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# Interval Valued Pythagorean Fuzzy Continuous Functions

[1]J Nivetha, [2]Dr K Mohana

[1]PG Scholar, [2]Assistant Professor
[1]Department of Mathematics,
[1]Nirmala College For Women, Coimbatore, India

*Abstract:* In the present paper, we introduce the concepts of interval valued Pythagorean fuzzy almost continuous mapping and interval valued Pythagorean fuzzy weakly continuous mapping in interval valued Pythagorean fuzzy topological space and we study some of their properties. We will also introduce and investigate interval valued Pythagorean fuzzy $\alpha$- continuous function between interval valued Pythagorean fuzzy topological spaces and establish their corresponding characterizations.

## I. INTRODUCTION

As a generalization of fuzzy set theory was first introduced by Zadeh[14] . Further Chang [3] introduced the fuzzy topological spaces with few results as continuity, closed and open sets. These spaces and their generalizations are later studied by several authors, one of which, developed by Sostak [12,8], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay [4] and by Ramadan [10].

In 1983, Atanassov introduced the concept of Intuitionistic fuzzy set with elements comprising membership and non-membership degree [1,11]. Using this type of generalized fuzzy set, Coker [5,9] introduced the concept of Intuitionistic fuzzy topological spaces and studied some notions such as continuity and compactness. The different notion and definition for fuzzy topological spaces were given by Lowen [6].

In 1996, Coker and Demirci [2] introduced the basic definitions and properties of Intuitionistic fuzzy topological spaces in Sostak's sense, which is a generalized form of fuzzy topological spaces developed by Sostak [12,8]. The concepts as fuzzy alpha open and closed sets and open map and continuous functions was developed by Rajvanshi and Singal. Yager [13] presented the concept of pythagorean fuzzy subset which is a typical fuzzy set. After the introduction of pythagorean fuzzysets, it was widely used in the field of decision making and was applied for the real life applications. Olgun [7] introduced pythagorean fuzzy topology along with the definition of continuity in pythagorean fuzzy topological spaces and their characterizations. Thus it was further developed with connected, compactness of pythagorean topological spaces.

In this study, we introduce the following concepts: interval valued Pythagorean fuzzy almost continuous mapping, interval valued Pythagorean fuzzy weakly continuous mapping and interval valued Pythagorean alpha open sets and closed sets and interval valued Pythagorean alpha continuity in interval valued Pythagorean fuzzy topological space.

## II. PRELIMINARIES

**Definition 2.1**

Let X be a nonempty fixed set and I the closed unit interval [0,1]. An Intuitionistic fuzzy set (IFS) A is an object having the form

$$A = \{< x, \mu_A(x), \nu_A(x)>: x \epsilon X\},$$

Where the mapping $\mu_A$ :X→ I and $\nu_A$ :X→ I denote the degree of membership and the degree of nonmembership of each element $x \epsilon$X to the set A and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \epsilon$X. The complement of the IFS A is $\bar{A} = \{< x , \nu_A(x), \mu_A(x) >: x \epsilon X\}$. obviously, every fuzzy set A on nonempty set X is an IFS having the form

$$A = \{< x, \mu_A(x), 1- \mu_A (x)>: x \epsilon X\},$$

For a given nonempty set X, denote the family of all IFSs in X by the symbol $\zeta^X$.

**Definition 2.2**

An intuitionistic fuzzy topology (IFT) in Chang's sense on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:

$(T_1)$ $0_{\sim}, 1_{\sim} \epsilon\, \tau$ , where $0_{\sim}$ = {< $x$,0,1>: $x$ ϵ X} and $1_{\sim}$ = {< $x$,1,0>: $x$ ϵ X};

$(T_2)$ $G_1 \cap G_2$ ϵ τ for any $G_1$ , $G_2$ ϵ τ;

$(T_3)$ $\cup\, G_i$ ϵ τ for any arbitrary family $\{G_i$: i ϵ J$\}$ ⊆ τ.

In this case, the pair (X,τ) is called Chang intuitionistic fuzzy topological space and each IFS in τ is known as intuitionistic fuzzy open set in X.

**Definition2.3**

An IFT in sostak's sense on a nonempty set X is an IFF τ on X satisfying the following axioms:

$(T_1)$ τ $(0_{\sim})$ = τ $(1_{\sim})$ = 1~;

$(T_2)$ τ (A ∩ B) ≥ τ (A) ^ τ (B) for any A B ϵ $\zeta^X$ ;

$(T_3)$ τ (∪ $A_i$) ≥ τ $(A_i)$ for any $\{A_i$ : iϵJ$\}$⊆ $\zeta^X$.

In this case, the pair (X,τ) is called an Intuitionistic fuzzy topological spaces in Sostak's sense (IFTS). For any A ϵ $\zeta^X$, the number $\mu_\tau$ (A) is called the openness degree of A, while $\nu_\tau$ (A) is called the nonopenness degree of A.

**Definition 2.4**

Let (X,τ) be an IFTS on X. Then the IFF $\tau^*$ is defined by $\tau^*$(A) = τ $(\bar{A})$. The number $\mu_{\tau^*}$(A) = $\mu_\tau(\bar{A})$ is called the closedness degree of A, while $\nu_{\tau^*}$(A) = $\nu_\tau(\bar{A})$ is called the nonclosedness degree of A.

**Definition 2.5**

Let (X,τ) be an IFTS and A be an IFS in X. Then the fuzzy closure and fuzzy interior of A are defined by

$cl_{\alpha,\beta}$ (A) = ∩ {K ϵ $\zeta^X$: A ⊆ K, $\tau^*$(K) ≥ <α,β>},

$int_{\alpha,\beta}$(A) =∪ {G ϵ $\zeta^X$: G ⊆ A, τ (G) ≥ <α,β>},

Where α ϵ $I_0$ = (0,1], β ϵ $I_1$= [0,1) with α + β ≤ 1.

**Definition 2.6**

Let (X,$\tau_1$) and (Y,$\tau_2$) be two IFTSs and f: X → Y be a mapping. then f is said to be

(i) intuitionistic fuzzy continuous if and only if $\tau_1(f^{-1}$(B)) ≥ $\tau_2$(B), for each B ϵ $\zeta^Y$;

(ii) intuitionistic fuzzy open if and only if $\tau_2$(f (A)) ≥ $\tau_1$(A)for each A ϵ $\zeta^X$.

## III. INTERVAL VALUED PYTHAGOREAN FUZZY ALMOST CONTINUOUS AND INTERVAL VALUED PYTHAGOREAN FUZZY WEAKLY CONTINUOUS MAPPING :

**Definition 3.1.**

Let A be an IVPFTS (X,τ). For α ϵ $I_0$ , βϵ$I_1$ with $[\alpha^b]^2$ +$[\beta^b]^2$ ≤ 1,

Where α = $[\alpha^a,\alpha^b]$ β = $[\beta^a, \beta^b]$,

A is called,

i) interval valued Pythagorean fuzzy regular open (IVPFRO) set of X if $int_{\alpha,\beta}(cl_{\alpha,\beta}$ A)= A

ii) interval valued Pythagorean fuzzy regular closed (IVPFRC) set of X if $cl_{\alpha,\beta}(int_{\alpha,\beta}$ A)= A

**THEOREM 3.2:**

Let A be an IVPFS in an IVPTS (X,τ).Then, for $\alpha^2$ ϵ $I_0$ , $\beta^2$ϵ$I_1$ with $[\alpha^b]^2$ +$[\beta^b]^2$ ≤ 1,

i)If A is IVPFRO (resp., IVPFRC),set then τ(A)≥ <α,β> (resp.,$\tau^*$ (A) ≥ <α,β>)

ii)A is IVPFRO set if and only if $\bar{A}$ is IVPFRC set.

**Proof:**

We will prove (ii) only

A is IVPFRO ⇔ $int_{\alpha,\beta}(cl_{\alpha,\beta}$ A)= A

⇔ $cl_{\alpha,\beta}(int_{\alpha,\beta}\bar{A}$ )= $\bar{A}$

⇔ $\bar{A}$ is IVPFRC

**Theorem 3.3**

Let (X,τ) be an IVPFTS. Then,

i)the union of two is IVPFRC sets is IVPFRC set.

ii)the intersection of two IVPFRO sets is IVPFRO set.

**Proof:**

i)Let A,B be any two IVPFRC sets. By theorem 3.2, we have $\tau^*(A) \geq$ <α,β>, $\tau^*(B) \geq$ <α,β> then $\tau^*(A \cup B) \geq \tau^*(A) \wedge \tau^*(B) \geq$ <α,β>, but $int_{\alpha,\beta}(A \cup B) \subseteq A \cup B$, this implies that $cl_{\alpha,\beta}(int_{\alpha,\beta}(A \cup B)) \subseteq cl_{\alpha,\beta}(A \cup B) = A \cup B$. now, $A = cl_{\alpha,\beta}(int_{\alpha,\beta}(A)) \subseteq cl_{\alpha,\beta}(int_{\alpha,\beta}(A \cup B))$ and $B = cl_{\alpha,\beta}(int_{\alpha,\beta}B)) \subseteq cl_{\alpha,\beta}(int_{\alpha,\beta}(A \cup B))$. Then $A \cup B \subseteq cl_{\alpha,\beta}(int_{\alpha,\beta}(A \cup B))$ .So, $cl_{\alpha,\beta}(int_{\alpha,\beta}(A \cup B)) = (A \cup B))$ and hence A∪B is IVPFRC set.

ii)It can be proved by the same manner.

**THEOREM 3.4:**

Let (X,τ) be an IVPFTS. Then,

i) If $A \in \zeta^X$ such that $\tau^*(A) \geq$ <α,β> then $int_{\alpha,\beta}(A)$ is (α,β)- IVPFRO set,

ii) If $B \in \zeta^X$ such that $\tau^*(B) \geq$ <α,β> then $cl_{\alpha,\beta}(A)$ is IVPFRC set.

**Proof:**

i) Let $A \in \zeta^X$ such that $\tau^*(A) \geq$ <α,β> clearly,

$int_{\alpha,\beta}(A) \subseteq int_{\alpha,\beta}(cl_{\alpha,\beta}(A))$ ;

This implies that

$int_{\alpha,\beta}(A) \subseteq int_{\alpha,\beta}(cl_{\alpha,\beta}(int_{\alpha,\beta}(A)))$

Now, since

$\tau^*(A) \geq$ <α,β>, then $cl_{\alpha,\beta}(int_{\alpha,\beta}(A)) \subseteq A$;

This implies that

$int_{\alpha,\beta}(cl_{\alpha,\beta}(int_{\alpha,\beta}(A))) \subseteq int_{\alpha,\beta}(A)$

Thus, $int_{\alpha,\beta}(cl_{\alpha,\beta}(int_{\alpha,\beta}(A))) = int_{\alpha,\beta}(A)$

Hence , $int_{\alpha,\beta}(A)$ is IVPFRO set.

ii) It can be proved by the same manner.

**DEFINITION 3.5:**

A mapping f: $(X,\tau_1) \to (Y,\tau_2)$ from an IVPFTS $(X,\tau_1)$ to another IVPFTS $(Y,\tau_2)$ is called

(i) Interval valued Pythagorean fuzzy strong continuous if and only if $\tau_1(f^{-1}(A)) = \tau_2(A)$, for each $A \in \zeta^Y$,

(ii) Interval valued Pythagorean fuzzy almost continuous if and only if $\tau_1(f^{-1}(A)) \geq$ <α,β>, for each (α,β)-IVPFRO set A of Y,

(iii) (α,β)- Interval valued Pythagorean fuzzy weekly continuous if and only if $\tau_2(A) \geq$ <α,β> implies $\tau_1(f^{-1}(A)) \geq$ <α,β>, for each $A \in \zeta^Y$.

**REMARK 3.6:**

From the above definition, it is clear that the following implications are true for $\alpha \in I_0$, $\beta \in I_1$ with $[\alpha^b]^2 + [\beta^b]^2 \leq 1$

Interval valued Pythagorean fuzzy almost continuous mapping

↑

Interval valued Pythagorean fuzzy continuous mapping

↓

Interval valued Pythagorean fuzzy weakly continuous mapping

But, reciprocal implications are not true in general.

**Theorem 3.7:**

Let f: $(X,\tau_1) \to (Y,\tau_2)$ be a mapping from an IVPFTS $(X,\tau_1)$ to another IVPFTS $(Y,\tau_2)$. Then the following statements are equivalent.

(i) f is (α,β)- Interval valued Pythagorean fuzzy almost continuous;

(ii) ${\tau^*}_1(f^{-1}(B)) \geq$ <α,β>, for each IVPFRC set B of Y;

(iii) $f^{-1}(B) \subseteq int_{\alpha,\beta} f^{-1}(int_{\alpha,\beta}(cl_{\alpha,\beta}(B)))$ for each $B \in \zeta^Y$ such that $\tau_2(B) \geq$ <α,β>;

(iv) $cl_{\alpha,\beta} f^{-1}(cl_{\alpha,\beta}(int_{\alpha,\beta}(B))) \subseteq f^{-1}(B)$ ,for each $B \in \zeta^Y$ such that $\tau_2(B) \geq$ <α,β> where $\alpha \in I_0$, $\beta \in I_1$ with $[\alpha^b]^2 + [\beta^b]^2 \leq 1$

**PROOF:**

(i)→ $(ii)$

Let B be IVPFRC set of Y.

Then by theorem 3.2, $\bar{B}$ is IVPFRO set.

By (i), we have ,$\tau_1(f^{-1}(\bar{B})) = \tau_1 \overline{f^{-1}(B)} = {\tau^*}_1(f^{-1}(B)) \geq$ <α,β>

(ii) → $(i)$

It is analogous to the proof of (ii) → $(i)$

(i)→ $(iii)$

Since that $\tau_2(B) \geq$ <α,β>, then $B = int_{\alpha,\beta}(B) \subseteq int_{\alpha,\beta}(cl_{\alpha,\beta}(B))$ and hence,

$f^{-1}(B) \subseteq f^{-1}(int_{\alpha,\beta}(cl_{\alpha,\beta}(B)))$.

Since, ${\tau^*}_2(cl_{\alpha,\beta}(B)) \geq$ <α,β> then by theorem 3.4

$int_{\alpha,\beta}(cl_{\alpha,\beta}(B))$ is (α,β)-IVPFRO set.

So, $\tau_1(f^{-1}(int_{\alpha,\beta}(cl_{\alpha,\beta}(B)))) \geq$ <α,β>. Then,

$f^{-1}$(B) ⊆ $f^{-1}$( $int_{\alpha,\beta}(cl_{\alpha,\beta}$ (B))) = $int_{\alpha,\beta}$ ($f^{-1}(int_{\alpha,\beta}(cl_{\alpha,\beta}$ (B)))).

(iii)→ $(i)$

Let B be is (α,β)-IVPFRO set of Y. Then, we have $f^{-1}$(B) ⊆ $int_{\alpha,\beta}$ ($f^{-1}(int_{\alpha,\beta}(cl_{\alpha,\beta}$ (B))))
= $int_{\alpha,\beta}(f^{-1}$(B));

This implies that $f^{-1}$(B) = $int_{\alpha,\beta}(f^{-1}$(B)), then

$\tau_1(f^{-1}$(B)) = $\tau_1$( $int_{\alpha,\beta}(f^{-1}$(B))) ≥ <α,β>

Hence, f is Interval valued Pythagorean fuzzy almost continuous.

(iii) →(iv) can similarly be proved.

**Theorem 3.8:**

Let f: (X,$\tau_1$)→ (Y,$\tau_2$) be a mapping from an IVPFTS (X,$\tau_1$) to another IVPFTS (Y,$\tau_2$). Then the following statements are equivalent.

i) f is Interval valued Pythagorean fuzzy weakly continuous;

ii) f($cl_{\alpha,\beta}$ (A)) ⊆ $cl_{\alpha,\beta}$ (f(A)) for each A ϵ $\zeta^X$.

**Proof:** (i)→(ii)

Let A ϵ $\zeta^X$. Then,

$f^{-1}$( $cl_{\alpha,\beta}$ (f(A))) = $f^{-1}$ [∩{K ϵ $\zeta^Y$: $\tau^*{}_2$ (K) ≥ <α,β>, K ⊇ f(A)}]

= $f^{-1}$ $\overline{[\cap\{K \epsilon \zeta^Y: \tau_2 (\bar{K}) \geq <\alpha,\beta>, K \supseteq f(A)\}]}$

⊇ $f^{-1}$ [∩{K ϵ $\zeta^Y$: $\tau^*{}_1$ ($f^{-1}$ (K)) =$\tau_1$( $\overline{f^{-1}\text{ (K) }}$ ) ≥ <α,β>, K ⊇ f(A)}]

⊇ ∩ {$f^{-1}$ (K):K ϵ $\zeta^Y$: $\tau^*{}_1$ ($f^{-1}$ (K)) ≥ <α,β>, $f^{-1}$(K) ⊇ A}

⊇ ∩ {$G$ ϵ $\zeta^X$: $\tau^*{}_1$ (G) ≥ <α,β>, G ⊇ A} =$cl_{\alpha,\beta}$ (A).

Then, f($cl_{\alpha,\beta}$ (A)) ⊆ $f(f^{-1}$( $cl_{\alpha,\beta}$ (f(A)))) ⊆ $cl_{\alpha,\beta}$ (f(A))

(ii)→(i)

Let B ϵ $\zeta^Y$ such that $\tau_2$ (B) ≥ <α,β>. Then $\tau_2^*$ ($\bar{B}$) = $\tau_2$ (B) ≥ <α,β>. So, we have $cl_{\alpha,\beta}$ ($\bar{B}$ ) = $\bar{B}$ .

Further, since f ($cl_{\alpha,\beta}(f^{-1}(\bar{B}$ ))) ⊆ $cl_{\alpha,\beta}$ (f($f^{-1}(\bar{B}$ ))) ⊆ $cl_{\alpha,\beta}$ ($\bar{B}$ ) = $\bar{B}$ ,

We have $cl_{\alpha,\beta}(f^{-1}(\bar{B}$ )) ⊆ $f^{-1}(\bar{B}$ ). then , $cl_{\alpha,\beta}(f^{-1}(\bar{B}$ )) = $f^{-1}(\bar{B}$ ).

This implies that $\tau_1^*(f^{-1}(\bar{B}$ ) ≥ <α,β>.

Therefore $\tau_1^*(\overline{f^{-1}(B)}) = \tau_1 f^{-1}(B)) \geq <\alpha,\beta>$ .Hence f is Interval valued Pythagorean fuzzy weakly continuous

**Theorem 3.9:**

Let f : X → Y be an Interval valued Pythagorean fuzzy continuous mapping with respect to the IVPFTS $\tau_1$ and $\tau_2$ respectively. Then for every IVPFS A in X, f($cl_{\alpha,\beta}$ (A)) ⊆ $cl_{\alpha,\beta}$ (f(A))

where α ϵ $I_0$ , βϵ $I_1$ with $[\alpha^b]^2$ +$[\beta^b]^2$ ≤ 1.

**Proof :**

Let f : X → Y be an Interval valued Pythagorean fuzzy continuous mapping with respect to $\tau_1$ and $\tau_2$ , and Let A ϵ $\zeta^X$. Then,

$f^{-1}$( $cl_{\alpha,\beta}$ (f(A))) = $f^{-1}$ [∩{K ϵ $\zeta^Y$: $\tau^*{}_2$ (K) ≥ <α,β>, f(A) ⊆ $K$ }]

= ∩ {$f^{-1}$ (K):K ϵ $\zeta^Y$: $\tau_2^*$ (K) ≥ <α,β>,A ⊆ $f^{-1}$(K) }

⊇ ∩ {$f^{-1}$ (K):K ϵ $\zeta^Y$: $\tau_1^*$ ($f^{-1}$(K) ≥ <α,β>,A ⊆ $f^{-1}$(K) }

⊇ ∩ {$G$ ϵ $\zeta^X$: $\tau_1^*$ (G) ≥ <α,β>, A ⊆ G} =$cl_{\alpha,\beta}$ (A).

This implies that f($cl_{\alpha,\beta}$ (A)) ⊆ $cl_{\alpha,\beta}$ (f(A)).

**Theorem 3.10:**

Let f : X → Y be an Interval valued Pythagorean fuzzy continuous mapping with respect to the IVPFTS $\tau_1$ and $\tau_2$ respectively. Then for every IVPFS A in Y, $cl_{\alpha,\beta}$ $f^{-1}$((A)) ⊆ $f^{-1}(cl_{\alpha,\beta}$ (A))

where α ϵ $I_0$ , βϵ $I_1$ with $[\alpha^b]^2$ +$[\beta^b]^2$ ≤ 1.

**Proof :**

Let A ϵ $\zeta^Y$ we get theorem 3.9

$cl_{\alpha,\beta}$ $f^{-1}$((A)) ⊆ $f^{-1}$ ( f($cl_{\alpha,\beta}$ ($f^{-1}$((A)))) ⊆ $f^{-1}(cl_{\alpha,\beta}$ (A)) ….(2.18)

Hence $cl_{\alpha,\beta}$ $f^{-1}$((A)) ⊆ $f^{-1}(cl_{\alpha,\beta}$ (A)) for every A ϵ $\zeta^Y$.

**Definition 3.11:**

A IVPFS P =$< x, \mu_{P,} \gamma_P >$ of a IVPFTS $(X, \tau )$ is called a interval valued Pythagorean fuzzy $\alpha$ open set if P ⊆ int (cl(int(P))). A IVPFS whose complement is a interval valued Pythagorean fuzzy $\alpha$ open set (IVPFαOS) is called a interval valued Pythagorean fuzzy $\alpha$ closed set (IVPFαCS).

**Proposition 3.12:**

Let $(X, \tau )$ be a IVPFS. Then arbitrary union of IVPFαOS is a IVPFαOS and arbitrary intersection of IVPFαC sets is an IVPFαCS.

**Proof:**

Let $\{P_i = < x, \mu_{P,} \gamma_P >/ i\epsilon\, I\}$ be a family of IVPFαO sets. Then for each $i\epsilon\, I$,

$P_i$ ⊆ int (cl(int($P_i$))). Thus

∪ $P_i$ ⊆ ∪ int (cl(int($P_i$)))

⊆ int (∪ cl(int($P_i$)))

= int ( cl(int ($\cup$ int($P_i$)))
$\subseteq$ int ( cl(int($\cup$ $P_i$)))

Hence $\cup$ $P_i$ is a IVPFαO set. If we take complement of this part, the consecutive will proved (ie. arbitrary intersection of IVPFαC is also a IVPFαC).

Every IVPFOS is a interval valued Pythagorean fuzzy $\alpha$ open set and every IVPFCS is a IVPFαC but the converse is not true.

**Definition 3.13:**

The interval valued Pythagorean fuzzy $\alpha$ closure of a IVPFS P in a IVPFS $(X,\tau)$ represented as $cl_\alpha$(P) and defined by $cl_\alpha$(P) = $\cap\{C_i/C_i$ is IVPFαC set and P $\subseteq C_i\}$.

**Proposition 3.14:**

In a IVPFTS $(X,\tau)$ a IVPFS P is IVPFαC if and only if P = $cl_\alpha$(P).

**Proof:**

Assume that P is a IVPFαC set. Then P $\epsilon$ $\{C_i/C_i$ is IVPFαC set and P $\subseteq C_i\}$

So P =$\cap\{C_i/C_i$ is IVPFαC set and P $\subseteq C_i\}$

= $cl_\alpha$(P)

Conversely, consider P = $cl_\alpha$(P),

P $\epsilon$ $\{C_i/C_i$ is IVPFαC set and P $\subseteq C_i\}$

Thus P is interval valued Pythagorean fuzzy $\alpha$- closed set.

**Proposition 3.15:**

In a IVPFTS $(X,\tau)$ the following hold for interval valued Pythagorean fuzzy $\alpha$- closure.

(1) $cl_\alpha(\underset{-}{0}) = \underset{-}{0}$
(2) $cl_\alpha$(P) is a IVPFαC in $(X,\tau)$ for every IVPFS P in X.
(3) $cl_\alpha$(P) $\subseteq$ $cl_\alpha$(R) whenever P $\subseteq$ R for every P and R in X.
(4) $cl_\alpha$( $cl_\alpha$(P)) = $cl_\alpha$(P) for every IVPFS P in X.

**Proof :**

(1) The proof is obvious.
(2) By preposition, P is IVPFαC if and only if P = $cl_\alpha$(P) we get $cl_\alpha$(P) is a IVPFαC for every P in X.
(3) By same preposition, we get P = $cl_\alpha$(P) and R = $cl_\alpha$(R). whenever P $\subseteq$ R, we have $cl_\alpha$(P) $\subseteq$ $cl_\alpha$(R).
(4) Let P be a IVPFS in X. we know that P = $cl_\alpha$(P)
$cl_\alpha$(P) = $cl_\alpha$( $cl_\alpha$(P))

Thus $cl_\alpha$( $cl_\alpha$(P)) = $cl_\alpha$(P) for every P in X.

**Definition 3.16:**

Let $(X,\tau_X)$ and $(Y,\tau_Y)$ be IVPFTS. A mapping f : X → Y is named as interval valued Pythagorean fuzzy $\alpha$- continuous (IVPFαCN) if the inverse image of each IVPFOS of Y is a IVPFαO set in X.

**Theorem 3.17:**

Let f : $(X,\tau_X) \to (Y,\tau_Y)$ be a mapping from a IVPFTS $(X,\tau_X)$ to a IVPFTS $(Y,\tau_Y)$. If f is interval valued Pythagorean fuzzy $\alpha$-continuous, then

(1) F $(cl(int(cl$(P)))) $\subseteq cl$(f (P)) for all IVPFS P in X.
(2) $cl(int(f^{-1}$(B))) $\subseteq f^{-1}cl$(B)) for all B in Y.

**Proof :**

Assume that f is a IVPFαCN mapping. Let P be a IVPFS in X. then

$cl$(f (P)) is a IVPFCS in Y and this $f^{-1}(cl$(f (P))) is a IVPFαC set in X. thus

$cl(int(cl$(P))) = $cl(int(cl(cl$(P))))

$\subseteq cl(int(cl(f^{-1}(cl$(f(P))))))

$\subseteq f^{-1}$ $(cl$(f (P)))

So f $(cl(int(cl$(P)))) $\subseteq cl$(f (P)).

Now let B be a IVPFS in Y.

Then $f^{-1}$(B) an IVPFS in X. by (i),

f $(cl(int(cl(f^{-1}$(B))))) $\subseteq cl$(f $(f^{-1}$(B))).

$\subseteq cl$(B)

Thus $cl(int(f^{-1}$(B))) $\subseteq f^{-1}cl$(B)).

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