq[\alpha+\beta] M_{y_{2 n-1}, y_{2 n}}(t) \cdot M_{y_{2 n-1}, y_{2 n+1}}(2 k t)
$$

And

$$
\begin{gather*}
N_{y_{2 n}, y_{2 n+1}}^{2}(k t) \nabla\left[N_{y_{2 n-1}, y_{2 n}}(k t) \cdot N_{y_{2 n}, y_{2 n+1}}(k t)\right] \leq[\alpha+\beta] N_{y_{2 n-1}, y_{2 n}}(t) . N_{y_{2 n-1}, y_{2 n+1}}(2 k t)  \tag{2kt}\\
M_{y_{2 n}, y_{2 n+1}}(k t) \cdot\left[M_{y_{2 n-1}, y_{2 n}}(k t) \star M_{y_{2 n}, y_{2 n+1}}(k t)\right] \geq[\alpha+\beta] M_{y_{2 n-1}, y_{2 n}}(t) \cdot M_{y_{2 n-1}, y_{2 n+1}}(2 k t) \tag{2kt}
\end{gather*}
$$

And

$$
N_{y_{2 n}, y_{2 n+1}}(k t) \cdot\left[N_{y_{2 n-1}, y_{2 n}}(k t) \nabla N_{y_{2 n}, y_{2 n+1}}(k t)\right] \leq[\alpha+\beta] N_{y_{2 n-1}, y_{2 n}}(t) \cdot N_{y_{2 n-1}, y_{2 n+1}}(2 k t)
$$

$$
M_{y_{2 n}, y_{2 n+1}}(k t) \cdot M_{y_{2 n-1}, y_{2 n+1}}(2 k t) \geq[\alpha+\beta] M_{y_{2 n-1}, y_{2 n}}(t) \cdot M_{y_{2 n-1}, y_{2 n+1}}(2 k t)
$$

And

$$
\begin{gathered}
N_{y_{2 n}, y_{2 n+1}}(k t) \cdot N_{y_{2 n-1}, y_{2 n+1}}(2 k t) \leq[\alpha+\beta] N_{y_{2 n-1}, y_{2 n}}(t) . N_{y_{2 n-1}, y_{2 n+1}}(2 k t) \\
M_{y_{2 n}, y_{2 n+1}}(k t) \geq M_{y_{2 n-1}, y_{2 n}}(t) \text { and } N_{y_{2 n}, y_{2 n+1}}(k t) \leq N_{y_{2 n-1}, y_{2 n}}(t)
\end{gathered}
$$

Similarly we can prove that,

$$
M_{y_{2 n+1}, y_{2 n+2}}(k t) \geq M_{y_{2 n}, y_{2 n+1}}(t) \text { and } N_{y_{2 n+1}, y_{2 n+2}}(k t) \leq N_{y_{2 n}, y_{2 n+1}}(t)
$$

For $k \in(0,1)$ and all $t>0$. Thus by lemma $2.8,\left\{y_{n}\right\}$ is a Cauchy sequence in $X$.
Now suppose $A B(X)$ is a complete. Note that the subsequence $\left\{y_{2 n+1}\right\}$ is contained in $A B(X)$ and has a limit in $A B(X)$ call it ' $z$ '. let $w \in A B^{-1}(z)$, then $A B w=z$. we shall use the fact that subsequence $\left\{y_{2 n}\right\}$ also converges to ' $z$ '.

By putting $x=w$ abd $y=x_{2 n+1}$ in (2) and taking limit as $n \rightarrow \infty$, we have

$$
M_{P w, z}^{2}(k t) \star\left[M_{z, P w}(k t) \cdot M_{z, z}(k t)\right] \geq\left[\alpha M_{z, P w}(t) \mp \beta M_{z, z}(t)\right] M_{z, z}(2 k t)
$$

And

$$
N_{P w, z}^{2}(k t) \nabla\left[N_{z, P w}(k t) \cdot N_{z, z}(k t)\right] \leq\left[\alpha N_{z, P w}(t)+\beta N_{z, z}(t)\right] N_{z, z}(2 k t)
$$

Thus if follows that,

$$
M_{z, P w}(k t) \geq\left[\alpha M_{z, P w}(t)+\beta\right] \text { and } N_{z, P w}(k t) \leq 0
$$

Also

$$
M_{z, P w}(k t) \geq \frac{\beta}{1-\alpha}=1 \text { and } N_{z, P w}(k t) \leq 0
$$

Hence $z=P w$. since $A B w=z$, thus we have $P w=z=A B w$ that is $w$ is coincidence point of $P$ and $A B w$. Since $P(X) \subset S T(X), P w=z$ implies that, $z \in S T(X)$. Let $v \in S T_{-}^{-1} z$ then $S T v=z$.

By putting $x=x_{2 n}$ and $y=v$ in (2) and taking $n \rightarrow \infty$ we have,

$$
M_{Z, Q v}^{2}(k t) \star\left[M_{z, z}(k t) \cdot M_{z, Q v}(k t)\right] \geq\left[\alpha M_{z, Z}(t)+\beta M_{z, Z}(t)\right] M_{z, Q v}(2 k t)
$$

And
$N_{z, Q v}^{2}(k t) \star\left[N_{z, Z}(k t) . N_{z, Q v}(k t)\right] \leq\left[\alpha N_{z, Z}(t)+\beta N_{z, Z}(t)\right] N_{z, Q v}(2 k t)$
Thus we have $M_{z, Q v}(k t) \geq 1$ and $N_{z, Q v}(k t) \leq 0$. Thus $z=Q v$, since $S T v=z$, we have $Q v=z=S T v$ that is $v$ is coincidence point of $Q$ and ST. this proves (b), the remaining two cases pertain essentially to the previous cases. Indeed if $P(x)$ or $Q(x)$ is complete, then by (1), $z \in P(X) \subset S T(X)$ or $z \in Q(X) \subset$ $A B(X)$, thus (a) and (b) are completely established.

Since the pair $(P, A B)$ is weakly compatible therefore $P$ and $A B$ commute at there coincidence point that is $P(A B w)=A B(P) w$, that is $P z=A B z$.

Since the pair $(Q, S T)$ is weakly compatible therefore $Q$ and $S T$ commute at there coincidence point that is $Q(S T v)=S T(Q) v$, that is $Q z=S T z$.

By putting $x=z$, and $y=x_{2 n+1}$ in (2) and taking limit at $n \rightarrow \infty$, we have

$$
M_{P Z, Z}^{2}(k t) \star\left[M_{A B z, P Z}(k t) \cdot M_{z, Z}(k t)\right] \geq\left[\alpha M_{A B Z, P Z}(t)+\beta M_{A B Z, Z}(t)\right] M_{A B Z, Z}(2 k t)
$$

And

$$
N_{P Z, Z}^{2}(k t) \star\left[N_{A B Z, P Z}(k t) . N_{z, Z}(k t)\right] \geq\left[\alpha N_{A B Z, P Z}(t)+\beta N_{A B Z, Z}(t)\right] N_{A B Z, Z}(2 k t)
$$

Thus we have $M_{z, P z}(k t) \geq 1$ and $N_{z, P z}(k t) \leq 0$. Thus $z=P z$. So $P z=A B z=z$.
By putting $x=x_{2 n}, y=z$ in (2) and taking limit at $n \rightarrow \infty$ we have $M_{Z, Q z}(k t) \geq 1$ and $N_{z, Q z}(k t) \leq$ 0 thus, $z=Q z$, so $Q z=S T z=z$.

By putting $x=z, y=T z$ in (2) and using (4) we have $M_{z, T z}(k t) \geq 1$ and $N_{z, T z}(k t) \leq 0$, thus $z=$ Tz. since $S T z=z$ therefore $S z=z$. to prove $B z=z$ we put $x=B z, y=z$ in (2) and using (4) we have $M_{B z, z}(k t) \geq 1$ and $N_{B z, Z}(k t) \leq 0$. Thus $z=B z$ since $A B z=z$, there fore $A z=z$. by combining the above results we have

$$
A z=B z=S z=T z=P z=Q z=z .
$$

That is $z$ is a common fixed point of $A, B, S, T, P, Q$.

## UNIQUENESS:-

Let ' $w$ ' is another fixed point of $A, B, S, T, P, Q$ different from ' $z$ ' then On taking $x=z$ and $y=w$ in (2), we have $M_{z, w}(k t) \geq 1$ and $N_{z, w}(k t) \leq 0$. Hence $z=w$ for all $x, y, \in X$ and $t>0$. Therefore ' $z$ ' is the unique common fixed point of $A, B, S, T, P, Q$.

This complete the proof.

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## Reference:-

1. L.A, Zadeh, Fuzzy sets, Inform Control, 8, (1965) 335-3534,.
2. Z. K. Deng, Fuzzy psedo-metric spaces. J. Math. Anal. Appl., 86, (1982) 74-95:
3. M. A. Ereeg. Metric space in fuzzy sets theory. J. Math. Anal. Appl., 69,( 1979) 338 -353,
4. J. X. Fang. On fixed point theorem in fuzzy metric spaces. Fuzzy sets and systems 46, (1992) 107-113::
5. A. George and P. Veeramani, On some results in fuzzy metric spaces, fuzzy sets and systems, 64, (1994) 395-339,
6. O. Kaleva and S. Seikkala, On fixed fuzzy metric spaces. Fuzzy sets and systems, 12, (1994) 215 -229,
7. I. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces. Kybernetica, 11(1975) 326- 334,
8. J. H. Park. Intuitionistic fuzzy sets. Fuzzy sets and systems, 20(1986)87-96,
9. Banach: S. "Surles operation dansles ensembles abstraites etleur application integrals" Fund. Math. 3(1922) 133-181
10. Brouwder, F.E. "Non-expansive non-linear operators in Banach spaces" Proc Nat.Acad. Sci. U.S.A. 54 (1965) 1041-1044
11. Ciric, L.B. "A generalization of Banach contraction principle" Proc. Amer. Math. Soc. 45 (1974), 267-273
12. Diminni C. , Gahler S. and White A. G. ' strictly Convex linear 2- normed spaces,Math, NAcher, 59 (1974), 319-324
13. Ehret R.' linear normed spaces, Dissertation, St. Louis university 1969
14. GuptaO.P and Badshah V.N "fixed Point theorem in Banach and 2-Banach spaces"Jnanabha 35 (2005)
15. Gahler S. ' 2-metrische Reume und iher topologische struktur' Math Nacher, 26 (1963/64) 115-148.
16. Gahler S. 'linear 2- normierte raume, Math NAcher, 28 (1965) 1-43
17. Hadzic, O. ' On common fixed point in 2-metric spaces.' Univ. Novom sadu. Zb. Rad. Prirod - Mat. Fak. Ser. Mat. 12(1982), 7-18
18. Hadzic, $O$. 'Some theorems on the fixed point in Probabilistic metric and random normed spaces, Bull. Union Mat. Ital. (6), 1-B(1982), 381-391.
19. Iseki K. ' fixed point theorem in Banach space' math. Sem. Notes kobe univ. 2 (1974), 11-13
20. Iseki K. 'on non-expansive mapping in strictly convex linear 2-normed space, this notes 3 (1975),

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21. Kanan M. ' fixed point theorem in 3-metric space' pure math. Manuscript vol. 5(1986) 77-82
22. Khan M.S. and Imdad M. " fixed and coincidence Points in Banach and 2- Banach spaces" Mathemasthical Seminar Notes. Vol 10 (1982)
23. Schweizer, B., Sklar, A., 'Probabilistic Metric Spaces' Elsevier North- Holland, 1983.
24. Sehgal, V., M., Bharucha-Reid, A., T., ' Fixed points of contraction mappings in probabilistic metric spaces', Math. Systems Theory6 (1972), 97-102.
25. White A.G. ' 2-Banach Space, Math NAcher, 42 (1969), 43- 60.

