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On Interval Valued Pythagorean Fuzzy Compactness

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Abstract: The concept of interval valued pythagorean fuzzy almost compactness and interval valued pythagorean fuzzy nearly compactness in interval valued pythagorean fuzzy topological spaces is introduced and studied. Besides giving characterizations of these spaces, we study some of their properties. Also, we investigate the behavior of interval valued pythagorean fuzzy compactness, interval valued pythagorean fuzzy almost compactness, and interval valued pythagorean fuzzy nearly compactness under several types of interval valued pythagorean fuzzy continuous mappings.

I. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh (1965)[12], and in 1968 Chang[2] defined the concept of fuzzy topological space and generalized some basic notions of topology such as open set, closed set, continuity and compactness to fuzzy topological space .Atanassov [1]introduced intuitionistic fuzzy set and Coker [3] developed intuitionistic fuzzy topological spaces and also introduced fuzzy compactness in intuitionistic fuzzy topological spaces. Lowen (1976) [9] also made different studies on fuzzy compactness .Yager[11] defined pythagorean fuzzy subsets. Besides in 2015 Peng.X and Yang.Y[10] introduced the Fundamental properties of interval valued pythagorean fuzzy aggregation operators.

In this paper some properties of interval valued pythagorean fuzzy compactness were investigated. We use the finite intersection property to give characterization of the interval valued pythagorean fuzzy compact spaces. Also we introduce interval valued pythagorean fuzzy almost compactness and interval valued pythagorean fuzzy nearly compactness and established the relationships between these types of compactness.

II.PRELIMINARIES

Definition 2.1

Let A be fixed set then a fuzzy set P in A can be defined as P={(a,$\lambda_P(a)$) / a$\in$A} where λ_P :A $\to$[0,1] is called membership degree of a$\in$A.

Definition 2.2

Let X be a nonempty fixed set and I the closed interval [0,1].An intuitionistic fuzzy set (IFS) A is an object of the following form: A=$\{\langle x, \mu_A(x), \nu_A(x)\rangle; x \in X\}$ where the mappings $\mu_A(x)$: X$\to$ I and $\nu_A(x)$: X $\to$ I denote the degree of membership, namely, $\nu_A(x)$, for each element x$\in$X to the set A respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in$ X.

Definition 2.3

Let (X,τ) be an IFTS and let A= $\{\langle x, \mu_A(x), \nu_A(x) \rangle; x \in X\}$ be an IFS in X . Then the intuitionistic fuzzy closure and intuitionistic fuzzy interior of X are defined by

(i) cl(A) =$\cap$ {C: C is an IFCS in X and C $\supseteq$ A};

(ii) int(A)= $\cup$ {D: D is an IFOS in X and D$\subseteq$ A}.

It can be also shown that cl(A) is an IFCS, int(A) is an IFOS in X and A is an IFCS in X if and only if cl(A); A is an IFOS in X if and only if int(A)=A.

Some of the properties of pythagorean fuzzy sets:

Definition 2.4

Let S and T be the two pythagorean fuzzy sets then S is the subset of T and T is the superset of S if it satisfies,

$\lambda_S(x) \leq \lambda_T(x)$ and $\eta_S(x) = \eta_T(x)$

Definition 2.5

Let S and T be the two pythagorean fuzzy sets then S and T are called similar sets if it satisfies,

$\lambda_S(x) = \lambda_T(x)$ or $\eta_S(x) = \eta_T(x)$

Definition 2.6

Let X and Y be the two pythagorean fuzzy sets then X and Y are called the comparable sets if it satisfies,

$\lambda_S(x) = \lambda_T(x)$ and $\eta_S(x) = \eta_T(x)$

Definition 2.7

Let S and T be the two pythagorean fuzzy sets then S and T are called equivalent sets if it satisfies the following condition hold,

$g : \lambda_S(x) \rightarrow \lambda_T(x)$ and $g : \eta_S(x) = \eta_T(x)$ both are bijective functions.

Definition 2.8

Let S be a universal set then an interval valued pythagorean fuzzy set T in S can be defined as

T $=\{(s, \mu_T(s), \nu_T(s)) / s \in S\}$

Where $\mu_T(s) = [\mu_T^a , \mu_T^b] \subset [0,1]$

$\nu_T(s) = [\nu_T^a , \nu_T^b] \subset [0,1]$

Also, $\mu_T^a = \inf \mu_T(s)$

$\mu_T^b = \sup \mu_T(s$

$\nu_T^a = \inf \nu_T(s)$

$\nu_T^b = \sup \nu_T(s)$

i.e) $0 \leq (\mu_T^b(k)^2 + (\nu_T^b(k)^2 \leq 1$

Definition 2.9

An interval valued pythagorean fuzzy sets on a universe X is an object of the form A=$\{\langle x, \mu_A(x), \nu_A(x)\rangle$ / x∈ X},where $\mu_A(x)$:X→ Int[0,1] and $\nu_A(x)$:X→Int[0,1] .(Int [0,1] stands for the set of all set of all closed subintervals of [0,1]) satisfies the condition ,for all x∈X

$\sup \mu_A(x) + \sup \nu_A(x) \leq 1.$

- The union of A and B is denoted by A∪ B where
  A ∪ B = $\{\langle x, [sup\ (\underline{\mu_A}(x), \underline{\mu_B}(x)), sup\ (\overline{\mu_A}(x), \overline{\mu_B}(x))], [inf\ (\underline{\nu_A}(x), \underline{\nu_B}(x)), inf\ (\overline{\nu_A}(x), \overline{\nu_B}(x))]\rangle \ | x \in X\}$
- The intersection of A and B is denotrd by A∩B where
  A∩ B = $\{\langle x, [inf\ (\underline{\mu_A}(x), \underline{\mu_B}(x)), inf(\overline{\mu_A}(x), \overline{\mu_B}(x))], [sup(\underline{\nu_A}(x), \underline{\nu_B}(x)), sup(\overline{\nu_A}(x), \overline{\nu_B}(x))]\rangle | x \in X\}$
- The complement of A is denoted by
  $A^C = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in X \}$

Definition 2.10

Let (X,τ) be an IVPFTS and let A= $\{\langle x, \mu_A(x), \nu_A(x) \rangle; x \in X\}$ be an IVPFS in X . Then the interval valued pythagorean fuzzy closure and inerval valued pythagoreant fuzzy interior of are defined by

(i) IVPF cl(A) =∩ {K: K is an IVPFCS in X and A ⊆ K};

(ii)IVPF int(A)= ∪ {G: G is an IVPFOS in X and G⊆ A}.

Note that for any IVPFs A in (X,τ) we have

IVPFs cl (A^C)=(IVPF int (A))^C and IVPF int(A^C)=(IVPF cl(A)^C)

Definition 2.11

A fuzzy function f:X → Y is called fuzzy irresolute if inverse image of each fuzzy open set is fuzzy open.

III.INTERVAL VALUED PYTHAGOREAN FUZZY COMPACTNESS

Definition 3.1

An IVPFTS B of (X,τ) is said to be IVPF compact relative to X if, for every collection $\{A_i$;i∈I} of IVPF open subsets of X such that

B⊆ U$\{A_i$; i∈I},there exists a finite subset I_0 of I such that B⊆U$\{A_i$; i∈$I_0\}$.

Definition 3.2

An IVPFTS of (X,τ) is said to be IVPF compact if it is IVPF compact as a subspace of X.

Definition 3.3

A family of IVPF closed sets $\{A_i$; i∈I} has the finite intersection property (in short FIP) if, for any subset I_0 of I,$\cap_{i \in I_0} A_i \neq$ 0.

Theorem 3.4

For an IVPFTS the following statements are equivalent.

(i) X is IVPF compact.

(ii) Any family of IVPF closed subsets of X satisfying the finite intersection property has a nonempty intersection.

Proof:

Let X be IVPF compact space and let $\{A_i;\ i\in I\}$ be a family of IVPF closed subsets of X satisfying the Finite intersection property. Suppose $\cap_{i\in I} A_i = 0$. Then $A_i \cup_{i\in I} \overline{A_i}= 1$. Since $\{\overline{A_i};\ i\in I\}$is a collection of IVPF open sets cover X, then from IVPF compactness of X there exists a finite subset I_0 of I such that $\cup_{i\in I_0} \overline{A_i}= 1$. Then $\cap_{i\in I_0} A_i = 0$,which gives a contradiction and therefore $\cap_{i\in I} A_i \neq 0$. Thus

(i) ⇒ (ii).

Let $\{A_i;\ i\in I\}$ be a family of IVPF open sets cover X . Suppose that for any finite subset I_0 of I we have $\cup_{i\in I_0} \overline{A_i} \neq 1$. Then $\cap_{i\in I_0} \overline{A_i} \neq 0$. Hence $\{A_i;\ i\in I\}$ satisfies the finite intersection property. Then, by hypothesis, we have $\cap_{i\in I} \overline{A_i} \neq 0$ which implies that $\cup_{i\in I} A_i \neq 1$ and contradicts that $\{A_i;\ i\in I\}$; is an IVPF open cover of X . Hence our assumption $\cup_{i\in I_0} A_i \neq 1$ is wrong. Thus $\cup_{i\in I_0} A_i = 1$ which implies that X is IVPF compact.Thus (ii) ⇒ (i).

Theorem 3.5

An interval valued pythagorean fuzzy closed subset of an interval valued pythagorean fuzzy compact space is interval valued pythagorean fuzzy compact relative to X.

Proof:

Let A be an IVPF closed subset of X . Let $\{G_i;\ i\in I\}$ be cover of A by IVPF open sets in X .Then the family $\{G_i;\ i\in I\}\cup \overline{A}$ is an IVPF open cover of X. Since X is IVPF compact, there is a finite subfamily $\{G_1, G_2,...,G_n\}$ of IVPF open cover, which also covers X . If this cover contains $\bar{A}$, we discard it. Otherwise leave the subcover as it is. Thus we obtained a finite IVPF open subcover of A . So A is IVPF compact relative to X.

Theorem 3.6

Let (X,τ) and (Y,σ) be interval valued pythagorean fuzzy topological spaces and let f:(X,τ) → (Y,σ) be interval valued pythagorean fuzzy irresolute, surjective mapping. If (X,τ) is IVPF compact space then so is (Y,σ).

Proof:

Let f : (X,τ)→(Y,σ) be interval valued pythagorean fuzzy irresolute mapping of an interval valued pythagorean fuzzy compact space (X,τ) onto an IVPFTS (Y,σ). Let $\{A_i;i\in I\}$ be any interval valued pythagorean fuzzy open cover of (Y,σ) .Then $\{f^{-1}(A_i):i\in I\}$ is a collection of interval valued pythagorean fuzzy open sets which covers X.Since X is interval valued pythagorean fuzzy compact, there exists a finite subset I_0 of I such that subfamily $\{f^{-1}(A_i):i\in I_0\}$of $\{f^{-1}(A_i):i\in I\}$ covers X . It follows that $\{A_i:i\in I_0\}$ is a finite subfamily of $\{A_i:i\in I\}$ which covers Y. Hence Y is interval valued pythagorean fuzzy compact.

Theorem 3.7

Let (X,τ) and (Y,σ) be interval valued pythagorean fuzzy topological spaces and let (X,τ) → (Y,σ) be interval valued pythagorean fuzzy irresolute mapping. If A is IVPF compact relative to X then f(A) is IVPF compact relative to Y .

Proof:

Let $\{A_i:i\in I\}$ be a family of IVPF open cover of X such that f(A) $\subseteq \cup_{i\in I} (A_i)$. Then A $\subseteq f^{-1}$(f(A)) $\subseteq f^{-1}(\cup_{i\in I} A_i)=\cup_{i\in I} f^{-1}(A_i)$. Since f is IVPF irresolute, $f^{-1}(A_i)$ is IVPF open cover of X. And A is IVPF compact in (X,τ); there exists a finite subset I_0 of I such that A $\subseteq \cup_{i\in I_0} f^{-1}(A_i)$. Hence f(A) $\subseteq$ f$(\cup_{i\in I_0} f^{-1}(A_i)) = \cup_{i\in I_0} f(f^{-1}(A_i)) \subseteq \cup_{i\in I_0} (A_i)$.Thus f(A) is IVPF compact relative to Y.

Theorem 3.8

An IVPF continuous image of IVPF compact space is IVPF compact.

Proof :

Let f : (X,τ) → (Y,σ) be an IVPF continuous from an IVPF compact space X onto IVPFTS . Let $\{A_i:i\in I\}$ be IVPF open cover of X . Then $\{f^{-1}(A_i);i\in I\}$ is IVPF open cover of X. Since X is IVPF compact, there exists finite subset I_0 of I such that finite family $\{f^{-1}(A_i);i \in I_0\}$ covers X.Since f is onto, $\{A_i;i \in I_0\}$ is a finite cover of Y.Hence Y is IVPF compact.

Definition 3.9

Let (X,τ) and (Y,σ) be two interval valued pythagorean fuzzy topological spaces. A mapping f : X → Y is said to be Interval valued pythagorean fuzzy strongly open if f(V) is IVPFOS of Y for every IVPFOS of X.

Theorem 3.10

Let f : (X,τ)→(Y,σ) be an IVPF strongly open, bijective function and Y is IVPF compact; then X is IVPF compact.

Proof :

Let $\{A_i;i \in I\}$ be IVPF open cover of X, and then $\{ f(A_i);i \in I\}$ is IVPF open cover of Y. Since Yis IVPF compact, there is a finite subset I_0 of I such that finite family $\{f(A_i);i \in I_0\}$ covers Y. But $1_{\sim X} = f^{-1}(1_{\sim Y}) = f^{-1}f(\cup_{i\in I_0}(A_i))=\cup_{i\in I_0}(A_i)$ and therefore is IVPF compact.

IV. INTERVAL VALUED PYTHAGOREAN FUZZY ALMOST COMPACTNESS AND IINTERVAL VALUED PYTHAGOREAN FUZZY NEARLY COMPACTNESS

In this section we investigate the relationships between IVPF compactness, IVPF almost compactness, and IVPF nearly compactness.

Definition 4.1

An IVPFTS (X, τ) is said to be IVPF almost compactness if and only if, for every family of IVPF open cover $\{A_i : i \in I\}$ of X, there exists a finite subset I_0 of I such that $\cup_{i \in I_0} cl\,(A_i) = 1_{\sim}$.

Definition 4.2

An IVPFTS (X, τ) is said to be IVPF nearly compactness if and only if, for every family of IVPF open cover $\{A_i : i \in I\}$ of X, there exists a finite subset I_0 of I such that $\cup_{i \in I_0} int(cl) = 1_{\sim}$.

Definition 4.3

An IVPFTS (X, τ) is said to be IVPF regular if, for each IVPF open set $A \in (X, \text{A}) = \cup\{A_i \in I^X | A_i\}$ is IVPF open, $cl(A_i) \subseteq A$.

Theorem 4.4

Let (X, τ) be IVPFT. Then IVPF compactness implies IVPF nearly compactness which implies IVPF almost compactness.

Proof:

Let (X, τ) be IVPF compact. Then for every IVPF open cover$\{A_i : i \in I\}$ of X, there exists a finite subset $\cup_{i \in I_0} (A_i) = 1_{\sim}$. Since A_i is an IVPFOS, for each $i \in I$, $A_i = int(A_i)$ for each $i \in I$. $A_i = int\,(A_i) \subseteq int\,(cl\,(A_i))$ for each $i \in I$. Here $1_{\sim} = \cup_{i \in I_0} A_i = \cup_{i \in I_0} int\ (A_i) \subseteq \cup_{i \in I_0} int\,(cl\,(A_i))$. Thus $\cup_{i \in I_0} int\,(cl\,(A_i)) = 1_{\sim}$. which implies that (X, τ) is IVPF nearly compactness. Now let (X, τ) be IVPF nearly compact. Then for every IVPF open cover $\{A_i : i \in I\}$ of X, there exists a finite subset $\cup_{i \in I_0} int\,(cl\,(A_i)) = 1_{\sim}$. Since $int(cl(A_i)) \subseteq cl\,A_i$ for each $i \in I_0$, $1_{\sim} = \cup_{i \in I_0} int(cl(A_i)) \subseteq \cup_{i \in I_0} cl\,(A_i)$. Thus $\cup_{i \in I_0} cl\,(A_i) = 1_{\sim}$. Hence (X, τ) is IVPF almost compact.

Remark 4.5

The Converse implications of the above theorem need not be true in general.

Example 4.6

Let X be a nonempty set. Then (X, τ) is IVPFTS, where $\tau = \{0, 1, \ldots A_n\}$, $n \in N$, where $A_n : X \to [0, 1]$ is defined by $A_n = \langle \square, 1 - 1/\square, 1/\square \rangle$; $x \in X$, $n \in N$. The collection $\{A_n : n \in N\}$ is IVPF open cover of X. But no finite subset of $\{A_n : n \in N\}$ covers X. Hence X is not IVPF compact. But $cl(A_n) = 1_{\sim}$ for $n \geq 3$. Thus there exists a finite subfamily $\{A_n : n \in N_0\}$ for $N_0 \subseteq N$ such that $\cup_{n \in N_0} cl(A_n) = 1_{\sim}$. Thus X is IVPF almost compactness. Also $int(cl(A_n)) = int(1_{\sim}) = 1_{\sim}$ for $n \geq 3$. Thus there exists a finite subfamily $\{A_n : n \in N_0\}$ for $N_0 \subseteq N$ such that $\cup_{n \in N_0} int(cl(A_n)) = 1_{\sim}$. Thus X is IVPF nearly compactness.

Theorem 4.7

Let (X, τ) be IVPFTS. If (X, τ) is IVPF almost compact and IVPF regular then (X, τ) is IVPF compact.

Proof:

Let $\{A_i : i \in I\}$ be IVPF open cover of X such that $\cup_{i \in I} A_i = 1_{\sim}$. Since (X, τ) is IVPF regular, $A_i = \cup\{B_i \in I^X | B_i$ is IVPF open, $cl(B_i) \subseteq A_i\}$ for each $i \in I$. Since $1_{\sim} = \cup_{i \in I}(\cup_{i \in I} B_i)$ and (X, τ) is IVPF almost compact there exists a finite set I_0 of I such that $\cup_{i \in I_0} cl(B_i) = 1_{\sim}$. But $cl(B_i) \subseteq A_i(int(clB_i) \subseteq cl\,(B_i))$. We have $\cup_{i \in I_0} A_i \supseteq \cup_{i \in I_0} cl(B_i) = 1_{\sim}$. Thus, $\cup_{i \in I_0} A_i = 1_{\sim}$. Hence (X, τ) is IVPF compact.

Theorem 4.8

Let (X, τ) be IVPFTS. If (X, τ) is IVPF nearly compact and IVPF regular then (X, τ) is IVPF compact.

Proof:

Let $\{A_i : i \in I\}$ be IVPF open cover of X such that $\cup_{i \in I_0} A_i = 1_{\sim}$. Since (X, τ) is IVPF regular, $A_i = \cup\{B_i \in I^X | B_i$ is IVPF open, $cl(B_i) \subseteq A_i\}$ for each $i \in I$. Since $1_{\sim} = \cup_{i \in I}(\cup_{i \in I} B_i)$ and (X,τ) is IVPF nearly compact there exists a finite set I_0 of I such that $\cup_{i \in I_0}$ $int\,(cl\,(B_i)) = 1_{\sim}$. But $int(cl\,(B_i)) \subseteq cl(B_i) \subseteq A_i$. We have $\cup_{i \in I_0} (A_i) \supseteq \cup_{i \in I_0} cl\,(B_i) \supseteq \cup_{i \in I_0} int(cl\,(B_i)) = 1_{\sim}$. Thus, $\cup_{i \in I} B_i = 1_{\sim}$. Hence (X, τ) is IVPF compact.

Theorem 4.9

Let (X, τ) and (Y, σ) be IVPFTS and let $f : (X, \tau) \to (Y, \sigma)$ be interval valued pythagorean fuzzy irresolute, surjective mapping. If (X, τ) is IVPF almost compact space then so is (Y, σ).

Proof:

Let $f : (X, \tau) \to (Y, \sigma)$ be interval valued pythagorean fuzzy irresolute mapping of an interval valued pythagorean fuzzy compact space (X, τ) onto an IVPFTS (Y, σ). Let $\{A_i : i \in I\}$ be any interval valued pythagorean fuzzy open cover of (Y, σ). Then $\{f^{-1}(A_i) : i \in I\}$ is an interval valued pythagorean fuzzy open cover of X. Since X is interval valued pythagorean fuzzy almost compact, there exists a finite subset I_0 of I such that $\cup_{i \in I_0} cl\,(f^{-1}(A_i)) = 1_{\sim X}$. And $f(1_{\sim X}) = f\,(\cup_{i \in I_0} cl\,(f^{-1}(A_i))) = \cup_{i \in I_0}$

$f(cl(f^{-1}(A_i))) = 1_{\sim Y}$. But $cl(f^{-1}(A_i)) \subseteq f^{-1}(cl(A_i))$ and from the surjectivity of f, f($cl(f^{-1}(A_i))) \subseteq f(f^{-1}(cl(A_i))) = cl(A_i)$. So $\cup_{i\in I_0} cl(A_i) \supseteq \cup_{i\in I_0} f(cl(f^{-1}(A_i))) = 1_{\sim Y}$.Thus $\cup_{i\in I_0} cl(A_i) = 1_{\sim Y}$. Hence (Y, σ) is IVPF almost compact .

Theorem 4.10

Let (X, τ) and (Y, σ) be interval valued pythagorean fuzzy topological spaces and let f : $(X, \tau) \rightarrow (Y, \sigma)$ be interval valued pythagorean fuzzy continuous, surjective mapping. If (X, τ) is IVPF almost compact space then (Y, σ) is IVPF almost compact.

Proof:

Let $\{A_i : i \in I\}$ be any interval valued pythagorean fuzzy open cover of (Y, σ). Then $\{f^{-1}(A_i) : i \in I\}$ is an interval valued pythagorean fuzzy open cover of X. Since X is interval valued pythagorean fuzzy almost compact, there exists a finite subset I_0 of I such that $\cup_{i\in I_0} cl(f^{-1}(A_i)) = 1_{\sim X}$ .And from the surjectivity of f, $1_{\sim Y} = f(1_{\sim X}) = f(\cup_{i\in I_0} cl(f^{-1}(A_i))) \subseteq \cup_{i\in I_0} f(cl(f^{-1}(A_i)))$ $\cup_{i\in I_0} cl\, f(f^{-1}(A_i)) \subseteq \cup_{i\in I_0} cl\, f(f^{-1}(A_i)) \subseteq \cup_{i\in I_0} cl(A_i)$ which implies that $\cup_{i\in I_0} cl(A_i) = 1_{\sim Y}$. Hence (Y, σ) is IVPF almost compact.

Definition 4.11

Let (X, τ) and (Y, σ) be two interval valued pythagorean fuzzy topological spaces. A function f : $X \rightarrow Y$ is said to be interval valued pythagorean fuzzy weakly continuous (IVPF weakly continuous) if, for each IVPFOS V in Y, $f^{-1}(V) \subseteq int(f^{-1}(cl(V)))$.

Theorem 4.12

A mapping f from an IVPFTS (X, τ) to an IVPFTS (Y, σ) is IVPF strongly open if and only if $f(int\, V) \subseteq int\, f(V)$.

Proof:

If f is IVPF strongly open mapping then $f(int\, V)$ is an IVPFOS in Y for IVPFOS V in X. Hence $f(int\, V) = int\, f(int\, V) = int\, f(V)$. Thus $f(int$ V) $\subseteq int\, f(V)$.

Conversely, let V be IVPFOS in X and then V= int V. Then by hypothesis, $f(V) = f(int\, V) \subseteq int\, f(V)$. This implies that $f(V)$ is IVPFOS in Y.

Theorem 4.13

Let (X, τ) and (Y, σ) be IVPFTS and let $f : (X, \tau) \rightarrow (Y, \sigma)$ be interval valued pythagorean fuzzy weakly continuous, surjective mapping. If (X, τ) is IVPF compact space then (Y, σ) is IVPF almost compact.

Proof:

Let $\{A_i : i \in I\}$ be IVPF open cover of Y such that $\cup_{i\in I_0} A_i = 1_{\sim Y}$. Then $\cup_{i\in I_0} f^{-1}(A_i) = f^{-1}(A_i) = \cup_{i\in I} f^{-1}(1_{\sim Y}) = 1_{\sim X}$.(X, τ) is IVPF compact, and there exists a finite subset I_0 of I such that $\cup_{i\in I_0} f^{-1}(A_i) = 1_{\sim X}$. Since f is IVPF weakly continuous, $f^{-1}(A_i) \subseteq int(f^{-1}(cl(A_i))) \subseteq f^{-1}(cl(A_i))$. This implies that $\cup_{i\in I_0} f^{-1}(cl(A_i)) \supseteq \cup_{i\in I_0} f^{-1}(A_i) = 1_{\sim X}$. Thus $\cup_{i\in I_0} f^{-1}(cl(A_i))$ $= 1_{\sim X}$. Since f is surjective, $1_{\sim Y} = f(1_{\sim X}) = f(\cup_{i\in I_0} f^{-1}(cl(A_i))) = \cup_{i\in I_0} f(f^{-1}(cl(A_i))) = \cup_{i\in I_0} cl(A_i)$.Hence $\cup_{i\in I_0} cl(A_i) = 1_{\sim Y}$. Hence (Y, σ) is IVPF almost compact.

Theorem 4.14

Let (X, τ) and (Y, σ) be interval valued pythagorean fuzzy topological spaces and let $f : (X, \tau) \rightarrow (Y, \sigma)$ be interval valued pythagorean fuzzy irresolute, surjective, and strongly open mapping. If (X, τ) is IVPF nearly compact space then so is (Y, σ).

Proof:

Let $\{A_i : i \in I$ be any interval valued pythagorean fuzzy open cover of (Y, σ). Since f is IVPF irresolute, then $\{f^{-1}(A_i) : i \in I\}$ is an interval valued pythagorean fuzzy open cover of X. Since (X, τ) is IVPF nearly compact, there exists a finite subset I_0 of I such that $\cup_{i\in I_0} int(cl f^{-1}(A_i)) = 1_{\sim X}$. Since f is surjective, $1_{\sim Y} = f(1_{\sim X}) = f(\cup_{i\in I_0} int(cl f^{-1}(A_i))) = \cup_{i\in I_0} f(int(cl f^{-1}(A)))$. Since f is IVPF strongly open,

$f(int(cl f^{-1}(A_i))) \subseteq int\, f(cl\, f^{-1}(A_i))$ for each $i \in I$. Since f is IVPF irresolute, then $f(cl\, f^{-1}(A_i)) \subseteq cl\, f(f^{-1}(A_i))$. Hence we have $1_{\sim Y} = \cup_{i\in I_0} f(int(cl f^{-1}(A_i))) \subseteq \cup_{i\in I_0} int\, f(cl\, f^{-1}(A_i)) \subseteq \cup_{i\in I_0} int(cl\, f(f^{-1}(A_i))) = \cup_{i\in I_0} int(cl(A_i))$. Thus $1_{\sim Y} = \cup_{i\in I_0} int(cl(A_i))$. Hence (Y, σ) is IVPF nearly compact.

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