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NON-SPONTANEOUS INVENTORY WITH EXPONENTIAL HOLDING COST UNDER INFLATION

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Abstract: This study evaluates the effect of inflation on an inventory model for non-instantaneous degenerating products with algebraic time sensitive demand and exponentially time-based holding cost under delay in payments. Shortages are tolerated and partially accumulated. The accumulation rate is an exponential function of the waiting duration for the renewal of inventory cycle. Mathematical formulation is framed to find optimal ordering policies to get the uttermost profit and optimal order quantity. This framework is divided under three cases based on the interval of non-decay, the duration on initial inventory consumption, trade credit period and inventory cycle span. The major purpose of this research is to get the maximum profit and to find the ideal total inventory. The findings of this model are illustrated by sensitivity investigation of parameters and demonstrated by numerical instances.

Index Terms: inflation, inventory model, non-instantaneous degenerating products, algebraic time sensitive demand, exponentially time-based holding cost, delay in payments.

I. INTRODUCTION

In a business, total profit mainly depends upon inventory strategy. The process of spoilage, damage, decay or becoming inferior in quality or condition of products is defines as deterioration. The time when deterioration process starts, classifies products as instantaneous and non-instantaneous deteriorating products. Many items such as fruits, vegetables etc. starts declining very soon, they could be categorized as spontaneous decaying items. On the other hand, some commodities for instance medicines, packed products, electronic gadgets start declining their value after a certain time of their packing, they all are categorized as non-immediate decaying products. So, study of both type of declines is essential. Similarly, there are many aspects such as cost of products, population density and weather conditions in particular area, financial status of that specific locality etc., that determines the demand of goods in a region. So, considering variable demand pattern, is more justified. As on hand inventory changes, the carrying cost also variates. Hence, taking account of varying holding cost is more realistic. To boost market profit game, trade credit policy is offered by supplier and sometime by retailer also and it plays an important role in financial management of a business. So, inventory management taking account of numerous characteristics such as Demand, decay, shortages, trade credit policy, inflation, various costs etc. is indispensable. Considering all these characteristics, a significant study has been made by many analysts. For non-spontaneous perishable items, Ai et al. (2017) introduced an EOQ model under shortages. For Weibull kind of declining products, Valliathal & Uthayakumar (2016) and Chakraborty et al. (2018) presented inventory models with ramp type demand and Singh et al. (2018) established inventory model for newly launched new age electronic items. Liao et al. (2020) focused on flawed standards of items whereas Ouyang et al. (2005) investigated cash deduction in their inventory systems respectively. Ouyang et al. (2006) discussed an EOQ framework under trade credit policy and later Chung (2009) offered flawless solution procedure for Ouyang et al. (2006)'s work.

Advertisement of products positively affects a business but its cost directly changes the inventory design. Shah et al. (2013) and Palanivel & Uthayakumar (2017) framed inventory models with advertisement dependent demand and later with similar kind of demand pattern, Udaykumar et al. (2020) presented EOQ model under permitted delay in payments and inflation. For non-spontaneous decaying commodities, Shaikh et al. (2017), Palanivel & Uthayakumar (2015), Ghoreishi et al. (2015) and Sundararajan et al. (2019) analysed EOQ models with stock and advertisement sensitive demand under inflation and shortages. Palanivel & Uthayakumar (2017a) developed inventory model with price and advertisement induced demand under inflation and shortages.

Liao (2008) and Jaggi et al. (2018) both emphasis that sometimes retailer also offers trade credit policy to his/her customers to raise the retail competitiveness. Manna et al. (2009) and Khanra et al. (2013) presented EOQ models with time sensitive demand. Chang & Dye (1999) studied the consequences of backlogging rate on the ordering policies. Tiwari et al. (2016), Palanivel & Uthayakumar (2016) and Meena et al. (2021) presented inventory frameworks under inflation and delay in payments for non-instantaneous perishable products. Tiwari et al. (2017) and Sharma & Bansal (2017) discussed inventory models with stock sensitive demands. Similarly, Singh et al. (2010), Kumar & Kumar (2016), Uthayakumar & Geetha (2009), Uthayakumar & Geetha (2009a), and Sharma et al. (2013) also established inventory models with stock induced demand rate under inflation. These models are specially designed for retail businesses. With stock-based consumption rate, Hou (2006), Hou et al. (2011) and Jaggi & Khanna (2010) also presented EOQ models for decaying products in view of the effect of inflation. Rajan & Uthayakumar (2017) presented a model for pricing and ordering strategy for spontaneous perishable products under inflation and permitted delay in payments and demonstrated that when shortages are fully accumulated, one gets more profit whereas under similar conditions, Sundararajan et al. (2021) offered these strategies for non-spontaneous decaying commodities and compare their model with the conditions of absence of inflation and trade credit policy, fully accumulated shortages and spontaneous decay. Yadav & Swami (2019) established two-warehouse inventory-framework for non-immediate decaying products with time proportional demand and carrying cost. Chen (1998) presented an EOQ model for Weibull decaying products under inflation. Rangarajan & Karthikeyan (2017) derived inventory patterns for non-immediate and immediate declining products with cubic demand under inflation and trade credit policy and illustrated these models with partial and fully accumulated shortages. Sundararajan et al. (2020) developed inventory model with price and time sensitive demand under inflation and shortages.

In present work, a profit maximization inventory model is established for non-immediate decline commodities with algebraic time sensitive demand pattern, constant decay rate and exponentially time induced carrying cost under partially accumulated shortages, allowed delay in payments and inflation.

The rest part of this present study is systematized in the following way: in Segment 2 of this paper accommodates hypothesis and symbols used in this work. in section 3 mathematical framework and solutions of this study is discussed. Section 4 and 5 contains solution methodology and numerical instances respectively. In Segment 6, sensitivity investigation and scrutiny are presented. This paper came to an end with conclusions and future research scope in section 7.

II. MODELLING ASSUMPTIONS AND NOTATIONS

2.1. Notations

The following symbols are used throughout this work

Table 1. symbols used in this study

Symbols	A	Ordering cost per order
	θ	The constant decay rate
	r	The rate of inflation
	p	Purchasing cost per item
	s	Selling price ($s > p$)
	δ	The backlogging parameter and $\delta > 0$
	c_2	Shortage cost per unit per order
	c_0	Opportunity cost of lost sales
	I_p	The interest charged per dollar per year
	I_e	The interest earned per dollar per year
	M	The trade credit period
	R	The retailer's initial order quantity
	P	The maximum amount of demand backordered
	t_1	Time at which items begins to decline
Decision variables	t_2	Time at which inventory vanishes within a replenishment cycle
	T	The cycle length
Functions	$D(t)$	The demand rate at time t
	c_h	The Holding cost
	$I_1(t)$	Inventory level in the time interval $0 \le t \le t_1$
	$I_2(t)$	Inventory level in the time interval $t_1 \le t \le t_2$
	$I_3(t)$	Inventory level in the time interval $t_2 \le t \le \mathrm{T}$

2.2. Modelling Assumptions

The following hypothesis adopted to establish the mathematical formulation of present work

1. The lead time is insignificant and the planning horizon is infinite.
2. The Demand rate is algebraically time dependent, that is $D(t) = at^2 + bt + c$, where a, b, c are constants and $a, b, c > 0$.
3. The Holding cost is considered as exponentially time induced, that is $c_h = he^{kt}$, where $h, k > 0$.
4. It is presumed that during the interval $[0, t_1]$, there is no perishing. At time t_1, deterioration of products starts with constant decay rate θ and at time t_2, shortages happens and backorder arises.
5. Shortages are permitted and partially accumulating rate is varying and based on the period of the hold on time for the succeeding recollection of stock and it is indicated by $B(t) = e^{-\delta(T-t)}$, where $\delta > 0$ and $t_2 \le t \le T$.
6. Refit or return for the damaged products is not permitted.
7. Delay in payments is allowed, that is, the retailer have a relaxation to clear the balance sheet with the supplier and he keeps the received sales revenue in an interest producing account. Interest is imposed on retailer by supplier after the end of permitted delay period M.

III. MATHEMATICAL FORMULATION

In this portion, the mathematical modelling of this inventory framework is presented. At the time $t = 0$, the retailer orders and collects R units of commodities from the supplier. During the period $[0, t_1]$, the inventory level I_1, reduces because of the demand only and there is no decay in this time interval. At the time $t = t_1$, perishing of items arises and during the time $[t_1, t_2]$, the inventory level I_2 shrinks due to demand and decay both and it vanishes at the time $t = t_2$. Shortages take place in the duration of $[t_2, T]$.

Thus, the variation in inventory level with respect to time during the time interval [0,t_2], can be specified by the following differential equations

(1)

(2)

with the boundary conditions $I_1(0) = R$ and $I_2(t_2) = 0$.

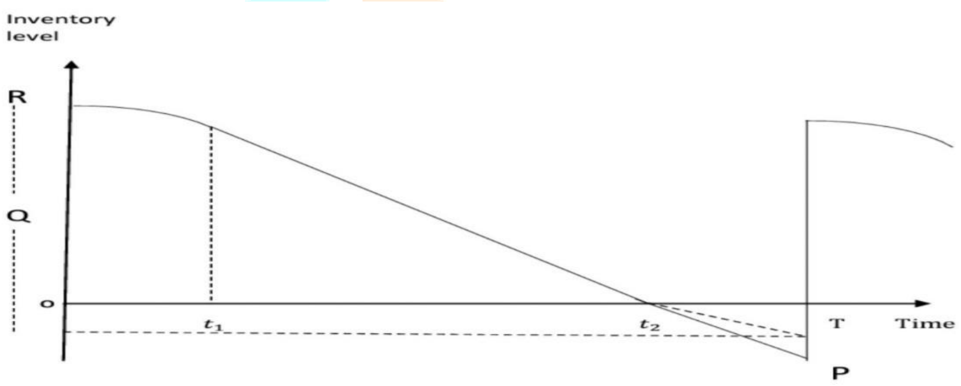

Figure 1 graphical portrayal of the inventory model

The solutions of equations (1) and (2) are

(3)

and

$I_2(t) = x_5 e^{-\theta t} - x_3 - x_6 t - x_7 t^2,$

where

$x_1 = 1/\theta$
$x_2 = (b - 2ax_1)x_1^2$
$x_3 = cx_1 - x_2$
$x_4 = ax_1 t_2^2 + \frac{x_2}{x_1} t_2$
$x_5 = e^{\theta t_2}(x_3 + x_4)$
$x_6 = \theta x_2$

(4)

Since, $I(t)$ is a continuous function of time. Hence at the time $t = t_1$, we have $I_1(t_1) = I_2(t_1)$. Therefore, from equations (3) and (4), we get

$R = x_5 e^{-\theta t_1} - x_8 + x_9,$

where $x_8 = x_3 + x_6 t_1 + x_7 t_1^2$ and $x_9 = \frac{a t_1^3}{3} + \frac{b t_1^2}{2} + c t_1$. (5)

Partial Backlogging Model

During the shortage in the time interval $[t_2, T]$, the inventory level $I_3(t)$ is governed by the differential equation

$$\frac{dI_3(t)}{dt} = -\frac{at^2+bt+c}{e^{\delta(T-t)}},\ t \in [t_2, T], \quad (6)$$

with the condition $I_3(t_2) = 0$. The solution of equation (6) is

$$I_3(t) = x_{10} - e^{-\delta(T-t)} \frac{[\delta^2(at^2+bt+c) - \delta(2at+b) + 2a]}{\delta^3},$$

where

$$x_{10} = e^{-\delta(T-t_2)} \frac{[\delta^2(at_2^2+bt_2+c) - \delta(2at_2+b) + 2a]}{\delta^3}. \quad (7)$$

On applying the condition $I_3(T) = -P$, we obtain the negative inventory

$$P = x_{11} - x_{10},$$

where

$$x_{11} = \frac{[\delta^2(aT^2+bT+c) - \delta(2aT+b) + 2a]}{\delta^3}. \quad (8)$$

Hence The Total inventory,

$$Q = R + P$$

$$Q = x_5 e^{-\theta t_1} - x_8 + x_9 + x_8 + x_{11} - x_{10}. \quad (9)$$

The total profit (TP) consists the following costs and earnings:

3.1. Ordering Cost (OC) is equal to A. (10)

3.2. Purchase Cost

$$PC = pR$$
$$= p\left(x_5 e^{-\theta t_1} - x_8 + x_9\right). \quad (11)$$

3.3. Holding Cost

$$HC = h\left[\int_0^{t_1} e^{kt} I_1(t) e^{-rt} dt + \int_{t_1}^{t_2} e^{kt} I_2(t) e^{-rt} dt\right]$$

$$= h\left\{\begin{array}{c} \frac{a}{3}\left\{e^{-t_1(r-k)}\left[\frac{t_1^3}{(r-k)} + \frac{3t_1^2}{(r-k)^2} + \frac{6t_1}{(r-k)^3}\right] + \frac{6}{(r-k)^4}\right\} \\ +\left(\frac{b}{2} - x_7\right)\left\{e^{-t_1(r-k)}\left[\frac{t_1^2}{(r-k)} + \frac{2t_1}{(r-k)^2} + \frac{2}{(r-k)^3}\right]\right\} \\ +\left\{e^{-t_2(r-k)}\left[\frac{t_2^2}{(r-k)} + \frac{2t_2}{(r-k)^2} + \frac{2}{(r-k)^3}\right] - \frac{b}{(r-k)^3}\right\} \\ +(c - x_6)\left\{e^{-t_1(r-k)}\left[\frac{t_1}{(r-k)} + \frac{1}{(r-k)^2}\right]\right\} \\ +\left\{e^{-t_2(r-k)}\left[\frac{t_2}{(r-k)} + \frac{1}{(r-k)^2}\right] - \frac{c}{(r-k)^3}\right\} \\ +(x_3 + R)\frac{\left[e^{-t_2(r-k)} - e^{-t_1(r-k)}\right]}{(r-k)} \\ +\frac{x_5 e^{-t_1[\theta+(r-k)]} - e^{-t_2[\theta+(r-k)]}}{\theta + (r-k)} \end{array}\right\}. \quad (12)$$

3.4. Sales Revenue

$$SR = s\left[\int_0^{t_2} D(t)e^{-rt}dt + e^{-rT}\int_{t_2}^{T} D(t)B(T-t)dt\right]$$

$$= s\left\{\begin{array}{c}\frac{a}{\delta^3}\left[x_{12} - x_{13}e^{-\delta(T-t_2)}\right] - \frac{b}{\delta^2}\left[x_{14} - x_{15}e^{-\delta(T-t_2)}\right] \\ +\frac{c}{\delta}\left[1 - e^{-\delta(T-t_2)}\right] + \frac{a}{r^3}\left[2 - x_{16}e^{-t_2 r}\right] \\ +\frac{b}{r^2}\left[1 - x_{17}e^{-t_2 r}\right] + \frac{c}{r}\left[1 - e^{-t_2 r}\right]\end{array}\right\},$$

where

$x_{12} = T^2\delta^2 - 2T\delta + 2,$
$x_{13} = t_2^2\delta^2 - 2t_2\delta + 2,$
$x_{14} = T\delta - 1,$
$x_{15} = t_2\delta - 1,$
$x_{16} = t_2^2 r^2 + 2t_2 r + 2,$
$x_{17} = t_2 r + 1.$ (13)

3.5. Shortage Cost

$$SC = c_2\int_{t_2}^{T}\left[-I_3(t)\right]e^{-rt}dt$$

$$= c_2\left\{\begin{array}{l}\frac{a}{\delta}\left\{e^{-x_{18}}\left[\frac{t_2^2}{(r-\delta)} + \frac{2t_2}{(r-\delta)^2} + \frac{2}{(r-\delta)^3}\right]\right. \\ \left.-e^{-Tr}\left[\frac{T^2}{(r-\delta)} + \frac{2T}{(r-\delta)^2} + \frac{2}{(r-\delta)^3}\right]\right\} \\ -\left(\frac{2a}{\delta^2} - \frac{b}{\delta}\right)\left\{e^{-x_{18}}\left[\frac{t_2}{(r-\delta)} + \frac{1}{(r-\delta)^2}\right]\right. \\ \left.-e^{-Tr}\left[\frac{T}{(r-\delta)} + \frac{1}{(r-\delta)^2}\right]\right\} \\ +\left(\frac{2a}{\delta^3} - \frac{b}{\delta^2} + \frac{c}{\delta}\right)\left[\frac{e^{-x_{18}} - e^{-Tr}}{(r-\delta)}\right] \\ +\frac{x_{10}\left(e^{-Tr} - e^{-t_2 r}\right)}{r}\end{array}\right\},$$

where $x_{18} = T\delta + t_2(r-\delta).$ (14)

3.6. Cost of Lost Sales

$$CLS = c_0\int_{t_2}^{T} D(t)\left[1 - e^{-\delta(T-t)}\right]e^{-rt}dt$$

$$= c_0\left\{\begin{array}{c}\frac{a}{r^3}\left[x_{19}e^{-Tr} - x_{20}e^{-t_2 r}\right] + \frac{b}{r^2}\left[x_{21}e^{-Tr} - x_{22}e^{-t_2 r}\right] \\ +\frac{c}{r}\left[e^{-Tr} - e^{-t_2 r}\right] - a\left\{\begin{array}{l}e^{-Tr}\left[\frac{T^2}{(r-\delta)} + \frac{2T}{(r-\delta)^2} + \frac{2}{(r-\delta)^3}\right] \\ -e^{-x_{18}}\left[\frac{t_2^2}{(r-\delta)} + \frac{2t_2}{(r-\delta)^2} + \frac{2}{(r-\delta)^3}\right]\end{array}\right\} \\ -b\left\{e^{-Tr}\left[\frac{T}{(r-\delta)} + \frac{1}{(r-\delta)^2}\right] - e^{-x_{18}}\left[\frac{t_2}{(r-\delta)} + \frac{1}{(r-\delta)^2}\right]\right\} \\ -\frac{c}{(r-\delta)}\left[e^{-Tr} - e^{-x_{18}}\right]\end{array}\right\},$$

where

$x_{19} = T^2 r^2 + 2Tr + 2,$
$x_{20} = t_2^2 r^2 + 2t_2 r + 2,$
$x_{21} = Tr + 1,$
$x_{22} = t_2 r + 1.$ (15)

3.7. Interest payable

3.7.1. When $0 \le M \le t_1$

$$IP_1 = pI_p\left[\int_M^{t_1} I_1(t)e^{-rt}dt + \int_{t_1}^{t_2} I_2(t)e^{-rt}dt\right]$$

$$= pI_p\left[\begin{array}{c}\frac{a}{3}\left(\frac{x_{23}e^{-t_1r}-x_{24}e^{-Mr}}{r^4}\right) + x_7\left(\frac{x_{27}e^{-t_2r}-x_{25}e^{-t_1r}}{r^3}\right)\\ +\frac{b}{2}\left(\frac{x_{25}e^{-t_1r}-x_{26}e^{-Mr}}{r^3}\right) + x_6\left(\frac{x_{30}e^{-t_2r}-x_{28}e^{-t_1r}}{r^2}\right)\\ +c\left(\frac{x_{28}e^{-t_1r}-x_{29}e^{-Mr}}{r^2}\right) - \frac{x_5\left(e^{-t_2(\theta+r)}-e^{-t_1(\theta+r)}\right)}{(\theta+r)}\\ +\frac{x_3(e^{-t_2r}-e^{-t_1r}) - R(e^{-t_1r}-e^{-Mr})}{r}\end{array}\right],$$

where

$x_{23} = t_1^3r^3 + 3t_1^2r^2 + 6t_1r + 6,$

$x_{24} = M^3r^3 + 3M^2r^2 + 6Mr + 6,$

$x_{25} = t_1^2r^2 + 2t_1r + 2,$

$x_{26} = M^2r^2 + 2Mr + 2,$

$x_{27} = t_2^2r^2 + 2t_2r + 2,$

$x_{28} = t_1r + 1,$

$x_{29} = Mr + 1,$

$x_{30} = t_2r + 1.$

(16)

3.7.2. When $t_1 \le M \le t_2$

$$IP_2 = pI_p\int_M^{t_2} I_2(t)e^{-rt}dt$$

$$= pI_p\left[\begin{array}{c}x_7\left(\frac{x_{27}e^{-t_2r}-x_{26}e^{-Mr}}{r^3}\right) + x_6\left(\frac{x_{30}e^{-t_2r}-x_{29}e^{-Mr}}{r^2}\right)\\ +\frac{x_3(e^{-t_2r}-e^{-Mr})}{r} - \frac{x_5\left(e^{-t_2(\theta+r)}-e^{-M(\theta+r)}\right)}{(\theta+r)}\end{array}\right]. \quad (17)$$

3.7.3. When $t_2 \le M \le T$

$IP_3 = 0.$

(18)

3.8. Interest earned

3.8.1 When $0 \le M \le t_1$

$$IE_1 = sI_e\int_0^M tD(t)e^{-rt}dt$$

$$= sI_e\left[\frac{a}{r^4}(6-x_{24}e^{-Mr}) + \frac{b}{r^3}(2-x_{26}e^{-Mr}) + \frac{c}{r^2}(1-x_{29}e^{-Mr})\right] \quad (19)$$

3.8.2 When $t_1 \le M \le t_2$

$$IE_2 = sI_e\int_0^M tD(t)e^{-rt}dt$$

$$= sI_e\left[a\left(\frac{6-x_{24}e^{-Mr}}{r^4}\right)+b\left(\frac{2-x_{26}e^{-Mr}}{r^3}\right)+c\left(\frac{1-x_{29}e^{-Mr}}{r^2}\right)\right] \quad (20)$$

3.8.3. When $t_2 \le M \le T$

$$IE_3 = sI_e\left[\int_0^{t_2} tD(t)e^{-rt}dt + (M-t_2)\int_0^{t_2} D(t)e^{-rt}dt\right]$$

$$= sI_e\left\{\begin{array}{c} a\left(\frac{6-x_{31}e^{-t_2r}}{r^4}\right)+[b+a(M-t_2)]\left(\frac{2-x_{27}e^{-t_2r}}{r^3}\right) \\ +[c+b(M-t_2)]\left(\frac{1-x_{30}e^{-t_2r}}{r^2}\right) \\ +c(M-t_2)\left(\frac{1-e^{-t_2r}}{r}\right)\end{array}\right\},$$

where $x_{31} = t_2^3r^3 + 3t_2^2r^2 + 6t_2r + 6.$ (21)

Hence, the total profit per unit time is defined as

$$TP = \begin{cases} TP_1; 0 \le t \le t_1 \\ TP_2; t_1 \le t \le t_2 \\ TP_3; t_2 \le t \le T \end{cases},$$

where

$$TP_1 = \frac{1}{T}[(SR + IE_1) - (OC + PC + HC + SC + CLS + IP_1)],$$

$$TP_2 = \frac{1}{T}[(SR + IE_2) - (OC + PC + HC + SC + CLS + IP_2)]$$

and

$$TP_3 = \frac{1}{T}[(SR + IE_3) - (OC + PC + HC + SC + CLS)]\,. \quad (22)$$

IV. SOLUTION PROCEDURE

In this problem, t_2 and T play the roles of decision variables and the total inventory Q is a dependent variable. The aim of this study is to maximize the total profit functions TP_1, TP_2 and TP_3 assuming t_2 and T as unknowns.

The maximum values of the total profit functions are obtained by applying the upcoming necessary and sufficient conditions:

$$\frac{\partial TP_i(t_2,T)}{\partial t_2} = 0 \text{ and } \frac{\partial TP_i(t_2,T)}{\partial T} = 0,$$

where $i = 1, 2, 3.$

The convexity of the total profit functions is checked by the Hessian matrix. The Hessian matrix of the total profit function for $i = 1, 2, 3$ is

$$H(t_2,T) = \begin{bmatrix} \frac{\partial^2 TP_i(t_2,T)}{\partial t_2^2} & \frac{\partial^2 TP_i(t_2,T)}{\partial t_2 \partial T} \\ \frac{\partial^2 TP_i(t_2,T)}{\partial T \partial t_2} & \frac{\partial^2 TP_i(t_2,T)}{\partial T^2} \end{bmatrix},$$

where $|H| > 0$, $\frac{\partial^2 TP_i(t_2,T)}{\partial t_2^2}|_{t=t_2^*,\ T=T^*} < 0$ and $\frac{\partial^2 TP_i(t_2,T)}{\partial T^2}|_{t=t_2^*,\ T=T^*} < 0.$

V. NUMERICAL EXAMPLES

Example 5.1
Case 1 $(0 \leq M \leq t_1)$
Let set the values of parameters as follows: $a = 17,\ b = 150,\ c = 2,\ \theta = 0.8,\ h = 8,\ k = 0.14,\ r = 0.16,\ \delta = 0.0001,\ A = 150,\ p = 18/unit,\ s = 118/unit,\ c_2 = 0.002/unit,\ c_0 = 0.02/unit,\ I_p = 0.011\ /year,\ I_e = 0.025/\ /year,\ t_1 = 0.05\ years\ and\ M = 0.004\ years$.
With the assistance of MATLAB software for equations (17) optimum solution are
$t_2^* = 0.4133\ years,\quad T^* = 1.4975\ years$
$TP_1^* = 11657/order/year$
$Q^* = 192.9961\ units.$

Example 5.2
Case 2 $(t_1 \leq M \leq t_2)$, The basic data are alike as first instance 5.1 except $a = 11,\ M = 0.06\ years$.
On solving, we obtain the optimal results in this manner
$t_2^* = 0.2368\ years,\quad T^* = 1.4842\ years$
$TP_2^* = 11199/order/year$
$Q^* = 180.5781\ units.$

Example 5.3
Case 3 $(t_2 < M)$
The basic data are alike as first instance except $a = 13, M = 0.3\ years$.
On solving, we obtain the upcoming optimal values
$t_2^* = 0.1597\ years,\quad T^* = 1.4842\ years$
$TP_3^* = 11338/order/year$
$Q^* = 182.4453\ units.$

VI. SENSITIVITY ANALYSIS AND OBSERVATIONS

In present part of the paper, sensitivity analysis is exhibited for the parameters $a, b, c, \theta,\ \delta,\ h,\ k,\ r,\ A,\ p,\ s,\ t_1\ and\ M$ used in this inventory system to find out the influence that changes in those parameters have on the expected total profit per unit time and the optimal values of t_2, T and Q. The sensitivity analysis is carried out by variate each parameter assuming single parameter as a variable at an instant and holding other parameters constant. Outcomes of this analysis is summarized in Table 2 and Figure 2 to Figure 5.

Table 2. sensitivity analysis of numerous parameters of this model for case 1 $(0 \leq M \leq t_1)$

Parameters	Change in parameters	t_2^*	T^*	TP_1^*	Q^*
a	15	0.2368	1.4842	11472	184.9453
	17	0.4133	1.4975	11657	192.9961
	19	0.4132	1.4978	11799	195.3203
	21	0.4115	1.5110	12032	201.1797
b	135	0.4133	1.4972	10616	175.8516
	150	0.4133	1.4975	11657	192.9961
	165	0.4011	1.5850	13344	235.7422
	180	0.1953	1.4844	13684	220.0508
c	2.1	0.2368	1.4842	11618	187.2656
	3.2	0.4140	1.4975	11768	194.8789
	4.3	0.4469	1.5348	12108	207.5313
	5.401	0.4119	1.5081	12045	201.1016
θ	0.5	0.4140	1.4975	11672	191.8938
	0.6	0.4468	1.5348	11909	202.8589
	0.71	0.2368	1.4842	11609	187.0785
	0.8	0.4133	1.4975	11657	192.9961
h	5	0.7409	1.6305	12274	246.9531
	8	0.4133	1.4975	11657	192.9961

	11	0.4133	1.4975	11649	192.9961
	14	0.1484	1.4842	11609	186.7773
k	0.03	0.2370	1.4844	11610	187.1680
	0.14	0.4133	1.4975	11657	192.9961
	0.26	0.2368	1.4842	11608	187.1250
	0.39	0.1945	1.7746	13513	271.6016
r	0.17	0.1367	1.6537	12524	234.0781
	0.20	0.5847	1.6848	12038	252.2422
	0.23	0.3905	1.4963	10547	192.1953
	0.26	0.9732	1.8316	11099	340.3750
A	150	0.4133	1.4975	11657	192.9961
	200	0.2382	1.4880	11601	188.1250
	250	0.4133	1.4975	11591	192.9961
	300	0.1108	1.4842	11507	186.7305
p	12	0.4143	1.4842	11634	189.5156
	14	0.2497	1.4842	11623	187.2148
	16.001	0.4135	1.4975	11680	193
	18	0.4133	1.4975	11657	192.9961
s	104	0.4125	1.4975	10236	192.9727
	118	0.4133	1.4975	11657	192.9961
	132	0.2368	1.4842	13006	187.1250
	146	0.4134	1.4975	14501	192.9961
M	0.00101	0.2368	1.4842	11609	187.1250
	0.002	0.4133	1.4976	11658	193.0195
	0.003	0.2368	1.4842	11608	187.1250
	0.004	0.4133	1.4975	11657	192.9961
t_1	0.036	0.4134	1.4975	11655	193.1859
	0.056	0.4122	1.4972	11657	192.8187
	0.076	0.1924	1.4846	11613	186.9516
	0.096001	0.5908	1.5776	12091	219.9241
δ	0.000100	0.4133	1.4975	11657	192.9961
	0.000233	0.4126	1.4977	11658	193.0239
	0.000255	0.2028	1.4770	11560	185.0518
	0.000277	0.3649	1.4509	11355	179.9133

Observations

From Table 1 and Figures 2 to Figure 5, we find these observations

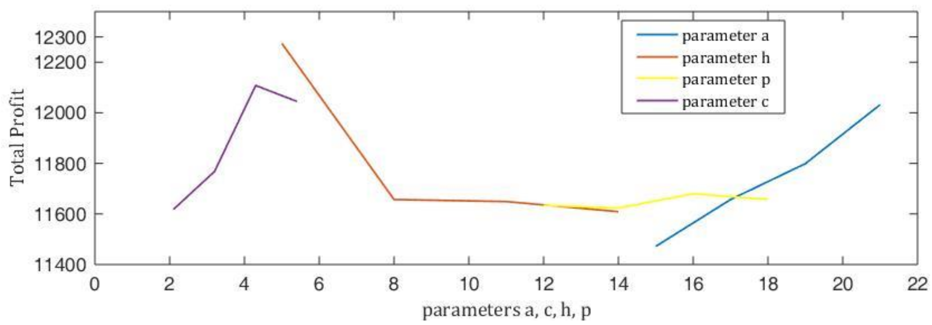

Figure 2 Effect of variation in parameters a, c, h & p on the optimal total profit for case 1

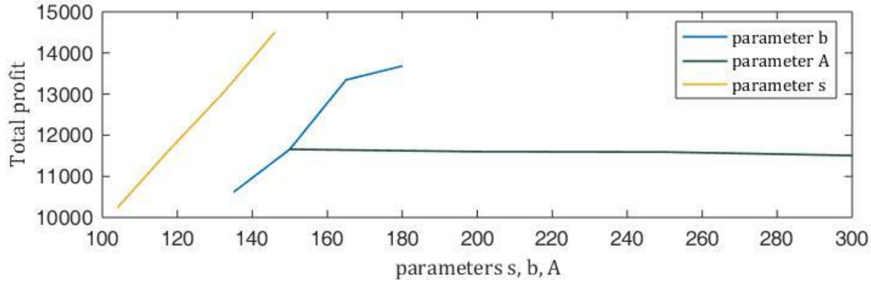

Figure 3 Effect of variation in s, b & A on the optimal total profit for case 1

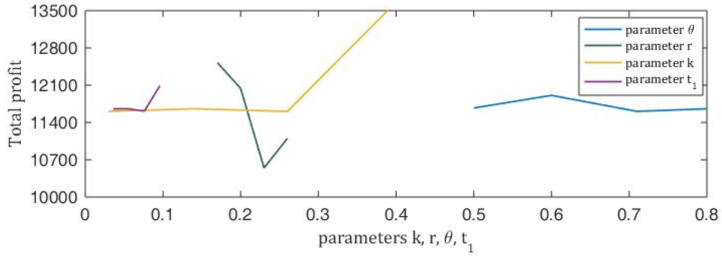

Figure 4 Effect of variation in k, r, θ & t_1 on the optimal total profit for case 1

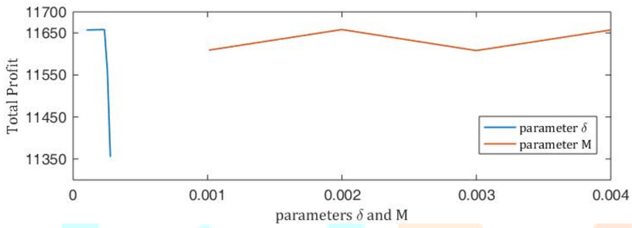

Figure 5 Effect of variation in δ & M on the optimal total profit for case 1

- Change in parameter a from 15 to 21, results slight increase in T^*, TP^* and Q^* and initially boosts t_2^* and then remains it almost unchanged.
- As we hike the parameter b from 135 to 180, at first T^* and Q^* boosts and then reduces, TP^* increases and at the beginning t_2^* remains almost constant and after that it declines.
- Rise in the parameter c from 2.1 to 5.401, consequences initial hike and then a bit decline in t_2^*, T^*, TP^* and Q^*.
- As we shrink the deterioration rate θ from 0.8 to 0.5, we observe slight declining and then rising and then again reducing behaviour of t_2^*, T^*, TP^* and Q^*.
- As we grow the holding cost parameter h from 5 to 14, t_2^*, T^*, TP^* and Q^* depletes.
- Changing in the parameter k from 0.03 to 0.39, outcomes initial slight change and then raise in T^*, TP^* and Q^* whereas at first rises t_2^* and after that depletes it.
- When the inflation rate r changes from 0.17 to 0.26, its consequences slight decline and then rise and then again reduce in t_2^*, T^*, Q^* and the total profit TP^* at start declines and then boosts.
- As the ordering cost per order A variates from 150 to 300, there is no major variation observed in T^*, TP^* and Q^* whereas t_2^* initially dips and then rise and after that again dips.
- When the purchasing cost per item p falls from 18 to 12, there is no extensive changes in T^*, TP^* and Q^* whereas at first, we observe decline in t_2^* and then it rises.
- As we raise the selling price per item s from 104 to 146, we get no significant variation in T^* and Q^* but the total profit TP^* improves and t_2^* initially shrinks and then rises.
- If the supplier reduces the permitted delay period M from 0.004 to 0.00101, T^*, TP^* and Q^* slightly variates and t_2^* initially falls and then rises and after that again boosts.
- If the non-decay duration t_1 rises from 0.036 to 0.096001, it initially diminishes and then extend t_2^*, T^*, TP^* and Q^*.
- When the backlogging rate δ boosts from 0.0001 to 0.000277, it slightly dips T^* and TP^* while t_2^*, at first depletes and then improves, on the other hand, Q^* shows opposite behaviour compare to t_2^*.

VII. CONCLUSION

In this study, an inventory model is established for non-immediate perishable commodities with time induced algebraic demand pattern, constant decay rate and time linked exponential carrying cost under the effect of inflation and permitted delay in payments. In this model, shortages are allowed and the backlogging rate depends on the awaiting time for the upcoming restock. The aim of this study is to obtain optimal ordering strategy to get maximum total profit. This model is partitioned into three parts upon the basis of durations of non-decay and permitted delay in payments, the time at which shortages take place and the cycle length. Numerical examples and sensitivity examination of key parameters is carried out to validate present work. The major findings of this work are:

- Rise of demand positively affect the total profit.
- Hike in Ordering cost per item reduces total profit.
- Rise of inflation rate declines total profit.
- Increased selling price per item results more profit.
- Increase in duration of non-decline consequences optimal profit.

The potential study can be conducted by assuming ramp type or advertisement linked demand rate or with variate decay rate or linear holding cost.

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