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Some Functions Via Interval Valued Pythagorean Fuzzy Topological Spaces

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Abstract: In this paper, we introduce some operations on interval valued Pythagorean fuzzy topological spaces. Also, the notions of interval valued Pythagorean fuzzy open(closed) functions and interval valued Pythagorean fuzzy homeomorphism and their basic properties are investigated.

I. INTRODUCTION

The fuzzy set theory was first introduced by Zadeh [13] in 1965. Fuzzy set theory was characterized by a membership function which assigns to each target a membership value between 0 to 1. Chang [4] introduced the fuzzy topological space and some basic notions of topology such as open set, closed set and continuity. Further Lowen [9,10] also made different studies on fuzzy topological spaces. Atanassov [1] introduced intuitionistic fuzzy set. Coker [5] introduced the concept of intuitionistic fuzzy topological spaces and studied some nations such as continuity and compactness. The different studies were carried out on intuitionistic fuzzy topological spaces [7,8] Yager [12] developed Pythagorean fuzzy set in, characterized by a membership degree and non- membership degree which satisfies the condition that the square sum of its membership and non- membership degree is less than or equal to 1. Atanassov and gergov[3] presented the idea of the interval valued intuitionistic fuzzy set and also introduced the notion of the interval valued intuitionistic fuzzy set

In this study, we investigated some basics notions of interval valued Pythagorean fuzzy topological spaces such as internal valued Pythagorean fuzzy interior, internal valued Pythagorean fuzzy closure, internal valued Pythagorean fuzzy boundary and internal valued Pythagorean fuzzy basic. Finally we also defined internal valued Pythagorean fuzzy open (closed) function and interval valued pythagorean fuzzy homeomorphism.

II. PRELIMINARIES

Definition 2.1 [13]

Let X be an universe. A fuzzy set A in X,A={x,$\mu_A(x)$): $x\epsilon X$}, where μ_A:X→[0,1] is the membership function of the fuzzy set A;$\mu_A(x)\epsilon[0,1]$ is the membership of $x\epsilon X$ in A.

Definition 2.2 [1]

Let X be a non empty fixed set. An Intuitionistic fuzzy set A in X is an object having the form A={<x,μ_A(x),ν_A(x)>: $x\epsilon X$} where the functions μ_A(x):X→[0,1] and ν_A(x) :X→[0,1] denote the degree of non membership of each element $x\epsilon X$ to the set A,and $0\leq \mu_A(x)+\nu_A(x) \leq 1$ for each $x\epsilon X$. The degree of indeterminacy $I_A=\sqrt{1-\mu_A(x)-\nu_A(x)}$

Definition 2.3 [12]

Let X be a universe of discourse.A Pythagorean fuzzy set P in X given by P={<x,$\mu_p(x)$,ν_p(x)>:$x\epsilon X$} where the functions $\mu_p(x)$:X→[0,1] denotes the degree of non-membership of the element $x\epsilon X$ to the set P,with the condition that 0≤ $(\mu_p(x))^2 +$ $(\nu_p(x))^2 \leq 1$. The degree of indeterminacy I_p=$\sqrt{1-(\mu_p(x))^2+(\nu_p(x))^2}$

Definition 2.4 [2]

An interval-valued fuzzy set A(over the basic set E) is specified by a function $X_A: E \rightarrow$ INT(X[0,1]) is the set of all intervals within [0,1], i.e for all $x\epsilon X$, X_A(x) is an interval [a,b], 0≤ a ≤ b ≤ 1

Definition 2.5[12]

Let P_1 ={<x,$\mu_{p_1}(x)$,$\nu_{p_1}(x)$>$x\epsilon X$} and P_2={<x,$\mu_{p_2}(x)$,$\nu_{p_2}(x)$>$x\epsilon X$} be two Pythagorean fuzzy sets over X. Then,

a) The Pythagorean fuzzy complement of P_1 is defined by

P_1^c={<x,$\nu_{p_1}(x)$,$\mu_{p_1}(x)$>:$x\epsilon X$}

b) The Pythagorean fuzzy intersection of P_1 and P_2 is defined by

$P_1 \cap P_2$={<x,min{$\mu_{p_1}(x)$,$\mu_{p_2}(x)$},max{$\nu_{p_1}(x)$,$\nu_{p_2}(x)$}>:$x\epsilon X$}

c) The Pythagorean fuzzy union of P_1 and P_2 is defined by

$P_1 \cup P_2$={<x,max{$\mu_{p_1}(x)$,$\mu_{p_2}(x)$},min{$\nu_{p_1}(x)$,$\nu_{p_2}(x)$}>:$x\epsilon X$}

d) We say P_1 is a Pythagorean fuzzy subset of P_2 and we write $P_1 \subseteq P_2$ if $\mu_{p_1}(x) \leq \mu_{p_2}(x)$ and $\nu_{p_1}(x) \geq \nu_{p_2}(x)$ for each $x\epsilon X$,

e) $0_X = \{< x, 0,1 >: x\epsilon X\}$ and 1_X={<x,0,1>: $x\epsilon X$}.

Definition 2.6 [11]

Let X≠ ∅ be a set and τ be a family of Pythagorean fuzzy subset of X. if

T1 0_X,$1_X \epsilon$ τ,

T2 for any P_1, P_2 ϵ τ, we have $P_1 \cap P_2 \epsilon$ τ,

T3 for any $\{P_i\}_{i\epsilon I} \subseteq$ τ,we have $\cup_{i\epsilon I} P_i \epsilon$ τ

Then τ is called a Pythagorean fuzzy topology on X and the pair (X,τ) is said to be a Pythagorean fuzzy topological space. Each member of τ is called a Pythagorean fuzzy open set .The complement of a Pythagorean fuzzy open set is called a Pythagorean fuzzy closed set.

Definition 2.7 [6]

Consider $(X,\tau)_P$ be a interval valued Pythagorean fuzzy topologically space and P={(x, $\mu_i(x), \nu_i(x)$): $x \in X$} be a IVPFS over X. then the interval valued Pythagorean fuzzy interior, IVPF closure of P are defined by;

a) int(P)=∪ {G: G is a IVPFOS in X and G ⊆ P}

b) cl(P)=⋂{K: K is IVPFCS in X and P ⊆ K}

III. Basic results

Definition 3.1

Let $\{P_i = \{(x, \mu_i(x), \nu_i(x)): x\epsilon X\}\}_{i\epsilon I}$ be a family of interval valued Pythagorean fuzzy sets over X

Where $\mu_i(x)$=[μ_i^a(x),μ_i^b(x)]

$\nu_i(x)$=[ν_i^a(x),ν_i^b(x)] Then,

a) $\cap_{i\epsilon I} P_i$={(x,inf{$\mu_i(x)$},sup{$\nu_i(x)$}):$x\epsilon X$}

b) $\cup_{i\epsilon I} P_i$={(x,sup{$\mu_i(x)$},inf{$\nu_i(x)$}):$x\epsilon X$}

Note that $\cap_{i\epsilon I} P_i$ and $\cup_{i\epsilon I} P_i$ are interval valued pythagorean fuzzy sets over X

We define $\cap_{i\epsilon I} P_i$={(x, α$\cap_{i\epsilon I} P_i$,β$\cap_{i\epsilon I} P_i$): x∈ X} such that α$\cap_{i\epsilon I} P_i$= inf {$\mu_i(x)$} and β$\cap_{i\epsilon I} P_i$=sup{$\nu_i(x)$} , $\cap_{i\epsilon I} P_i$ to be interval valued pythagorean fuzzy set we must have that $\alpha^2 \cap_{i\epsilon I} P_i + \beta^2 \cap_{i\epsilon I} P_i \leq 1$. we see since $\beta^2 \cap_{i\epsilon I} P_i$= sup{${\nu_i}^2$(x)}

Then $\beta^2 \cap_{i\epsilon I} P_i$= sup {${\nu_i}^2$(x)} =sup {${r_i}^2$-${\mu_i}^2$,${r_i}^2$-${\nu_i}^2$}

≤ sup {${r_i}^2$-inf {${\mu_i}^2$,${\nu_i}^2$}, ${r_i}^2$-inf {${\mu_i}^2$, ${\nu_i}^2$}}

$\beta^2 \cap_{i\epsilon I} P_i$ (x) ≤ sup{1-inf {${\mu_i}^2$,${\nu_i}^2$}, 1-inf {${\mu_i}^2$,${\nu_i}^2$}}

≤ 1-inf {${\mu_i}^2$,${\nu_i}^2$}

Where $\mu_i^2 + \nu_i^2 = r_i^2$ for every i∈ I. from this we see that

$\alpha^2 \cap_{i\epsilon I} P_i + \beta^2 \cap_{i\epsilon I} P_i \leq$ inf $\{\mu_i^2, \nu_i^2\}$+1 -inf $\{\mu_i^2, \nu_i^2\} \leq 1$. thus ,$\cap_{i\epsilon I} P_i$ is a interval valued pythagorean fuzzy set. The proof is trivial for $\cup_{i\epsilon I} P_i$

Theorem 3.2

Let $\{P_i = \{(x, \mu_i(x), \nu_i(x)): x \in X\}\}_{i\epsilon I}$ be a family of interval valued Pythagorean fuzzy sets over X, where $\mu_i(x)=[\mu_i^a(x), \mu_i^b(x)]$,

$\nu_i(x)=[\nu_i^a(x), \nu_i^b(x)]$

Then,(i) $\overline{\cap_{i\epsilon I} P_i} = \cup_{i\epsilon I} \overline{P_i}$

(ii) $\overline{\cup_{i\epsilon I} P_i} = \cap_{i\epsilon I} \overline{P_i}$

Proof

We have $\cap_{i\epsilon I} P_i = \{(x, \inf\{\mu_i(x)\}, \sup\{\nu_i(x)\}): x \in X\}$

Then $\overline{\cap_{i\epsilon I} P_i} = \{(x, \sup\{\nu_i(x)\}, \inf\{\mu_i(x)\}): x \in X\}$ and $\overline{P_i} = \{(x, \nu_i(x), \mu_i(x)): x \in X\}$ and so

$\cup_{i\epsilon I} \overline{P_i} = \{(x, \sup\{\nu_i(x)\}, \inf\{\mu_i(x)\}): x \in X\}$

i.e $\overline{\cap_{i\epsilon I} P_i} = \cup_{i\epsilon I} \overline{P_i}$

ii) It is proved similar to (i)

Definition 3.3

Consider $(X, \tau)_P$ be a interval valued Pythagorean fuzzy topologically space and P=$\{(x, \mu_i(x), \nu_i(x)): x \in X\}$ be a IVPFS over X. then the interval valued Pythagorean fuzzy boundary of P are defined by;

Fr(P)=cl(P)∩cl(P^c)

Example 3.4

Let x= {x_1, x_2, x_3}. τ= {1x, 0x, P_1, P_2, P_3, P_4} where.

P_1 = {(x_1 [0.5, 0.7],[0.7, 0.9], (x_2, [0.6, 0.8],[0.5, 0.7]),(x_3, [0.2, 0.4],[0.1, 0.3])}

P_2 = {(x_1, [0.6, 0.8],[0.8, 0.9]),(x_2, [0.1, 0.3],[0.4, 0.6]),(x_3, 0.1, 0.2],[0.8, 0.9])}

P_3 = {(x_1, [0.6, 0.8], [0.7, 0.9], (x_2, [0.6, 0.8], [0.4, 0.6]), (x_3, [0.2, 0.4], [0.1, 0.3])}

P_4 = {(x_1 [0.5, 0.7], [0.8, 0.9]), (x_2, [0.1, 0.3], [0.5, 0.7]), (x_3, [0.1, 0.2], [0.8, 0.9])}

It is clear that $(X,\tau)_p$ is a IV Pythagorean fuzzy topological space. Now assume that,

P = {(x_1, [[0.7, 0.9], [0.4, 0.6]), (x_2, [0.8, 0.9], , [0.2, 0.4]), (x_3, [0.3, 0.4], [0.1, 0.2])} is a IVPF subset over X.

Then int (P) = O_X UP_1 UP_2 UP_3 UP_4

= P_3

P_3 = {(x_1, [0.6, 0.8], [0.7, 0.9], (x_2, [0.6, 0.8], [0.4, 0.6]), (x_3, [0.2, 0.4], [0.1, 0.3])} on the other hand, in order to find the IVPF closure of P, it necessary to determine the interval valued Pythagorean fuzzy closed sets over X, then

P_1^C = {(x_1, [0.7, 0.9], [0.5, 0.7]), (x_2, [0.5, 0.7], [0.6, 0.8]), (x_3, [0.1, 0.3], [0.2, 0.4])}

P_2^C = {(x_1, [0.8, 0.9], [0.6, 0.8]), (x_2, [0.4, 0.6], [0.1, 0.3]), (x_3, [0.8, 0.9], [0.1, 0.2])}

P_3^C = {(x_1, [0.7, 0.9], [0.6, 0.8]), (x_2, [0.4, 0.6], [0.6, 0.8]), (x_3, [0.1, 0.3], [0.2, 0.4])}

P_4^C = {(x_1, [0.8, 0.9], [0.5, 0.7]), (x_2, [0.5, 0.7], [0.1, 0.3]), (x_3, [0.8, 0.9], [0.1, 0.2])}.

Hence, Cl (P) = 1_X

Similarly to find the interval valued Pythagorean fuzzy boundary of P,

P^C = {(x_1, [0.4, 0.6], [0.7, 0.9]), (x_2, [0.2, 0.4], [0.8, 0.9]), (x_3, [0.1, 0.2], [0.3, 0.4])}.

Cl (P^C) = 1_X ∩P_1^C ∩P_2^C ∩P_3^C ∩P_4^C

P_3^C = {(x_1, [0.7, 0.9], [0.6, 0.8]), (x_2, [0.4, 0.6], [0.6, 0.8]), (x_3, [0.1, 0.3], [0.2, 0.4])}

F_r (P) = Cl (P) ∩ Cl (P^C)

= 1_X ∩ P_3^C

= {(x_1, [0.7, 0.9], [0.6, 0.8]), (x_2, [0.4, 0.6], [0.6, 0.8]), (x_3, [0.1, 0.3], [0.2, 0.4])}.

Preposition 3.5

Consider $(X,\tau)_P$ be a IVPFTS and P,P_1, P_2 be IVPFS over X. then the following properties hold.

i) int(P)⊆P

ii) int(int(P))=int(P)

iii) $P_1 \subseteq P_2 \Rightarrow \text{int}(P_1) \subseteq \text{int}(P_2)$

iv) int($P_1 \cap P_2$)=int(P_1)∩int(P_2)

v) $\text{int}(1_x) = 1_x$, $\text{int}(0_x) = 0_x$

Proof

i,ii,iii) and v) can be easily obtained from the definition of interval valued Pythagorean fuzzy interior.

iv) From int($P_1 \cap P_2$) ⊆ int(P_1) and int($P_1 \cap P_2$) ⊆ int(P_2)

We obtain int($P_1 \cap P_2$) ⊆ int(P_1)∩ int(P_2) on the other hand from the facts int(P_1) ⊆ P_1 and int(P_2) ⊆ P_2 ⇒int(P_1) ∩ int(P_2) ⊆ $P_1 \cap P_2$ and int(P_1)∩ int(P_2)ϵτ. We have int(P_1)∩int($P_1 \cap P_2$)

Hence the proof.

Theorem 3.6

Let J:IVPFS(x)→IVPFS(x) be a mapping the family

τ={P ∈ IVPFS(x):J(P)=P} is a interval valued Pythagorean fuzzy topology over X,if the mapping J satisfies the following condition :

i) J(P) ⊆ P

ii) $J(1_x) = 1_x$

iii) J(J(P))=J(P)

iv) J($P_1 \cap P_2$)= J(P_1)∩J(P_2)

Also J(P)= int (P) for each IVPFS P in the IVPFTS.

Proof

i,ii,iii) can be easily obtained from the definition of the IVPF interior.

iv)From J($P_1 \cap P_2$)⊆ J(P_1) and J($P_1 \cap P_2$)⊆ J(P_2) .

we obtain J($P_1 \cap P_2$)⊆ J(P_1)∩J(P_2) and J(P_1)∩J(P_2) ∈ τ.

we have J(P_1)∩J(P_2) ⊆ J($P_1 \cap P_2$).

Hence the proof

Preposition 3.7

Consider $(X,\tau)_P$ be a IVPFTS and P,P_1, P_2 be IVPFS over X. then the following properties hold.

i) P⊆ cl(P)

ii) cl(cl(P) = cl(P)

iii) $P_1 \subseteq P_2 \Rightarrow \text{cl}(P_1) \subseteq \text{cl}(P_2)$

iv) cl($P_1 \cup P_2$)=cl(P_1)∪$cl(P_2)$

v) $\text{cl}(1_x) = 1_x$, $\text{cl}(0_x) = 0_x$

Proof

i,ii,iii) and v) can be easily obtained from the definition of the IVPF closure

iv) From cl(P_1) ⊆ cl($P_1 \cup P_2$) and cl(P_2) ⊆ cl($P_1 \cup P_2$) we obtain

cl(P_1)∪$cl(P_2)$ ⊆ cl($P_1 \cup P_2$) on the other hand from the fact P_1 ⊆cl(P_1) and P_2 ⊆cl(P_2) ⇒ $P_1 \cup P_2$ ⊆ cl(P_1)∪$cl(P_2)$ and cl(P_1)∪$cl(P_2)$ ∈ IVPFCS .we have cl($P_1 \cup P_2$)⊆cl(P_1)∪$cl(P_2)$.

Hence the proof

Theorem 3.8

Let C:IVPFS(x)→IVPFS(x) be a mapping the family τ={P∈ IVPFS(x):C(P^c)=P^c} is IVPF topology over X, if the mapping C satisfies the following conditions

i) P⊆ C(P)

ii) $C(0_x) = 0_x$

iii) C(C(P))=C(P)

iv) $C(P_1 \cup P_2)$= $C(P_1) \cup C(P_2)$

Also C(P)=cl(P) for each IVPF set P in this IVPFTS

Proof

i,ii and iii) can be easily obtained from the definition of the IVPF closure

iv)From $C(P_1) \subseteq C(P_1 \cup P_2)$ and $C(P_2) \subseteq C(P_1 \cup P_2)$ we obtain $C(P_1) \cup C(P_2) \subseteq C(P_1 \cup P_2)$ on the other hand from the facts $P_1 \subseteq C(P_1)$ and $P_2 \subseteq C(P_2) \Rightarrow P_1 \cup P_2 \subseteq C(P_1) \cup C(P_2)$ and $C(P_1) \cup C(P_2)$ ∈IVPFCS. We have $C(P_1 \cup P_2) \subseteq C(P_1) \cup C(P_2)$.

Hence the proof.

Theorem 3.9

Let $(X, \tau)_P$ be a IVPFTS and P be IVPFS over X. then

a) cl(P^c)= $(int(p))^c$

b) int(P^c)= $(cl(p))^c$

Proof

a) Let P={(x,μ(x), ν(x)):x ∈ X}

Where μ(x)=[μ^a(x),μ^b(x)] , ν(x)=[ν^a(x),ν^b(x)] and assume that the family of IVPFS contained in P are indexed by the family $\{P_i = \{(x, \mu_i(x), \nu_i(x)): x \epsilon X\}\}_{i \epsilon I}$ Where $\mu_i(x)$=[μ_i^a(x),μ_i^b(x)]

$\nu_i(x)$=[ν_i^a(x),ν_i^b(x)]

Then we see that int(P)={(x,sup{ $\mu_i(x)$},inf{ $\nu_i(x)$}:x ∈ X} and hence $(int(p))^c$={(x,inf{ $\nu_i(x)$},sup{ $\mu_i(x)$}): x ∈ X}.since P^c={(x, $\nu_i(x), \mu_i(x)$): $x \in X$} and $\mu_i(x) \leq$ μ(x) ,$\nu_i(x) \leq$ ν(x) for each i∈I, we obtain that $\{P_i = \{(x, \mu_i(x), \nu_i(x)): x \in X\}\}_{i \epsilon I}$is the family of IVPFS containing P^c, i.e cl(P^c)={(x,inf{ $\nu_i(x)$},sup{ $\mu_i(x)$}): x ∈ X}

Therefore cl(P^c)= $(int(p))^c$ immediately.

b) This analogous to (a)

Preposition 3.10

Consider $(X, \tau_1)_P$ and $(Y, \tau_2)_P$ be two IVPFTS and f:X→Y be a IVPF function, then the following are equivalent to each other.

a) f is a Interval valued Pythagorean fuzzy continuous function

b) f(cl(P)) ⊆ cl(f [P]) for each IVPFS P in X

c) cl($f^{-1}[k]$) ⊆ f^{-1}[cl(k)] for each IVPFS K in Y

d) f^{-1}[int(k)]⊆ int(f^{-1}[k]) for each IVPFS K in Y

Proof

a) ⇒ b) let f:X→Y be a IVPF continuous function and P be IVPFS over X. then f [P]⊆ cl(f [P]) and P⊆ f^{-1}[cl(f [P])]. since cl(f [P]) is a IVPF closed set in Y and f is a IVPF continuous function, f^{-1}[cl(f [P])] is a IVPF closed set in X on the other hand ,if cl(P) is the smallest interval valued Pythagorean fuzzy closed set containing P,then

cl(P) ⊆ f^{-1}[cl(f [P])] and so,f [cl(P)]⊆ cl(f [P])

b) ⇒ c) suppose that P=$f^{-1}[k]$

from b) f [cl(P)]=f^{-1}[cl(k)] ⊆ cl)(f [P])=cl(f [$f^{-1}[k]$)⊆ cl(k)

then cl($f^{-1}[k]$)=cl(P) ⊆ f^{-1}[f [cl(P)]]⊆ f^{-1}[cl(k)]

c) ⇒ d) since int (k) =(cl(k^c))^c, then

cl($f^{-1}[k]$)=cl(P)⊆ f^{-1}[f [cl(f [P])]⊆ f^{-1}[cl(k)].Assume that,G is a IVPF open set in Y. then int(G)=G from (d)

f^{-1}[G]=f^{-1}[int(G)]⊆ int(f^{-1}[G]) ⊆ f^{-1}[G].Therefore f is a IVPF continuous function

Definition 3.11

Consider $(X, \tau_1)_P$ be a IVPFTS

a) A subfamily Γ of τ is a called a IVPF basic for τ ,if for each P∈ τ, P=0_x or these exist Γ'⊆ Γ such that P=∪ Γ'

b) A subfamily Φ of τ is called IVPF sub base for τ, if the family

$\Gamma = \{\cap \Phi' : \Phi'$ is a finite subset of $\Phi\}$ is a IVPF sub base for τ

Theorem 3.12

Consider $(X, \tau_1)_P$ and $(Y, \tau_2)_P$ be two IVPFTS and f:X→Y be a IVPF function, then

i) f is a IVPF continuous function iff for each B∈ Γ we have $f^{-1}[B]$ is a IVPF open subset of X such that Γ is a IVPF basic for τ_2

ii) f is a IVPF continuous function iff for each K ∈ Φ we have $f^{-1}[k]$ is a IVPF open subset of X such that Φ is a IVPF sub base for τ_2

Proof

i) Let f be a IVPF continuous function. Since each B∈ $\Gamma \subseteq \tau_2$ and f is IVPF continuous function ,then $f^{-1}[B] \in \tau_1$

Conversely ,suppose that Γ is a IVPF basic for τ_2 and $f^{-1}[B] \in \tau_1$

for each B∈ Γ then arbitrary a IVPF open set P∈ τ_2

$f^{-1}[P] = f^{-1}[\cup_{B\in\Gamma} B] = \cup_{B\in\Gamma} f^{-1}[B] \in \tau_1$, that is f is IVPF continuous function

ii) Let f be a IVPF continuous function .since each k∈ $\Phi \subseteq \tau_2$ and f is IVPF continuous function then $f^{-1}[k] \in \tau_1$, conversely assume that Φ is IVPF sub base for τ_2 and $f^{-1}[k] \in \tau_1$ for each k∈ Φ. then for arbitrary a IVPF open set P ∈ τ_2

$f^{-1}[P] = f^{-1}[\cup_{i_j\in\Gamma}(k_{i_1} \cap k_{i_2} \cap \ldots \cap k_{i_n}]$

$= \cup_{i_j\in\Gamma} (f^{-1}[k_{i_1}] \cap f^{-1}[k_{i_2}] \cap \ldots \cap f^{-1}[k_{i_n}]) \in \tau_1$

That is f is IVPF continuous function

Definition 3.13

Consider $(X, \tau_1)_P$ and $(Y, \tau_2)_P$ be two IVPFTS and f:X→Y be a IVPF function, then

a) F is called a IVPF open function if f[P] is a IVPF open set over Y for every IVPF open set P over X

b) f is called IVPF closed function if f[k] is IVPF closed set over Y for every IVPF closed set over X

Theorem 3.14

Let $(X, \tau_1)_P$ and $(Y, \tau_2)_P$ be two IVPFTS and f:X→Y be a IVPF function,then

i) f is a interval valued pythagorean fuzzy open function if f [int (p)]⊆ int (f[p]) for each interval valued pythagorean fuzzy set P over X.

ii) f is a interval valued pythagorean fuzzy closed function if cl(f [P]) ⊆ f [cl(P)] for each interval valued pythagorean fuzzy set P over X.

Proof

i) Let f be a interval valued pythagorean fuzzy open function and P be IVPFS over X. then int(P) is a interval valued pythagorean fuzzy open set and int(P) ⊆ P. Since f is interval valued pythagorean fuzzy open function, f [int (p)] is a IVPFOS over Y and f [int (p)]⊆ (f[p]). Thus f [int (p)]⊆ int (f[p]) is obtained.

Conversely,suppose that P is any IVPFOS over X. Then P=int (P). from the condition of theorem, we have f [int (p)]⊆ int (f[p]). then f[P]=f [int(P)] ⊆ int (f [P]) ⊆f [P]. this implies that f [P]=int(f [P]). that is, f is a interval valued pythagorean fuzzy open function.

ii) Let f be a interval valued pythagorean fuzzy closed function and P be a IVPFS over X. since f is a interval valued pythagorean fuzzy closed function then f [cl(P)] is a interval valued pythagorean fuzzy closed set over Y and f[P] ⊆ f [cl(P)].thus cl(f [P]) ⊆ f[cl(P)] is obtained.

Conversely, assume that P is any interval valued pythagorean fuzzy closed set over X. then P= cl(P). from the condition of theorem, we have cl(f [P]) ⊆ f [cl(P)] =f [P] ⊆ cl(f [P]). this means that, cl(f [P]) =f [P]. That is, f is a interval valued pythagorean fuzzy closed function.

Definition 3.15

Consider $(X, \tau_1)_P$ and $(Y, \tau_2)_P$ be two IVPFTS and f:X→Y be a IVPF function, then f is called a IVPF homeomorphism, if

i) f is a bijection

ii) f is a interval valued Pythagorean fuzzy continuous function,

iii) f^{-1} is a interval valued Pythagorean continuous function.

Theorem 3.16

If f: $(X, \tau_1) \rightarrow (Y, \tau_2)$ is called a IVPF closed mapping and g: $(Y, \tau_2) \rightarrow (Z, \tau_3)$ is an IVPF closed mapping, then gof: $(X, \tau_1) \rightarrow (Z, \tau_3)$is an IVPF closed mapping.

Proof

Let v be any interval valued pythagorean fuzzy closed set in (X, τ_1). Then f(v) is an interval valued pythagorean fuzzy closed set in (Y, τ_2), because f is an IVPF closed mapping. Now g(f (v)) is an IVPFCS in (Z, τ_3) because g is an IVPF closed mapping. Therefore gof: $(X, \tau_1) \rightarrow (Z, \tau_3)$is an IVPF closed mapping.

Definition 3.17

A mapping f:$(X, \tau_1) \rightarrow (Y, \tau_2)$ is called an interval valued pythagorean fuzzy open mapping, if for interval valued pythagorean fuzzy open set v of (X, τ_1), f(v) is an interval valued pythagorean fuzzy open set in Y

Theorem 3.18

For any bijection mapping f:$(X, \tau_1) \rightarrow (Y, \tau_2)$ the following statements are equivalent

i) The inverse map f^{-1}:$(Y, \tau_2) \rightarrow (X, \tau_1)$ is interval valued pythagorean fuzzy continuous

ii) f is an interval valued pythagorean fuzzy open mapping

iii) f is an interval valued pythagorean fuzzy closed mapping

Proof

i)⇒ ii) let v be an IVPFOS in X. since f^{-1} is IVPF continuous ,the inverse image of v under f^{-1} namely f(v) is an IVPFOS in Y and so f is an IVPF open mapping

ii) ⇒ iii) let v be an IVPFCS in X, then v^c is an IVPFOS in X. since f is an IVPF open, then f(v^c) is an IVPFOS in Y . but f(v^c) =Y - f(v) and so f(v) is an IVPFCS in Y. therefore f is an IVPF closed mapping.

iii) ⇒ i) let v be any IVPFCS in X. then the inverse image of v under f^{-1},namely f(v) is IVPFCS in Y. since f is an IVPF closed mapping.

Therefore f^{-1} is an IVPF continuous mapping.

Theorem 3.19

Let f: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be bijective mapping and IVPF continuous. Then the following statements are equivalent

i) f is an IVPF open mapping

ii) f is an IVPF homeomorphism

iii) f is an IVPF closed mapping

Proof

i) ⇒ ii) Given f:$(X, \tau_1) \rightarrow (Y, \tau_2)$ is bijective, IVPF continuous and interval valued pythagorean fuzzy open. Then by def f is an IVPF homeomorphism

ii) ⇒ iii) given f:$(X, \tau_1) \rightarrow (Y, \tau_2)$ is bijective and IVPF homeomorphism by previous theorem f is an IVPF closed mapping

iii) ⇒ i) given f:$(X, \tau_1) \rightarrow (Y, \tau_2)$ is bijective and IVPF closed. By previous theorem , f is an IVPF closed mapping

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