



Role of Transverse Magnetic Field on 3D Rotating Flow of Maxwell Nanofluid Over a Stretching Sheet with Chemical Reaction.

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Abstract:

In this article, an attempt has been made to study the heat transfer characteristics of Maxwell-nanofluid flow over a stretching sheet with transverse magnetic field in the presence of chemical reaction. The nonlinear governing equations with suitable boundary conditions are initially cast into dimensionless form by similarity transformations and then the resulting highly nonlinear coupled equations are solved via the optimal homotopy analysis method. The impact of all sundry parameters like Magnetic parameter, Deborah number, Prandtl number, Schmidt number, Rotation parameter, Velocity slip parameter, Chemical reaction parameter, Thermophoresis parameter, Brownian motion parameter, Heat sources per sink parameter, Biot number, Eckert number on the velocity, temperature and concentration field are analyzed through graphs and tables.

Keywords: MHD; Deborah number ; Optimal Homotopy Analysis Method (OHAM).

1. Introduction

Over the decades, the study of boundary layer flow and heat transfer over a stretching sheet has attracted the attention of several researchers due to its various industrial and engineering applications such as annealing and tinning of copper wires, manufacturing of plastic and rubber sheets, crystal growing, continuous cooling, extrusion of polymer, wire drawing, in textiles and glass fiber production, paper production etc. In view of these applications, beginning with the pioneering work by Sakiadis [1] initiated the study of boundary layer flow past a stretching surface. Crane [2] examined the flow and heat transfer characteristics of Sakiadis' flow. The significance of heat transfer over a continuous stretching surface with variable temperature is mentioned in ref [3-4].

Many researchers at present are engaged in exploring the flows of non-Newtonian fluids at various aspects. Non-Newtonian fluids are very popular due to their importance in our daily life. Examples of such fluids include sugar solutions, tomato ketchup, lubricants, apple sauce, shampoos, cosmetic products, and many others. The non-Newtonian fluids frequently appear in industry and in engineering. Due to the distinct physical structures of fluids, that can describe all the rheological properties of such fluids. In spite of this, the non-Newtonian fluids have been classified into three types: (i) the differential type, (ii) the rate type, (iii)

the integral type. There is a simplest subclass of rate type fluids known as upper-convected Maxwell (UCM) fluid. This model can easily predict the relaxation time phenomena. Recent contribution on the flow analysis of UCM fluid include those of Abbas et al. [5], Hayat et al. [6], Abel et al. [7], Nadeem et al. [8], Awasi et al. [9], Shafique et al. [10]. In recent years, some interest has been given to analyze the convective transport of nanofluids. Its main advantage is thermal conductivity of the fluids plays an important role on the heat transfer coefficient between the heat transfer medium and heat transfer surface. Nano-scale particle added fluids are called nanofluid, which is firstly coined by Choi [11]. Choi was the finally revealed to the society about the enhancement of thermal conductivity of fluids with nanoparticles. Many researchers investigated an excellent work on nanofluid over a stretching sheet with distinct physical aspects such as presence of magnetic field, slip effect condition, convective heat transfer and viscous dissipation. Examined the boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition Ref. [12-13]. Study of convective heat transfer in the flow of viscous Ag-water and Cu-water nanofluid over a stretching surface excellent work done by Vajravelu et al. [14].

The study of heat generation and absorption on heat transfer is very important because its effects are crucial in controlling the heat transfer. Bataller [15] explained the heat transfer of a viscoelastic fluid over a stretching sheet in the presence of heat source/sink. Ramesh and Gireesha [16] explore the importance of heat source/sink on a Maxwell fluid over a stretching sheet with convective boundary condition in the presence of nanoparticles. Several researchers investigated the effect of viscous dissipation and heat generation and absorption over a stretching sheet and shrinking sheet Ref. [17-19]. In above cited papers they are shown that for effective cooling of stretching, heat source/sink should used. Andersson [20] examined the slip flow past a stretching sheet. Ibrahim and Shankar [21] studied the MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity slip. Sui et al. [22] investigated the effect of heat and mass transfer with cattaneo-christov double-diffusion in UCM nanofluid past a stretching sheet with slip velocity. Rout et al. [23] reported the heat and mass transfer of chemical reaction fluid flow over a moving vertical plate in presence of magnetic field with heat source and convective surface boundary condition. Anjanna Matta and Nagaraju Gajjela [24] numerically studied the order of chemical reaction and convective boundary condition effects on micropolar fluid flow over a stretching sheet.

In this article, to explore the influences of magnetic field and heat source/sink on rotating flow of non-Newtonian Maxwell nanofluid over an stretching surface with chemical reaction and velocity slip. The governing nonlinear coupled system of equations for flow and heat transfer have been lessened to nonlinear coupled differential equations are solved by Optimal Homotopy Analysis Method (OHAM) [25-26]. Solutions for Horizontal velocity, transverse velocity, temperature distribution and concentration distribution are developed. Pictorial illustrations for Horizontal velocity, transverse velocity, temperature and concentration are presented to emphasize the physical effects of sundry parameters on the solutions. Numerical values of Horizontal skin friction, transverse skin friction, local Nusselt number and local Sherwood number for a range of parameters are tabulated.

2. Mathematical Modeling of the Problem

Consider a three-dimensional, steady boundary layer flow of a viscous incompressible non-Newtonian Maxwell-nanofluid by an elastic surface subjected to the convective boundary condition and zero mass flux nanoparticle concentration. A Cartesian coordinate system is chosen such that the surface is stretched in the x -direction with linearly varying velocity of the form $u_w(x) = ax + \lambda \frac{\partial u}{\partial z}$ which induces flow in the neighboring layers of the fluid. The uniform magnetic flux B_0 is acting along y -axis. Let Ω be the constant angular velocity of the rotating fluid. Geometry of the problem.

The problem under consideration is governed by the following boundary layer equations of Maxwell fluid along nano-particles in the presence of heat source per sink and chemical reaction which are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - 2\Omega v = \nu \left(\frac{\partial^2 u}{\partial z^2} \right) - \lambda_1 \left[u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + w^2 \frac{\partial^2 u}{\partial z^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} + 2vw \frac{\partial^2 u}{\partial y \partial z} + 2uw \frac{\partial^2 u}{\partial x \partial z} - 2\Omega \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) + 2\Omega \left(v \frac{\partial u}{\partial x} - u \frac{\partial u}{\partial y} \right) \right] - \frac{\sigma B_0^2}{\rho} \left[u + \lambda_1 w \frac{\partial u}{\partial z} \right] \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} - 2\Omega u = \nu \left(\frac{\partial^2 v}{\partial z^2} \right) - \lambda_1 \left[u^2 \frac{\partial^2 v}{\partial x^2} + v^2 \frac{\partial^2 v}{\partial y^2} + w^2 \frac{\partial^2 v}{\partial z^2} + 2uv \frac{\partial^2 v}{\partial x \partial y} + 2vw \frac{\partial^2 v}{\partial y \partial z} + 2uw \frac{\partial^2 v}{\partial x \partial z} + 2\Omega \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) + 2\Omega \left(v \frac{\partial v}{\partial x} - u \frac{\partial v}{\partial y} \right) \right] - \frac{\sigma B_0^2}{\rho} \left[v + \lambda_1 w \frac{\partial v}{\partial z} \right] \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha \frac{\partial^2 T}{\partial z^2} + \frac{Q}{\rho C_p} (T - T_\infty) + \frac{\mu}{\rho C_p} \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right] + \left\{ \tau \left[D_B \left(\frac{\partial T}{\partial z} \cdot \frac{\partial C}{\partial z} \right) + \frac{DT}{T_\infty} \left(\frac{\partial T}{\partial z} \right)^2 \right] \right\} \quad (4)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = D_B \left[\frac{\partial^2 C}{\partial z^2} \right] + \frac{D_T}{T_\infty} \left[\frac{\partial^2 T}{\partial z^2} \right] - k_1 [C - C_\infty] \quad (5)$$

where u, v and w are the fluid velocity components in the x, y and z directions respectively.

The appropriate boundary conditions are given as:

$$u = u_w(x) = ax + k_1 \frac{\partial u}{\partial z}, \quad v = 0, \quad w = 0,$$

$$-k \frac{\partial T}{\partial z} = h_f(T_f - T), D_B \frac{\partial C}{\partial z} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial z} = 0 \quad \text{at } z = 0, \quad (6)$$

$$u \rightarrow 0, v \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \quad \text{as } z \rightarrow \infty.$$

Introducing the

following set of dimensionless similarity transformations

$$\eta = \sqrt{\frac{a}{\nu}} z, \quad u = axf'(\eta), \quad v = axg(\eta), \quad w = -\sqrt{a\nu}f(\eta),$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty},$$

(7) Using the Eq.

(7), we observe that Eq. (1) is satisfied. Substituting these transformations into (2) to (5) by simplification can be presented as

$$f''' + ff'' - f'^2 + 2\lambda[g - \beta fg'] + \beta[2ff'f - f^2f'''] - Mn[f' - \beta ff''] = 0 \quad (8)$$

$$g'' + fg' - f'g - 2\lambda[f' + \beta(f'^2 - ff'' + g^2)] + \beta[2ff'g' - f^2g''] - Mn[g - \beta fg'] = 0 \quad (9)$$

$$\frac{1}{Pr}\theta'' + f\theta' + \alpha\theta + Ec[f''^2 + g'^2] + Nb\theta'\phi' + Nt\theta'^2 = 0 \quad (10)$$

$$\phi'' + Scf\phi' + \left(\frac{Nt}{Nb}\right)\theta'' - Sc\delta\phi = 0 \quad (11)$$

Subjected to the transformed conditions

$$\text{at } \eta \rightarrow 0: \quad f(0) = g(0) = 0, \quad f'(0) = 1 + Kf''(0),$$

$$\theta'(0) = -\gamma[1 - \theta(0)], \quad Nb\phi'(0) + Nt\theta'(0) = 0.$$

$$\text{as } \eta \rightarrow \infty: \quad f'(\infty) = g(\infty) = \theta(\infty) = \phi(\infty) = 0. \quad (12)$$

Where the non-

dimensional quantities like $Mn, \beta, \lambda, Pr, \alpha, Ec, Sc, \delta, Nt, Nb, K$ and γ are respectively the Magnetic number, Deborah number, Rotation parameter, Prandtl number, Heat source sink parameter, Eckert number, Schmidt number, Chemical reaction, Thermophoresis parameter, Brownian motion parameter, Velocity slip parameter and Biot number are defined as follows:

$$Mn = \frac{\sigma B_0^2}{\rho a}, \quad \beta = \lambda_1 a, \quad \lambda = \frac{\Omega}{a}, \quad Pr = \frac{\alpha}{\nu}, \quad K = k_1 \sqrt{\frac{a}{\nu}},$$

$$\alpha = \frac{Q}{\rho c_p a}, \quad Ec = \frac{a^2 x^2}{c_p (T_w - T_\infty)}, \quad Sc = \frac{\nu}{D_B}, \quad \gamma = \frac{h}{k} \sqrt{\frac{\nu}{a}},$$

$$\delta = \frac{k_c}{a}, \quad Nb = \frac{\tau D_B}{\nu} (c_w - c_\infty), \quad Nt = \frac{\tau D_T}{T_\infty \nu} (T_w - T_\infty).$$

(13) The quantities

with core physical interest are by using Fourier law can be to define local Nusselt number Nu_x and Fick's law can be used to define local Sherwood number Sh_x . These are as follows:

3.Method of solution

OHAM which is totally based on the concept of homotopy, it's derived from topology. This method plays an important tool where a non-linear problem is transformed into an infinite number of linear sub-problems its very effective method in Semi-Numerical Methods We have great freedom take intial guess and Linear Opertaor then by graph concluding the results accurately(18-21).

4.Results and discussion

In this article, The system of Eq. (8) to Eq. (11) are highly non-linear and coupled ordinary differential equations. Exact analytical solutions for the complete set of equations subject to the boundary conditions (12) are solved analytically via efficient OHAM. The results are demonstrated through pictorially and tabular form. Figures are drawn for Horizontal velocity $f'(\eta)$, Transverse velocity $g(\eta)$, temperature field $\theta(\eta)$ and concentration field $\phi(\eta)$. The impact of the physical sundry parameters such as the magnetic parameter Mn , the Deborah number β , the rotation parameter λ , the Prandtl number Pr , the heat sources sink parameter α , the Biot number γ , the Eckert number Ec , the Schmidt number Sc , the chemical reaction δ , the velocity slip parameter K , the thermophoresis parameter Nt and Brownian motion parameter Nb on the $f'(\eta)$, $g(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ are analyzed graphically in Figures 2(a)-2(d). In Fig. 2(a-d) demonstrate the effect of increasing values of β and Mn on $f'(\eta)$, $g(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ respectively. In Fig. 2(a) it is clear that increasing β and Mn corresponds to decreases in the case of Horizontal velocity this is due to the fact that retarding force called the Lorentz force. But in the opposite trend is observed in the case of transverse velocity $g(\eta)$, temperature field $\theta(\eta)$ and concentration field $\phi(\eta)$ (see fig 2(b),2(c),2(d)).

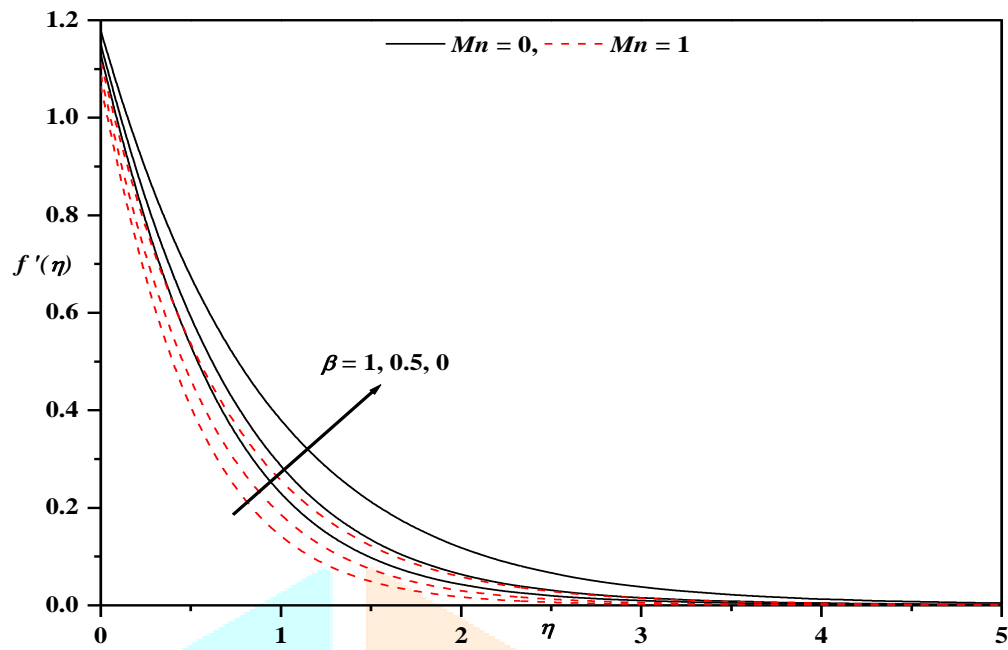


Fig.2(a):Horizontal velocity profile for different values of Mn and β with $Pr = 1.09$
 $\gamma = 0.5, \lambda = 0.2, Nt = 0.3, Nb = 0.5, Ec = 0.02, \alpha = 0.1, Sc = 0.22, \delta = 0.5, K = 0.2.$

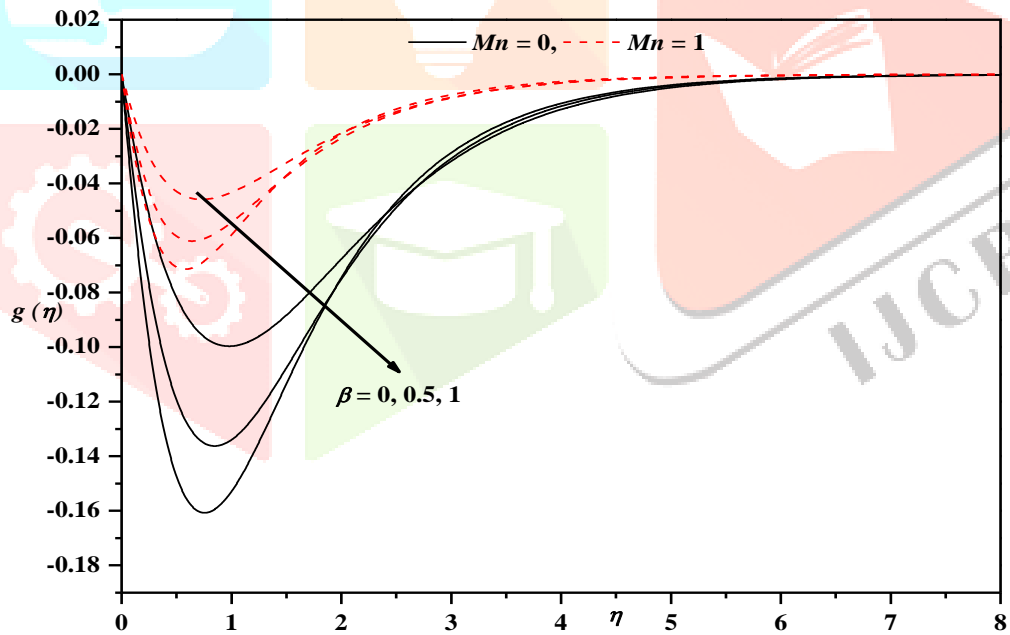


Fig.2(b):Transverse velocity profile for different values of Mn and β with $Pr = 1.09$
 $\gamma = 0.5, \lambda = 0.2, Nt = 0.3, Nb = 0.5, Ec = 0.02, \alpha = 0.1, Sc = 0.22, \delta = 0.5, K = 0.2.$

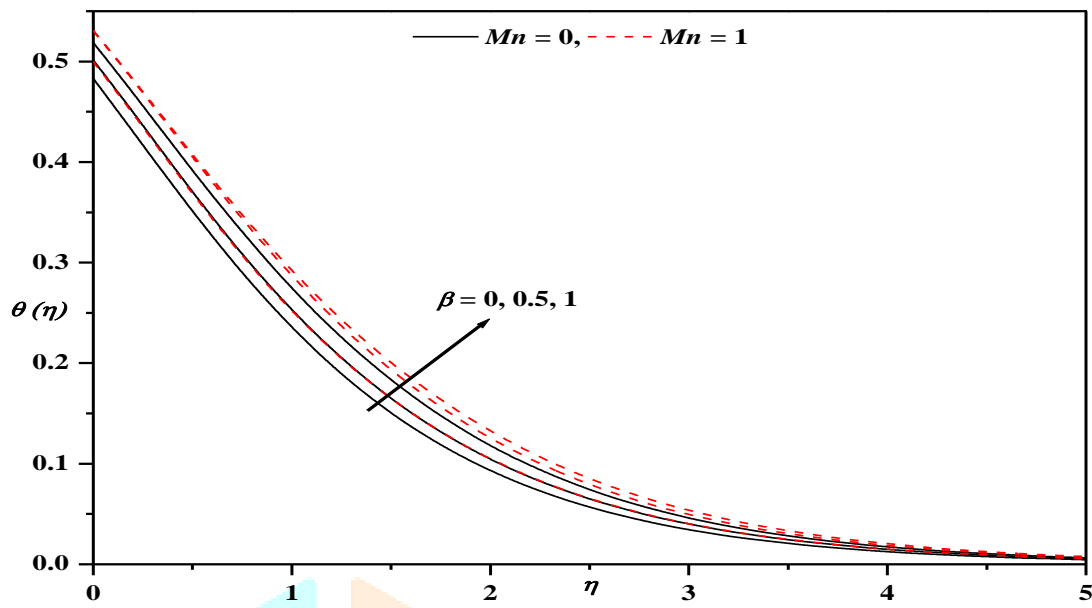


Fig.2(c):Temperature profile for different values of Mn and β with $Pr = 1.09$
 $Ec = 0.02, \gamma = 0.5, \lambda = 0.2, Nt = 0.3, Nb = 0.5, \alpha = 0.1, Sc = 0.22, \delta = 0.5, K = 0.2$

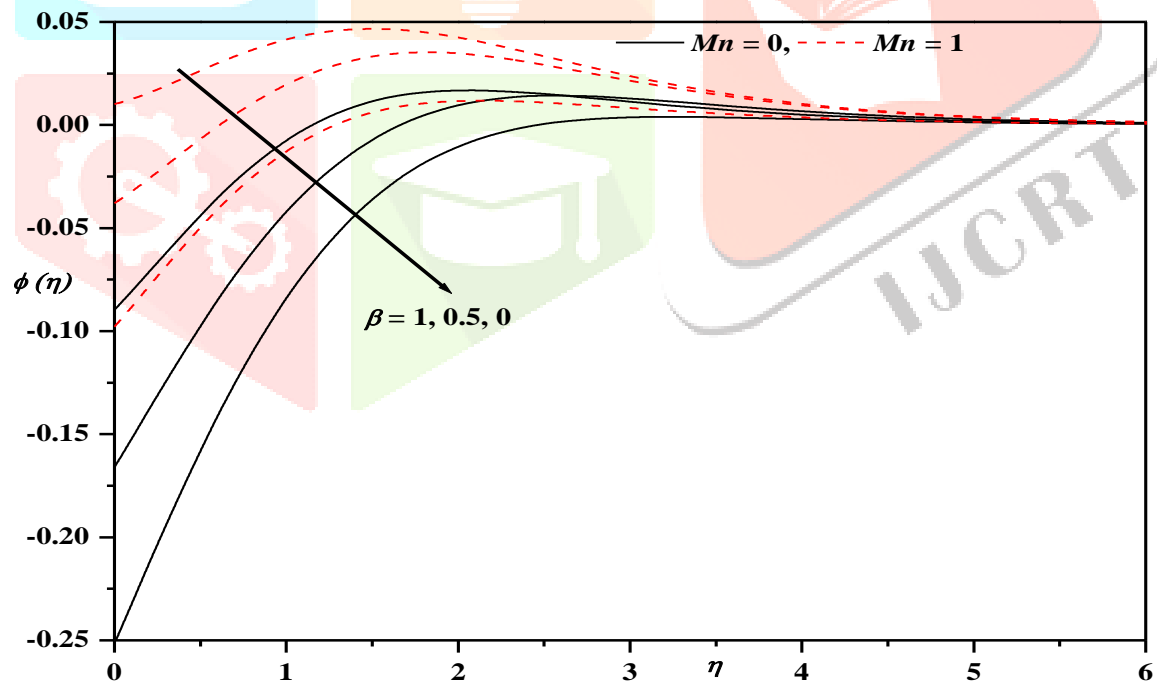


Fig.2(d):Concentration profile for different values of Mn and β with $Pr = 1.09$,
 $\gamma = 0.5, \lambda = 0.2, Nt = 0.3, Nb = 0.5, Ec = 0.02, \alpha = 0.1, Sc = 0.22, \delta = 0.5, K = 0.2$.

5. Conclusion:

The following are the some of the important findings:

- ❖ For higher values of magnetic parameter and Deborah number, horizontal velocity field is declined.
- ❖ The effect of Thermophoresis and Brownian motion parameter enhances the thermal conductivity.

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