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Improved Correlation Coefficients of Intuitionistic Fuzzy Sets for MADM

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Abstract: In this paper we have studied the improved correlation coefficients of intuitionistic fuzzy sets and investigate its properties. Further we have applied this concept in multiple attribute decision making methods with intuitionistic fuzzy environment. Finally we illustrated an example in the above proposed method to the multiple attribute decision making problems.

Key Words – Fuzzy Sets, Intuitionistic Fuzzy Sets, Improved Correlation Coefficients.

I. INTRODUCTION

Fuzzy sets were introduced by Zadeh [1] in 1965 which allows the membership function valued in the interval [0,1] and also it is an extension of classical set theory. Fuzzy set helps to deal the concept of uncertainty, vagueness and imprecision which is not possible in the cantorian set. As an extension of Zadeh's fuzzy set theory intuitionistic fuzzy set (IFS) was introduced by Atanassov [2] in 1986, which consists of degree of membership and degree of non membership and lies in the interval of [0,1]. IFS theory widely used in the areas of logic programming, decision making problems, medical diagnosis etc.

Florentin Smarandache introduced the concept of Neutrosophic set in 1995 which provides the knowledge of neutral thought by introducing the new factor called indeterminacy in the set. Therefore neutrosophic set was framed and it includes the components of truth membership function (T), indeterminacy membership function (I), and falsity membership function (F) respectively. Neutrosophic sets deals with non standard interval of]−01+[. Since neutrosophic set deals the indeterminacy effectively it plays an vital role in many applications areas include information technology, decision support system, relational database systems, medical diagnosis, multicriteria decision making problems etc.,

To deal the real world problems, Wang [14] (2010) introduced the concept of single valued neutrosophic sets (SVNS) which is also known as an extension of intuitionistic fuzzy sets and it became a very new hot research topic now. Rajashi Chatterjee.,et al proposed the concept of Quadripartitioned single valued neutrosophic sets which is based on Belnap's four valued logic and Smarandache's four numerical valued logic. In (QSVNS) indeterminacy is splitted into two functions known as 'Contradicition' (both true and false) and 'Unknown' (neither true nor false) so that QSVNS has four components T,C,U,F which also lies in the non standard unit interval $]^{-}0\ 1^{+}[$. Further, R. Radha and A. Stanis Arul Mary [10] defined a new hybrid model of Quadripartitioned Neutrosophic Pythagorean Sets in 2021.

Correlation coefficient is a effective mathematical tool to measure the strength of the relationship between two variables. So many researchers pay the attention to the concept of various correlation coefficients of the different sets like fuzzy set, IFS, SVNS, QSVNS. In 1999 D.A Chiang and N.P.L [4] in proposed the correlation of fuzzy sets under fuzzy environment. Later D.H.Hong [5] (2006) defined fuzzy measures for a correlation coefficient of fuzzy numbers under Tw (the weakest t-norm) based fuzzy arithmetic operations. Correlation coefficients plays an important role in many real world problems like multiple attribute group decision making, clustering analysis, pattern recognition, medical diagnosis etc., Jun Ye defined the improved correlation coefficients of single valued neutrosophic sets andinterval neutrosophic sets for multiple attribute decision making to overcome the drawbacks of the correlation coefficients of single valued neutrosophic sets (SVNSs).

In this paper, We have discussed some of its properties and decision making method using the improved correlation coefficient with intuitionistic fuzzy environment. Additionaly, an illustrative example is given in above proposed correlation method particularly in multiple criteria decision making problems.

II. PRELIMINARIES

Definition 2.1

Let X be a universe. An Intuitionistic fuzzy set A in X is defined as object of following form:

$$A = \{\langle x, M_A(x), N_A(x)\rangle : x \in X\}$$

Where $M_A: X \to [0,1]$, $N_A: X \to [0,1]$ define the degree of membership and degree of non-membership of element $x \in X$ respectively.

$$0 \leq M_A(x) + N_A(x) \leq 1 \text{ for any } x \in X$$

Here, $M_A(x)$ and $N_A(x)$ is the degree of membership and non-membership of the element of x respectively.

Definition 2.2

Let X be universe set. Then a Pythagorean fuzzy set A which is set of ordered pairs over X

$$A = \{\langle x, M_A(x), N_A(x)\rangle : x \in X\}$$

Where $M_A: X \to [0,1]$, $N_A: X \to [0,1]$ denote the degree of membership and degree of non-membership of element $x \in X$ to the set A which is a subset of X and

$$0 \leq (M_A(x))^2 + (N_A(x))^2 \leq 1 \text{ for any } x \in X$$

$M_A(x)$ and $N_A(x)$ is the degree of membership and non-membership of the element of x respectively.

Definition 2.3

Let A and B be Intuitionistic fuzzy sets in a topological space X of the form $A = \{\langle x, M_A(x), N_A(x)\rangle : x \in X\}$, $B = \{\langle x, M_B(x), N_B(x)\rangle : x \in X\}$

$$A \cup B = \{x, \max(M_A(x), M_B(x)), \min(N_A(x), N_B(x)) | x \in X\}$$
$$A \cap B = \{x, \min(M_A(x), M_B(x)), \max(N_A(x), N_B(x)) | x \in X\}$$
$$A^C = \{(x, N_A(x), M_A(x)) | x \in X\}$$

III. IMPROVED CORRELATION COEFFICIENTS

Definition 3.1

Let P and Q be any two Intuitionistic fuzzy sets in the universe of discourse $R = \{r_1, r_2, r_3, \ldots, r_n\}$, then the improved correlation coefficient between P and Q is defined as follows

$$K(P,Q) = \frac{1}{2n}\sum_{k=1}^{n}[\lambda_k (1 - \Delta M_k) + \mu_k(1 - \Delta N_k)] \quad (1)$$

Where,

$$\lambda_k = \frac{1 - \Delta M_k - \Delta M_{max}}{1 - \Delta M_{min} - \Delta M_{max}},$$
$$\mu_k = \frac{1 - \Delta N_k - \Delta N_{max}}{1 - \Delta N_{min} - \Delta N_{max}},$$
$$\Delta M_k = |M_P(r_k) - M_Q(r_k)|,$$
$$\Delta N_k = |N_P(r_k) - N_Q(r_k)|,$$
$$\Delta M_{min} = \min_k |M_P(r_k) - M_Q(r_k)|,$$
$$\Delta N_{min} = \min_k |N_P(r_k) - N_Q(r_k)|,$$
$$\Delta M_{max} = \max_k |M_P(r_k) - M_Q(r_k)|,$$
$$\Delta N_{max} = \max_k |N_P(r_k) - N_Q(r_k)|,$$

For any $r_k \in R$ and $k = 1,2,3,\ldots,n$.

Theorem 3.2

For any two Intuitionistic fuzzy sets P and Q in the universe of discourse $R = \{r_1, r_2, r_3, \ldots, r_n\}$, the improved correlation coefficient K(P,Q) satisfies the following properties.

(i) $K(P,Q) = K(Q,P)$;
(ii) $0 \leq K(P,Q) \leq 1$;
(iii) $K(P,Q) = 1$ iff $P = Q$.

Proof:

(i) It is obvious and straightforward.

(ii) Here, $0 \leq \lambda_k \leq 1$, $0 \leq \mu_k \leq 1$,
$0 \leq (1 - \Delta M_k) \leq 1$, $0 \leq (1 - \Delta N_k) \leq 1$,
Therefore the following inequation satisfies
$0 \leq \lambda_k(1 - \Delta M_k) + \mu_k(1 - \Delta N_k) \leq 2$.
Hence we have $0 \leq K(P,Q) \leq 1$.

(iii) If $K(P,Q) = 1$, then we get $\lambda_k(1 - \Delta M_k) + \mu_k(1 - \Delta N_k) = 2$
Since $0 \leq \lambda_k(1 - \Delta M_k) \leq 1$, $0 \leq \mu_k(1 - \Delta N_k) \leq 1$,
there are $\lambda_k(1 - \Delta M_k) = 1$, $\mu_k(1 - \Delta N_k) = 1$.
And also since $0 \leq \lambda_k \leq 1$, $0 \leq \mu_k \leq 1$
$0 \leq (1 - \Delta M_k) \leq 1$, $0 \leq (1 - \Delta N_k) \leq 1$.
We get $\lambda_k = \alpha_k = \mu_k = 1$ and
$1 - \Delta M_k = 1 - \Delta N_k = 1$.

This implies, $\Delta M_k = \Delta M_{min} = \Delta M_{max} = 0$, $\Delta N_k = \Delta N_{min} = \Delta N_{max} = 0$.

Hence $M_P(r_k) = M_Q(r_k)$, $N_P(r_k) = N_Q(r_k)$ for any $r_k \in R$ and $k = 1,2,3,\ldots,n$. Hence $P = Q$.

Conversely, assume that $P = Q$, this implies $M_P(r_k) = M_Q(r_k)$, $N_P(r_k) = N_Q(r_k)$ for any $r_k \in R$ and $k = 1,2,3,\ldots,n$.

Thus $\Delta M_k = \Delta M_{min} = \Delta M_{max} = 0$, $\Delta N_k = \Delta N_{min} = \Delta N_{max} = 0$.

Hence we get K(P,Q) = 1.

The improved correlation coefficient formula which is defined is correct and also satisfies these properties in the above theorem .When we use any constant $\varepsilon > 2$ in the following expressions

$$\lambda_k = \frac{\varepsilon-\Delta M_k-\Delta M_{max}}{\varepsilon-\Delta M_{min}-\Delta M_{max}},$$

$$\alpha_k = \frac{\varepsilon-\Delta H_k-\Delta H_{max}}{\varepsilon-\Delta H_{min}-\Delta H_{max}},$$

$$\mu_k = \frac{\varepsilon-\Delta N_k-\Delta N_{max}}{\varepsilon-\Delta N_{min}-\Delta N_{max}}$$

Example 3.3

Let A = {r,0,0} and B = {r,0.4,0.2} be any two Intuitionistic fuzzy sets in R. Therefore by equation (1) we get K(A,B) = 0.7. It shows that the above defined improved correlation coefficient overcome the disadvantages of the correlation coefficient.

In the following, we define a weighted correlation coefficient between Intuitionistic fuzzy sets since the differences in the elements are considered into an account. Let w_k be the weight of each element r_k(k = 1,2,…,n), $w_k \in [0,1]$ and $\sum_{k=1}^{n} w_k=1$, then the weighted correlation coefficient between the Intuitionistic fuzzy sets A and B.

$$K_w(A,B) = \frac{1}{2}\sum_{k=1}^{n} w_k[\lambda_k (1-\Delta M_k) + \mu_k(1-\Delta N_k)] \qquad (2)$$

Theorem 3.4

Let w_k be the weight of each element r_k(k = 1,2,…,n), $w_k \in$ [0,1] and $\sum_{k=1}^{n} w_k=1$, then the weighted correlation coefficient between the Intuitionistic fuzzy sets A and B which is denoted by $K_w(A,B)$ defined in equation (2) satisfies the following properities.

1) $K_w(A,B) = K_w(B,A)$;
2) $0 \le K_w(A,B) \le 1$;
3) $K_w(A,B) = 1$ iff A = B.

It is similar to prove the properities in theorem (3.2).

IV. DECISION MAKING USING THE IMPROVED CORRELATION COEFFICIENT OF INTUITIONISTIC FUZZY SETS

Multiple criteria decision making problems refers to make decisions when several attributes are involved in real-life problem. For example one may buy a dress by analysing the attributes which is given in terms of price, style, safety,comfort etc.,

Here we consider a multiple attribute decision making problem with intuitionistic fuzzy information and the characteristic of an alternative A_i(i=1,2,…,m) on an attribute C_j(j=1,2,…,n) is represented by the following Intuitionistic fuzzy set:

$$A_i = \{C_j, M_{A_i}(C_j), N_{A_i}(C_j) / C_j \in C, j = 1,2,\ldots n\}$$

where, $M_{A_i}(C_j), N_{A_i}(C_j) \in [0,1]$ and

$0 \le M_{A_j}(C_j) + N_{A_j}(C_j) \le 1$

for $C_j \in C$, j = 1,2,…n and i = 1,2,…m.

To make it convenient, we are considering the following two functions $M_{A_i}(C_j), N_{A_i}(C_j)$ in terms of Intuitionistic fuzzy value.

$$d_{ij} = (m_{ij}, n_{ij}) \quad (i = 1,2,\ldots,m;\ j = 1,2,\ldots n)$$

Here the values of d_{ij} are usually derived from the evaluation of an alternative A_i with respect to a criteria C_j by the expert or decision maker. Therefore we got a intuitionistic fuzzy decision matrix $D=(d_{ij})_{m\times n}$.

In the case of ideal alternative A^* an ideal intuitionistic fuzzy sets can be defined by $d_j^* = m_j^*, n_j^* = (1,0)$(j =1,2,…,n) in the decision making method, Hence the weighted correlation coefficient between an alternative A_i(i=1,2,…,m) and the ideal alternative A^* is given by,

$$K_w(A_i,A^*) = \frac{1}{2}\sum_{j=1}^{n} w_j[\lambda_{ij} (1-\Delta m_{ij}) + \mu_{ij}(1-\Delta n_{ij})] \qquad (3)$$

Where,

$$\lambda_{ij} = \frac{1-\Delta m_{ij}-\Delta m_{imax}}{1-\Delta m_{imin}-\Delta m_{imax}},$$

$$\mu_{ij} = \frac{1-\Delta n_{ij}-\Delta n_{imax}}{1-\Delta n_{imin}-\Delta n_{imax}},$$

$$\Delta m_{ij} = |m_{ij} - m_j^*|,$$

$$\Delta n_{ij} = |n_{ij} - n_j^*|,$$

$$\Delta m_{imin} = \min_j|m_{ij} - m_j^*|,$$

$$\Delta n_{imin} = \min_j|n_{ij} - n_j^*|,$$

$$\Delta m_{imax} = \max_j|m_{ij} - m_j^*|,$$

$$\Delta n_{imax} = \max_j|n_{ij} - n_j^*|,$$

For i =1,2,…,m and j = 1,2,…,n.

By using the above weighted correlation coefficient we can derive the ranking order of all alternatives and we can choose the best one among those.

Example 4.1

This section deals the example for the multiple attribute decision making problem with the given alternatives corresponds to the criteria allotted under intuitionistic fuzzy environment. For this example which we will discuss here is about the best mobile phone among all available alternatives based on various criteria. The alternatives A_1, A_2, A_3 respectively denotes the Samsung, Vivo, Redmi. The customer must take a decision according to the following four attributes that is (1) C_1 is the cost (2) C_2 is the storage space (3) C_3 is the camera quality (4) C_4 is the looks. According to this attributes we will derive the ranking order of all alternatives and based on this ranking order customer will select the best one.

The weight vector of the above attributes is given by $w = (0.2, 0.35, 0.25, 0.20)^T$. Here the alternatives are to be evaluated under the above four attributes by the form of Intuitionistic fuzzy sets. In general the evaluation of an alternative A_i with respect to an attribute C_j(i = 1,2,3; j =1,2,3,4) will be done by the questionnaire of a domain expert. In particularly, while asking the opinion about an alternative A_1 with respect to an attribute C_1, the possibility he (or) she say that the statement true is 0.2 and the statement false is 0.5. It can be denoted in intuitionistic notation as d_{11}= (0.2, 0.5). Continuing this procedure for all three alternatives with respect to four attributes we will get the following intuitionistic fuzzy decision value table.

A_iC_j	C_1	C_2	C_3	C_4
A_1	[0.2,0.5]	[0.1,0.2]	[0.3,0.2]	[0.4,0.1]
A_2	[0.1,0.2]	[0.3,.0.5]	[0.8,0.1]	[0.3,0.4]
A_3	[0.2,0.4]	[0.2,0.7]	[0.6,0.1]	[0.4,0.1]

Then by using the proposed method we will obtain the most desirable alternative. We can get the values of the correlation coefficient K_w (A_i, A^*)(i= 1,2,3) by using Equation(3).Hence K_w (A_1,A^*) = 0.411, K_w(A_2,A^*) = 0.8625, K_w(A_3,A^*) = 0.3875. Therefore the ranking order is, $A_2 > A_1 > A_3$.The alternative A_2 (Vivo) Mobile phone is the best choice among all the three alternatives.

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