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On $(g\alpha r)^{**}$-Closed Sets in Topological Spaces

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Abstract: The purpose of this paper is to introduce a new class of sets called generalized α regular-star-star-closed sets (briefly $(g\alpha r)^{**}$-closed set) in topological spaces. We compare $(g\alpha r)^{**}$-closed sets with the other existing sets and also their characterizations are analyzed.

I. Introduction

The concept of generalization of closed set was introduced by Levine [3] in 1970. Further investigation on generalization closed set has lead to significant contribution to the theory of separation axiom, generalization of continuity and covering properties. The concepts of generalized b-closed sets in topological spaces was introduced by A.A.Omari and M.S.M.Noorani [1]. In this paper the non-empty topological space is denoted by (X,τ), (Y,σ), and (Z,η) or X, Y and Z on which no separation axioms are assumed unless otherwise explicitly stated. In this paper we introduce a new class of sets called generalized α regular-star-star-closed sets in topological spaces. Also their characterizations are analyzed and it is compared with the other existing sets.

II. Preliminaries

Definition 2.1

Let A be an subset of topological space (X, τ). Then A is called

1. α-open set [6] if A ⊆ int(cl(intA)).
2. generalized closed set (briefly g-closed set) [3] if cl(A) ⊆ U whenever A ⊆ U and U is open.
3. generalized * closed set (briefly g^*-closed set) [7] if cl(A) ⊆ U whenever A ⊆ U and U is g-open.
4. generalized α-closed set (briefly gα-closed set) [5] if αcl(A) ⊆ U whenever A ⊆ U and U is α-open.
5. α-generalized closed set (briefly αg-closed set) [4] if αcl(A) ⊆ U whenever A ⊆ U and U is open.
6. generalized pre regular-closed set (briefly gpr-closed set) [2] if pcl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
7. generalized α regular-closed set (briefly gαr-closed set) [9] if αcl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
8. regular closed set [10] if A = cl(int(A)).
9. gsp-closed set [11] if spcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
10. generalized α regular-star-closed set (briefly $(g\alpha r)^*$-closed set) [8] if cl(A) ⊆ U whenever A ⊆ U and U is (gαr)-open in X.

III. GENERALIZED α REGULAR-STAR-STAR-CLOSED SETS IN TOPOLOGICAL SPACES

Definition 3.1

A subset A of a topological space (X,τ) is called generalized α regular-star-star-closed set (briefly $(g\alpha r)^{**}$-closed set) if cl(A) ⊆ U whenever A ⊆ U and U is $(g\alpha r)^*$-open in X.

Definition 3.2

A subset A of a topological space (X,τ) is called $(g\alpha r)^{**}$-open set if and only if A^C is $(g\alpha r)^{**}$-closed in X.

Theorem 3.3:

Every $(g\alpha r)^{**}$-closed set is g-closed set.

Proof:

Let A be any $(g\alpha r)^{**}$-closed set in X such that $A \subseteq U$, where U is open. Since every open set is $(g\alpha r)^{*}$-open in X. Therefore A is $cl(A) \subseteq U$. Hence A is g-closed set in X.

The converse of above theorem need not to true as shown from the following example.

Example 3.4:

Let X={a,b,c,d} with τ={X,∅,{a},{b},{a,b}}. Here, g-closed set={X, ∅,{c},{d},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{b,c,d},{a,c,d},{a,b,d}} and $(g\alpha r)^{**}$-closed set={X, ∅,{c},{d},{a,c},{a,d},{b,c},{c,d},{a,b,c},{b,c,d},{a,c,d}},{a,b,d}}.

Then the set {b,d} is g-closed set but not $(g\alpha r)^{**}$-closed set.

Theorem 3.5:

Every $(g\alpha r)^{**}$-closed set is gα-closed set.

Proof:

Let A be any $(g\alpha r)^{**}$-closed set in X. Let $A \subseteq U$, where U is α-open set. Since every α-open set is $(g\alpha r)^{*}$-open in X. Then U is gαr-open. Therefore $\alpha cl(A) \subseteq cl(A) \subseteq U$. Hence A is gα-closed set.

Theorem 3.6:

Every $(g\alpha r)^{**}$-closed set is αg-closed set.

Proof:

Let A be any $(g\alpha r)^{**}$-closed set in X. Let $A \subseteq U$ and U is open set. Since every open set is $(g\alpha r)^{*}$-open. Then U is $(g\alpha r)^{**}$-open. Therefore $\alpha cl(A) \subseteq cl(A) \subseteq U$. Hence A is αg-closed set.

The converse of above theorem need not be true as shown from the following example.

Example 3.7:

Let X = {a,b,c,d} with τ = {X,∅,{a},{b},{a,b}}. Then the set {b,d} is αg-closed set but not $(g\alpha r)^{**}$-closed set.

Theorem 3.8:

Every $(g\alpha r)^{**}$-closed set is gpr-closed set.

Proof:

Let A be any $(g\alpha r)^{**}$-closed set in X and U be any regular open set containing A. Since every regular open set is $(g\alpha r)^{*}$-open. Then $pcl(A) \subseteq cl(A) \subseteq U$.Therefore $pcl(A) \subseteq U$. Hence A is gpr-closed set.

Theorem 3.9:

Every $(g\alpha r)^{**}$-closed set is g^{*}-closed set.

Proof:

Let A be any $(g\alpha r)^{**}$-closed set in X and U be any g-open set containing A. Since every g-open set is $(g\alpha r)^{*}$-open. Therefore $cl(A) \subseteq U$. Hence A is g^{*}-closed set.

Theorem 3.10:

Every regular closed set is $(g\alpha r)^{**}$-closed set.

Proof:

Let A be any regular closed set in X such that $A \subseteq U$ where U is $(g\alpha r)^{*}$-open. Since A is regular closed $cl(int(A)) = A$, Therefore $cl(A) \subseteq cl(int(A)) = A \subseteq U$.Therefore $cl(A) \subseteq U$. Hence A is $(g\alpha r)^{**}$-closed set in X.

The converse of above theorem need not be true as shown from the following example.

Example 3.11:

Let X = {a,b,c,d} with τ = {X,∅,{a},{b},{a,b}}. Then the set {c} is $(g\alpha r)^{**}$-closed set but not regular closed set.

Theorem 3.12:

Every $(g\alpha r)^{**}$-closed set is gsp-closed set.

Proof:

Let A be any $(g\alpha r)^{**}$-closed set in X and U be any open set containing A. Since every open set is $(g\alpha r)^{*}$-open, $spcl(A) \subseteq cl(A) \subseteq U$.Therefore $spcl(A) \subseteq U$. Hence A is gsp-closed set.

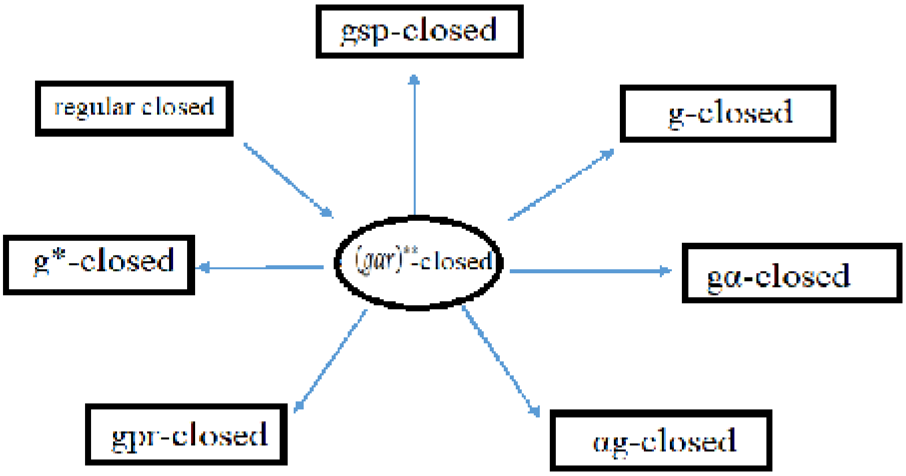
The converse of above theorem need not be true as shown from the following example.

Example 3.13:

Let X = {a,b,c,d} with τ = {X,∅,{a},{b},{a,b}}. Then the set {b} is gsp-closed set but not $(g\alpha r)^{**}$-closed set.

Remark 3.14:

From the above theorem and examples we have the following diagrammatic representation.



Theorem 3.15:

If A and B are $(g\alpha r)^{**}$-closed set in X then A∪B is $(g\alpha r)^{**}$-closed set in X.

Proof:

Let A and B are $(g\alpha r)^{**}$-closed set in X and U be any $(g\alpha r)^{*}$-open set such that A∪B⊆ U. Therefore cl(A) ⊆ U, cl(B) ⊆ U. Hence cl(A∪B) = cl(A) ∪ cl(B) ⊆ U. Therefore A∪B is $(g\alpha r)^{**}$-closed set in X.

Theorem 3.16:

If a set A is $(g\alpha r)^{**}$-closed set then cl(A)−A contains no non empty $(g\alpha r)^{**}$-closed set.

Proof:

Let F be $(g\alpha r)^{*}$-closed set in X. Such that F ⊆ cl(A)−A. F⊆ cl(A)∩ A^c ⇒F ⊆ cl(A) and F⊆ A^c, A⊆ F^c. Then A ⊆ X−F. Since A is $(g\alpha r)^{**}$-closed set and X−F is $(g\alpha r)^{*}$-open then cl(A) ⊆ X−F. (i.e.) F⊆ X−cl(A). So F⊆(X−cl(A))∩(cl(A)-A). Therefore, F=∅.

Theorem 3.17:

If B is $(g\alpha r)^{**}$-closed set and B ⊆ A ⊆ cl(B) then A is $(g\alpha r)^{**}$-closed.

Proof:

Let B be $(g\alpha r)^{**}$-closed and O be any $(g\alpha r)^{*}$-open set such that A⊆ O. Then B⊆ O which implies cl(A)⊆ cl(B) ⊆ O. Hence A is $(g\alpha r)^{**}$-closed.

Theorem 3.18:

A is any $(g\alpha r)^{**}$-open set if and only if B⊆ int (A) where B is $(g\alpha r)^{**}$-closed and B ⊆ A.

Proof: Let A be any $(g\alpha r)^{**}$-open set. Let B be $(g\alpha r)^{*}$-closed and B⊆A. Then A^c ⊆ B^c which implies cl(A^c) ⊆ B^c. Since A^c is $(g\alpha r)^{**}$-closed set and B^c is $(g\alpha r)^{*}$-open. Therefore we have B⊆ int (A). Conversely, assume that B⊆ int (A). Whenever B is $(g\alpha r)^{**}$-closed and B⊆A. Let O be any $(g\alpha r)^{*}$-open. The O^c is $(g\alpha r)^{**}$-closed. Therefore by assumption, O^c ⊆ int (A) which implies cl(A^c) ⊆ O. Hence A is $(g\alpha r)^{**}$-open.

Theorem 3.19:

If int(A)⊆B ⊆A and A is $(g\alpha r)^{**}$-open then B is $(g\alpha r)^{**}$-open.

Proof:

int(A)⊆B ⊆A implies A^c ⊆ B^c ⊆ cl(A^c). Since A is $(g\alpha r)^{**}$-open, A^c $(g\alpha r)^{**}$-closed. Therefore by theorem 3.17, B^c is $(g\alpha r)^{**}$-closed. Hence B is $(g\alpha r)^{**}$-open.

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