



Effect of Partial Slip and Radiation on MHD Nanofluid over an Exponentially Stretching Sheet Embedded in Double Stratified Medium

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Abstract: In this paper, we presented MHD boundary layer nanofluid flow and heat transfer towards an exponentially stretching sheet embedded in double stratified medium. Using similarity transformation technique, the governing system of partial differential equations is transformed into a system of ordinary differential equations and are solved numerically by using the well-known implicit finite difference scheme known as the Keller Box method. The velocity, temperature and nanoparticle volume fraction profiles are expressed graphically for different flow pertinent parameters, namely the Magnetic parameter (M), Radiation parameter (R), Thermal stratification parameter (St), Solutal stratification parameter (Sd), Suction parameter (S), Prandtl number(Pr), Eckert number(Ec), Lewis number(Le), Thermophoresis parameter (Nt), Brownian parameter (Nb). Moreover comparative study between the previously results and the present study is made and they show good agreement.

Index Terms - Nanofluid, MHD, Slip effect, Stratified medium.

I. INTRODUCTION

The study of Magnetohydrodynamics (MHD) plays an important role in agriculture, engineering and petroleum industries. MHD has won practical applications, for instance, it may be used to deal with problems such as cooling of nuclear reactors by liquid sodium and induction flow water which depends on the potential differencing the fluid direction perpendicular to the motion goes to the magnetic field. A nanofluid is a fluid containing nano meter-sized particles, called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid. The nanoparticles used in nanofluid are typically made of metals, oxides, carbides, or carbon nanotubes. Common base fluids include water, ethylene glycol and oil. Nanofluids have novel properties that make them potentially useful in many applications in heat transfer, including microelectronics, fuel cells, pharmaceutical processes, and hybrid-powered engines, engine cooling/vehicle thermal management, domestic refrigerator, chiller, heat exchanger, in grinding, machining

and in boiler flue gas temperature reduction. They exhibit enhanced thermal conductivity and the convective heat transfer coefficient compared to the base fluid. The term Nanofluid was first introduced by Choi¹. Nanofluids are the suspensions of various nano particles in the base fluids which in turn evolved as a challenge for the scientist in thermal sciences². They find wide range of applications in heat transfer and a deep research is under progress at various research institutions to enhance the heat transfer mechanism. The nanofluid synthesized by using chemical solutions have high conductivity enhancement and excellent stability as compared with other methods. T. Hayat et al.³ solved with the use of convergent series by the Homotopy analysis method the MHD effects on a thermo-Solutal stratified nanofluid flow on an exponentially radiating stretching sheet. P. Loganathan and C.Vimala⁴ was studied MHD flow of nanofluid over an exponentially stretching sheet embedded in a stratified medium with suction and radiation effects by using shooting technique. Nanofluids has many applications are primarily used as coolant in heat transfer equipment such as heat exchangers, electronic cooling system (such as flat plate) and radiators. the concept of Brownian motion and thermophoresis for horizontal stretching surface. Currently, an extensive literature can be found for both Newtonian and non-Newtonian fluid with various physical models in the presence of nanoparticle.

The boundary layer flow on a continuous stretching sheet has attracted considerable attention during the last few decades due to its numerous applications in engineering and industrial manufacturing processes such as metal spinning, hot rolling and polymer extrusion. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products. Crane⁵ investigated a flow past a stretching sheet whose velocity is proportional to the distance from the slit. This occurs in the drawing of plastic film. Magyari and Keller⁶ extended the work of Crane by assuming stretching sheet need not be linear but it can be is of exponential and they solved this situation by using the both analytical and numerical methods. Radiation and mass transfer Radiation effects on MHD boundary Layer flow due to an exponentially Stretching Sheet with Heat Source studied by G Vijaya Lakshmi et.al⁷.

The effect of stratification is an important aspect in heat and mass transfer. Thermal stratification may arise when there is a continuous discharge of the thermal boundary layer into the medium, for example, the thermal stratification of lakes refers to a change in the temperature at different depths in the lake, and is due to the change in water's density with temperature. This phenomenon occurs due to a change in temperature or concentration, or variations in both, or due to the presence of fluids with different densities. M Swathi⁸ investigated on MHD flow of a viscous fluid induced by an exponentially stretching sheet in a thermally stratified medium and the numerical solutions are obtained by using shooting method.

Later this work was extended by G Vijaya Lakshmi et.al⁹ by investigating the effects of Joule heating and viscous dissipation on MHD boundary layer flow of an exponentially stretching sheet embedded in thermally stratified medium and the results are obtained by using implicit finite difference method. Rizwan Ul Haq et al¹⁰ studied mixed convection flow of thermally stratified MHD nanofluid over an exponentially stretching surface with viscous dissipation effect by using Finite difference method. The flow direction is towards the sheet, and then it is a stagnation flow phenomenon. In fluid dynamics, Stagnation points exist at the surface of objects in the flow field, where the fluid is brought to rest by the object. Hiemenz¹¹ was first studied about

stagnation point flow. Ibrahim et al¹² studied MHD Stagnation Point Flow and Heat Transfer due to Nanofluid Towards a Stretching Sheet and numerically solved using fourth order Runge–Kutta method along with shooting technique. G Vijaya Lakshmi, L Anand Babu ,K Srinivasa Rao¹³ investigated the chemical reaction effect on Heat and mass transfer in MHD stagnation point nanofluid flow over stretching sheet in porous medium with and prescribed surface heat flux by sing finite difference method.

The no-slip condition for viscous fluids assumes that at a solid boundary, the fluid will have zero velocity relative to the boundary interface. But in the existence of slip-flow, the flow velocity at the solid wall is non-zero. The fluids that exhibit the boundary slip have important technological applications such as in the polishing of the artificial heart valves and internal cavities. Partial slips occur for fluid with particulate such as emulsion suspensions, foams and polymer solutions. Swathi M¹⁴investigated the slip effects on an MHD boundary layer flow past an exponentially stretching sheet with suction/blowing and thermal radiation solved by shooting method. MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal and Solutal slip boundary conditions was investigated by Wubshet Ibrahim and Bandari Shankar¹⁵ numerically solved using fourth order Runge–Kutta method along with shooting technique. Effects of slip and heat generation/absorption on MHD stagnation flow of nanofluid past a stretching/shrinking surface with convective boundary conditions studied by Samir Kumar Nandy and Tapas Ray Mahapatra B¹⁶.MHD partial slip flow of Ag-water nanofluid over a stretching sheet was studied by Preeti Agarwala R Khare¹⁷.Effects of MHD on Boundary Layer Flow in Porous Medium due to Exponentially Shrinking Sheet with Slip conditions explained by Shalini Jain and Rakesh Choudhary¹⁸.

The objective of this present work is to extend the flow and heat transfer analysis in boundary layer over an exponentially stretching sheet embedded in a stratified medium. The present study is to extend the work of Khan and Pop¹⁹ for the effect of partial slip on MHD stagnation flow of Nanofluid over an exponentially stretching sheet embedded in temperature and concentration stratified medium. Using suitable similarity transformations, a third order ordinary differential equation corresponding to the momentum equation and a second order differential equation corresponding to the heat equation and diffusion equations are derived. Numerical calculations up to desired level of accuracy were carried out for different values of dimensionless parameters of the problem under consideration for the purpose of illustrating the results graphically. The analysis of the results obtained shows that the flow field is influenced appreciably by the stratification parameter, stagnation point and slip conditions. Estimation of heat transfer coefficient which is very important from the industrial application point of view is also presented in this analysis. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies. Stagnation-Point Flow by an Exponentially Stretching Sheet in the Presence of Viscous Dissipation and Thermal Radiation studied by Z. Iqbal et .al²⁰.Mixed convection flow of thermally stratified MHD nanofluid over exponentially stretching sheet using finite difference method was explained by Prabhakar et al²¹.MHD effects on a Thermo-Solutal Stratified Nanofluid Flow on an exponentially radiating stretching sheet was explained by T. Hayat et.al²²

2. FLOW ANALYSIS AND MATHEMATICAL FORMULATION:

Consider the steady nanofluid flow of an incompressible viscous electrically conducting fluid over an exponentially stretching sheet coinciding with the plane $y = 0$. The flow is confined to $y > 0$. Two equal and opposite forces are applied along the x -axis, so that the wall is stretched keeping the origin fixed. A variable magnetic field $B(x) = B_0 e^{\frac{x}{2L}}$ is applied normal to the sheet and B_0 is a constant. Buoyancy forces are also considered for thermal and concentration to deal double stratified phenomena. The sheet is of temperature $T_w(x)$ and is embedded in a thermally stratified medium of variable ambient temperature $T_\infty(x)$ where $T_w(x) > T_\infty(x)$. It is assumed that $T_w(x) = T_0 + a e^{\frac{x}{2L}}$, $C_w(x) = C_0 + b e^{\frac{x}{2L}}$, $T_\infty(x) = T_0 + c e^{\frac{x}{2L}}$ and $C_\infty(x) = C_0 + d e^{\frac{x}{2L}}$, where a, b, c and d are constants respectively ($a \geq 0, b \geq 0, c \geq 0$ and $d \geq 0$).

The continuity, momentum, energy and concentration equations of governing such type of flow are written

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho_f} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{(\rho c_p)_p}{(\rho c_p)_f} \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

Where velocity components are given by u and v in x and y directions. Kinematic viscosity is given by ν , electric conductivity is σ , magnetic induction is by B , fluid density is ρ , temperature of the fluid is T , thermal conductivity of the fluid is k , specific heat at constant pressure is C_p , coefficient of fluid viscosity is μ , radiative heat flux is q_r .

The boundary conditions of the problem is given by

$$u = U_w = U_0 e^{\frac{x}{2L}} + \alpha_0 \frac{\partial u}{\partial y}, v = -V(x), T = T_w(x) = T_0 + a e^{\frac{x}{2L}}, C = C_w(x) = C_0 + b e^{\frac{x}{2L}} \text{ at } y = 0 \quad (5)$$

$$u \rightarrow U_e = U_\infty e^{\frac{x}{2L}}, v \rightarrow 0, T \rightarrow T_\infty(x) = T_0 + c e^{\frac{x}{2L}}, C \rightarrow C_\infty(x) = C_0 + d e^{\frac{x}{2L}} \text{ as } y \rightarrow \infty \quad (6)$$

here, stretching velocity is $U(x) = U_0 e^{\frac{x}{2L}}$, reference velocity U_0 , velocity of suction is $V(x) > 0$ and velocity of blowing $V(x) < 0$, a special type of velocity at the wall is considered as $V(x) = V_0 e^{\frac{x}{2L}}$ and the initial strength of suction is V_0 .

Introducing the suitable transformations as

$$\eta = \left(\frac{U_0}{2\nu L} \right)^{\frac{1}{2}} e^{\frac{x}{2L}} y, u = U_0 e^{\frac{x}{2L}} f'(\eta),$$

$$v = - \left(\frac{\nu U_0}{2L} \right)^{\frac{1}{2}} e^{\frac{x}{2L}} (f(\eta) + \eta f'(\eta)), \frac{T - T_\infty}{T_w - T_0} = \theta(\eta), \frac{C - C_\infty}{C_w - C_0} = \varphi(\eta) \quad (7)$$

Substituting the Eq. (7) in Equations (2) - (5) the governing equations are transformed to

$$f''' + f f'' - 2f'^2 - 2M f' = 0 \quad (8)$$

$$(1 + 4/3R)\theta'' + Pr f \theta' - Pr f' \theta - Pr St f' + Pr Nb \theta' \varphi' + Pr Nt \theta'^2 + Pr Ec (f'')^2 = 0 \quad (9)$$

$$\varphi'' + Le(Pr f \varphi' - Pr f' \varphi - Pr Sd f') + \frac{Nt}{Nb} \theta'' = 0 \quad (10)$$

and the boundary conditions take the following form:

$$f = S, f' = 1 + \lambda f'', \theta = 1 - St, \varphi = 1 - Sd \text{ at } \eta = 0$$

$$f' \rightarrow 0, \theta = \varphi = 0 \text{ at } \eta \rightarrow \infty \tag{11}$$

Where the prime denotes differentiation with respect to η , $M = \sqrt{\frac{2 \sigma B_0^2 L}{\rho U_0}}$ is the magnetic parameter, $Ec = \frac{U_0^2}{(T_w - T_0) c_p}$ is the Eckert Number, $S = \frac{V_0}{\sqrt{\frac{U_0 v}{2L}}}$ (or < 0) is the suction (or blowing) parameter, Prandtl number is $Pr = \frac{\mu c_p}{k}$, thermal stratification parameter is $St = \frac{c}{a}$, concentration stratification parameter is $Sd = d/b$, Stably stratified environment is for $Sm > 0$ and if $Sm = 0$ corresponds to an unstratified environment, $Le = \frac{v}{D_B}$ is the Lewis number, the Brownian motion parameter is $Nb = D_B \tau \frac{(c_w - c_\infty)}{v}$, the thermophoresis parameter is $Nt = \frac{D_T}{T_\infty} \tau \frac{(T_w - T_\infty)}{v}$, $\tau = \frac{(\rho c_p)_p}{(\rho c_p)_f}$, $\lambda = L \sqrt{\frac{U_0}{2L}}$

Table I. Comparison of the values of $-\theta'(0)$ for different values of physical parameters Pr and M when $Nb=Nt=St=Sd=Ec=0$

M	Pr	Magyari and Keller ⁶	Bidin, Nazar ²³	Ishak ¹⁹	Present Study
0	1	0.954782	0.9548	0.9548	0.9546
0	2		1.4714		1.4714
0	3	1.869075	1.8691	1.8691	0.8687
0	5	2.500135		2.5001	2.4997
0	10	3.660379		3.6604	3.6604
1	1			0.8611	0.7911

Table II. Computation of values for various parameters where the fixed values $Pr=0.7, Le=M=S=R=0.5$

St	Sd	Nt	Nb	Ec	S	λ	$-\theta'(0)$	$-\varphi'(0)$
0.3	0.3	0.3	0.3	0.1	0.3	0.3	0.6185	0.9873
0.5	0.3	0.3	0.3	0.1	0.3	0.3	0.5456	0.9561
0.7	0.3	0.3	0.3	0.1	0.3	0.3	0.4619	0.9437
0.7	0.5	0.3	0.3	0.	0.3	0.3	0.4472	0.7636
0.7	0.7	0.3	0.3	0.1	0.3	0.3	0.4339	0.7198
0.7	0.7	0.5	0.3	0.1	0.1	0.1	0.4368	0.5896
0.7	0.7	0.7	0.7	0.1	0.1	0.1	0.4394	0.3384
0.7	0.7	0.7	0.3	0.2	0.1	0.1	0.4182	0.6119
0.7	0.7	0.7	0.5	0.3	0.1	0.2	0.3548	0.8003
0.7	0.7	0.7	0.3	0.3	0.1	0.3	0.2634	0.6384
0.7	0.7	0.7	0.3	0.3	0.2	0.1	0.2993	0.7119
0.7	0.7	0.7	0.5	0.3	0.3	0.1	0.3548	0.8003

3. NUMERICAL METHOD:

The nonlinear ordinary differential Equations (8)-(10) together with boundary conditions (11) are solved numerically by Keller box method as mentioned by Cebeci and Bradshaw.²⁴

According to Vajravelu et al,²⁵ to obtain the numerical solutions, the following steps are considered in this method.

- Reduce the ordinary differential equations to a system of first order equations.
- Write the difference equations for ordinary differential equations using central differences.
- Linearize the algebraic equations by Newton's method, and write them in matrix vector form.
- Solve the linear system by the block tri-diagonal elimination technique.

4. RESULTS AND DISCUSSION:

From the Figures 2 (a), 2(b) and 2(c) we observed that the effect of magnetic parameter (M) on the velocity profile and is reduced with the increasing value of M due to Lorentz force. But in the case of the temperature and concentration profiles, as increases magnetic field increases the temperature and concentration profiles.

In Figure 3, the relationship between Kinematic energy (K.E) and enthalpy is nothing but Eckert number Ec and it is clear that as increasing the values of Ec in turn increases temperature profile.

From Figure 4, the Prandtl number (Pr) which is the ratio of the momentum diffusivity and the thermal diffusivity of base fluid. For the increasing values of Pr the temperature profile and thermal boundary layer thickness will decreases quickly and there will be higher heat transfer effects. Thus the base fluid plays an important role in the heat transfer of the nanofluid.

Figure 5(a) and 5(b) denotes the effect of Brownian motion parameter (Nb) on temperature profile and concentration profile. Brownian motion depends on the size of nanoparticle. As Brownian motion parameter (Nb) increases the temperate profile increases and it is clear that the increasing value of Nb decreases the nanoparticle volume fraction in turn it also decreases the nanoparticle volume boundary layer thickness.

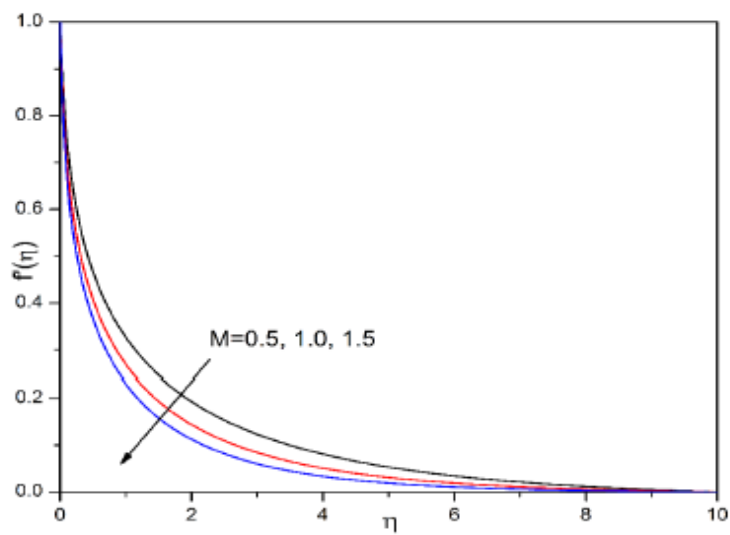


Fig 2(a) Effect of magnetic parameter (M) on velocity profile

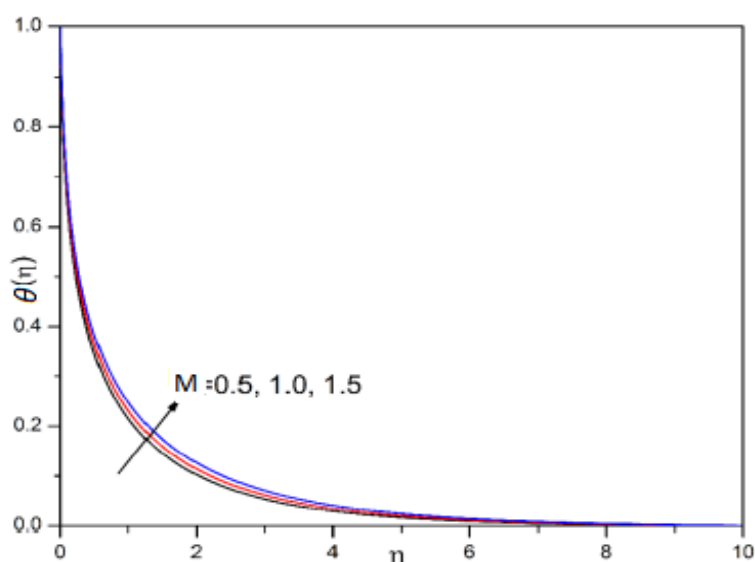


Fig 2 (b) Effect of magnetic parameter (M) on tempuratre profile.

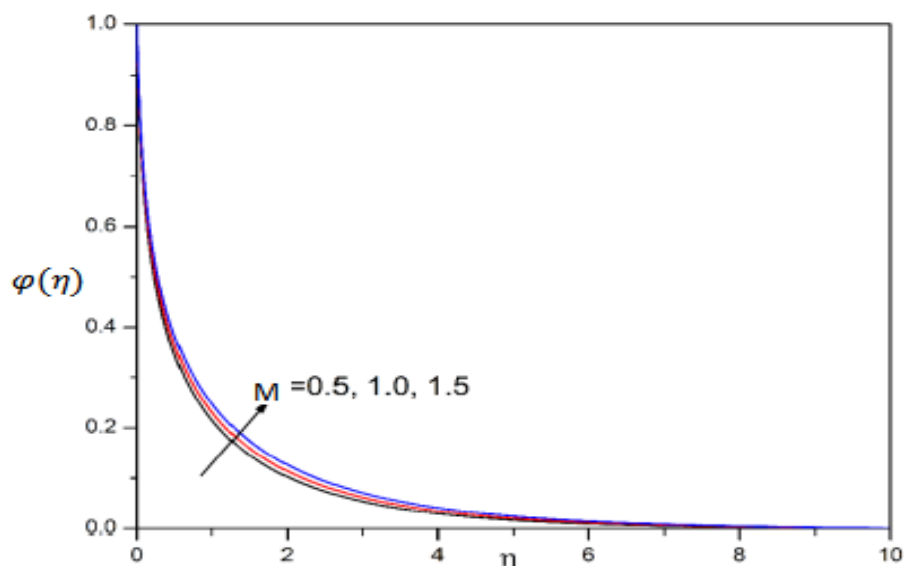


Fig 2(c) Effect of magnetic parameter (M) on concentartion profile.

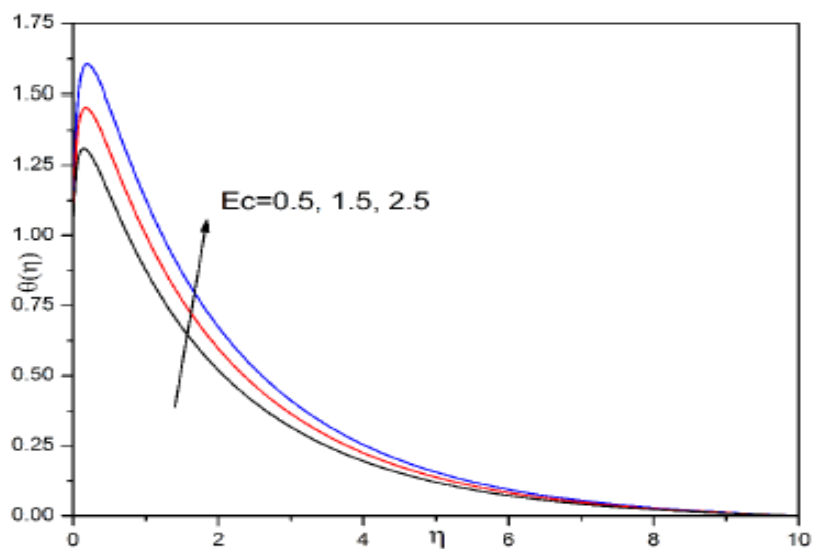


Fig. 3. Effect of Eckert parameter (E_c) on temperature profile

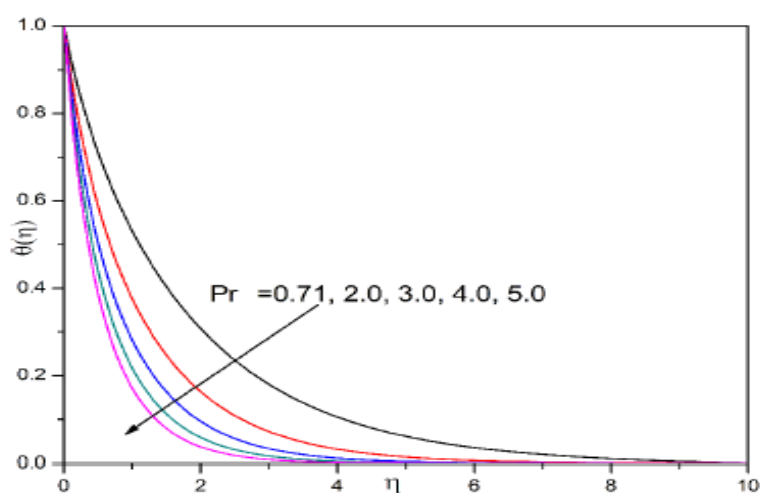


Fig. 4. Effect of Prandtl number (Pr) on temperature profile

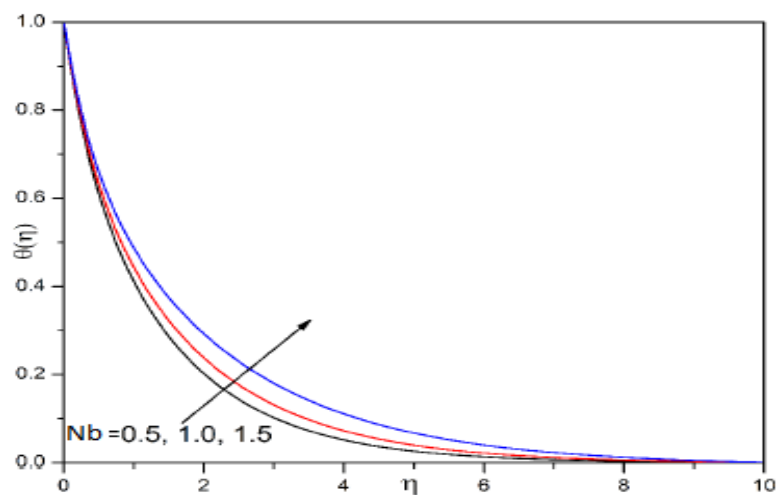


Fig.5. (a) Effect of Brownian motion parameter (N_b) on temperature profile.

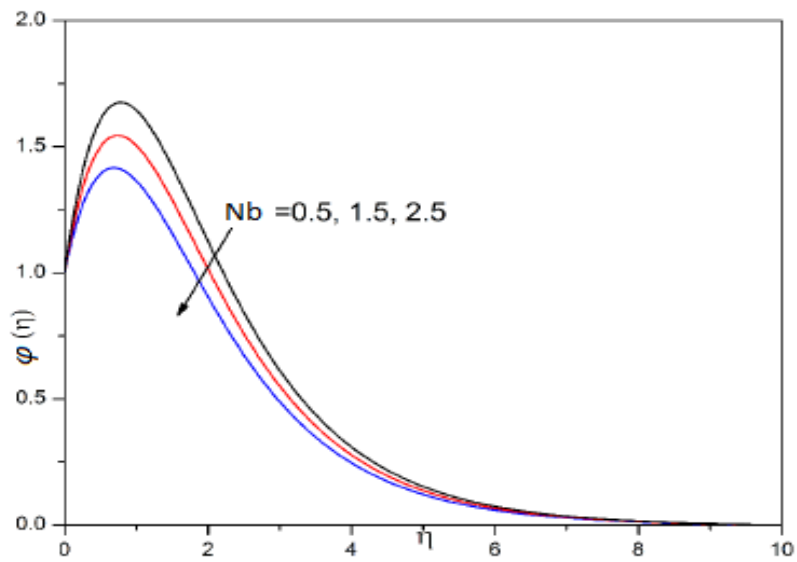


Fig 5 (b) Effect of Brownian motion parameter (Nb) on concentration profile.

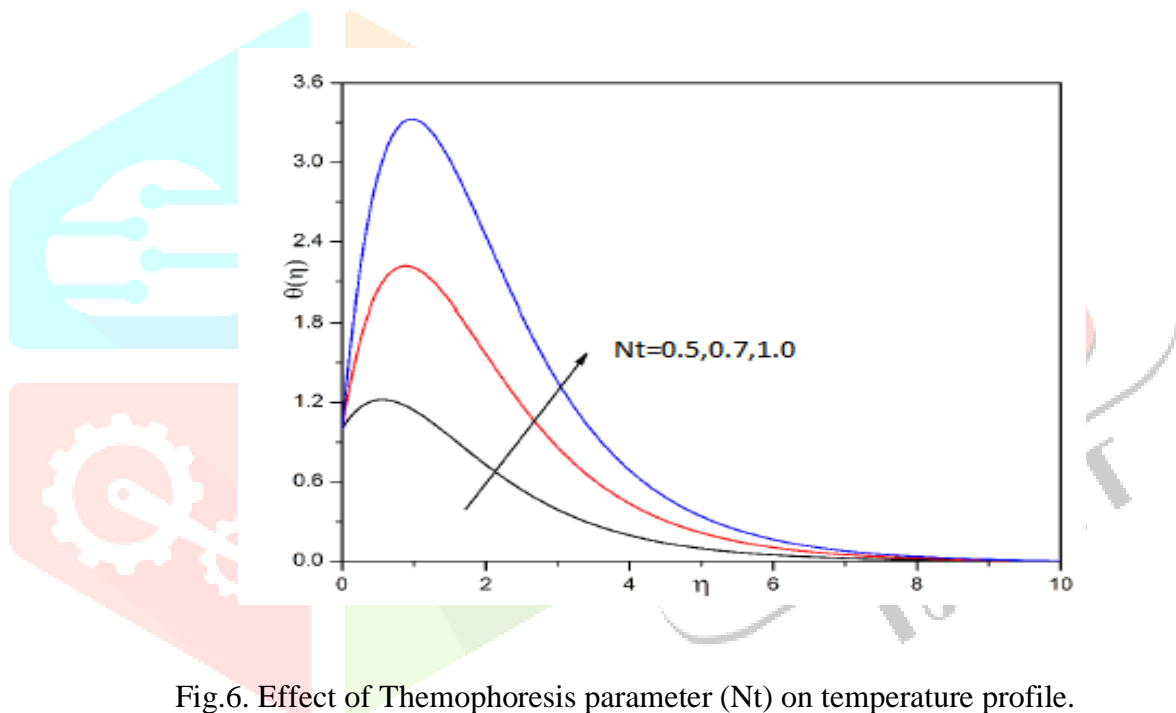


Fig.6. Effect of Themophoresis parameter (Nt) on temperature profile.

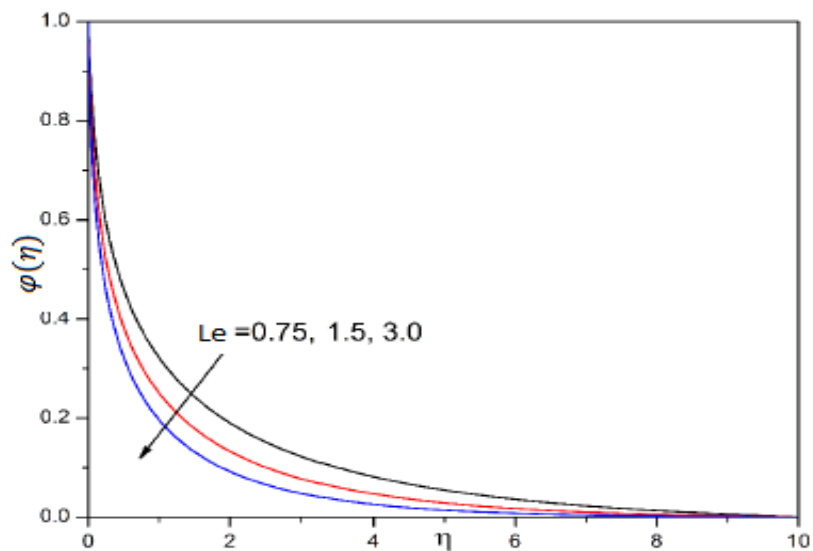


Fig.7. Effect of Lewis (Le) on concentration profile.

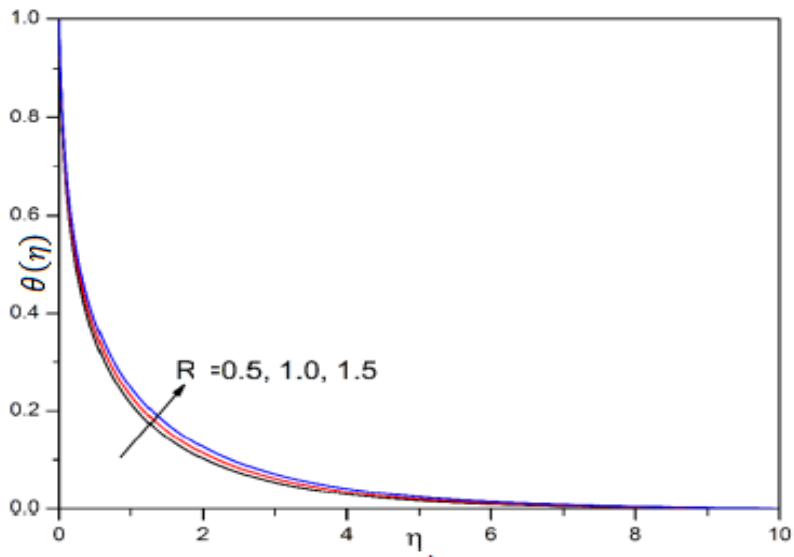


Fig.8 Effect of Radiation parameter (R) on temperature profile

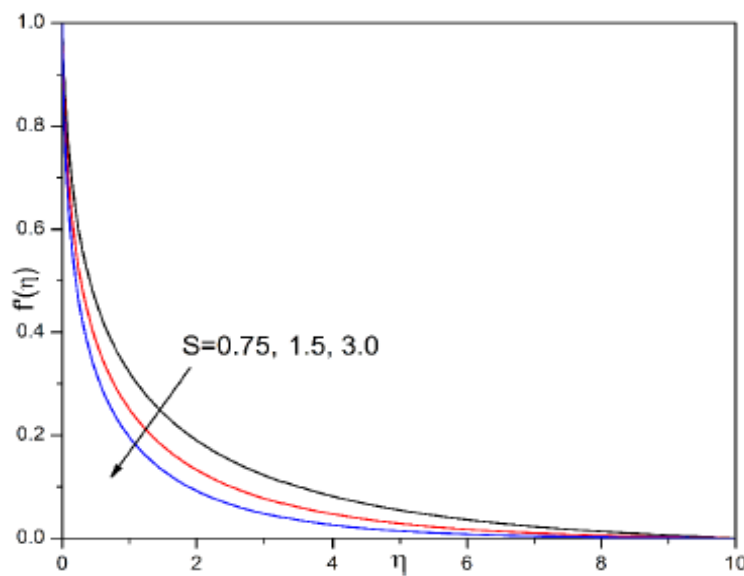


Fig.9(a) Effect of Suction parameter (S) on velocity profile.

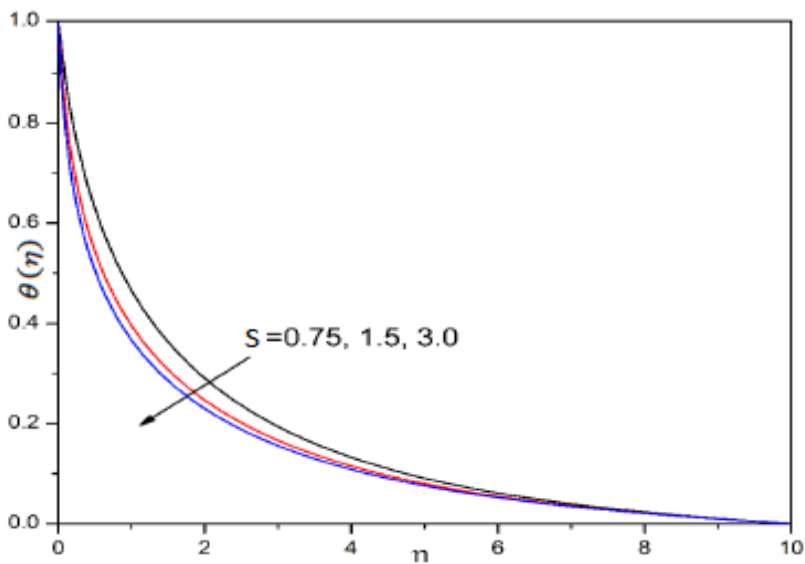


Fig 9(b) Effect of Suction parameter (S) on temperature profile.

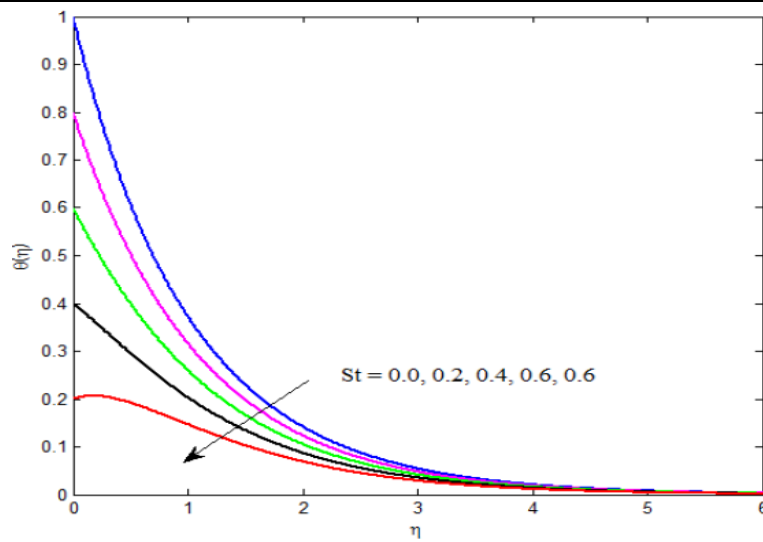


Fig 10 Effect of thermal stratification parameter (St) on temperature profile

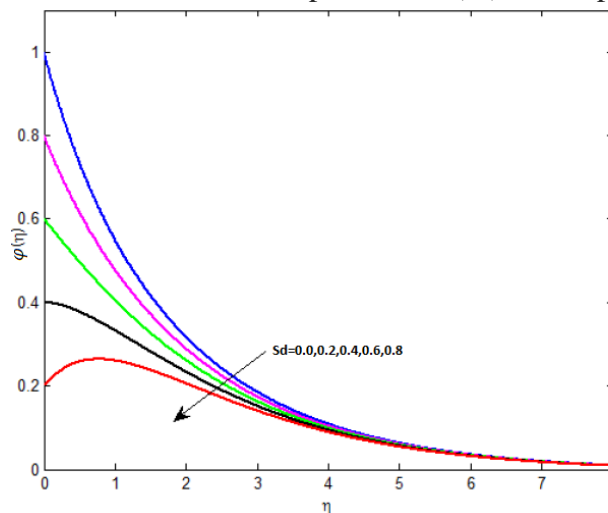


Fig.11 Effect of Solutal stratification parameter (Sd) concentration profile.

The effect of thermophoresis parameter (Nt) was studied on the temperature parameter $\theta(\eta)$ and is shown in Figure 6. Increasing the value of thermophoresis parameter increases the temperature distribution. From Figure 7 it is clear that with the increase in the value of Lewis number Le decreases concentration profile and the thickness of the nanoparticle volume boundary layer. Figure 8 describes the Radiation effect (R) on temperature profile as radiation parameter increases the temperature profile increases.

As shown in the Figures 9(a) & 9(b) as velocity and temperature profile increases the influence of wall transpiration (i.e. suction / injection) parameter (S) decreases.

In Figure 10 and 11 the thermal stratification and Solutal stratified parameter have opposite effects on Nusselt and Sherwood number. As Thermal stratified number St increases the Nusselt number behaviour decreases where as Solutal stratified parameter Sd increases the Sherwood number behaviour decreases.

Table II illustrates the variations of Nusselt number $-\theta'(0)$ and Sherwood number $-\phi'(0)$

With variable values of St , Sd , Nt , Nb , Ec , S , λ . From the table it is clear that increases in the values of the partial slip parameter reduces the Nusselt number. We can conclude that Sherwood number is reduced with increasing the values of Le , Nb and R

5. CONCLUSIONS:

The following conclusions are drawn.

- As increasing magnetic parameter M , velocity profile decreases but in the case of temperate and concentration profile it increases.
- Slip parameter of velocity increases which in turn enhances the velocity profile and reduces the concentration profile.
- As increasing the thermal and concentration stratification parameters there is reduction in the temperature and concentration profile too.

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