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Mathematical Analysis of Waiting Time in a Feedback Queuing Model With Three Servers and Chances of Revisits of Customer Atmost Once to Any Server

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Abstract

A queuing model has been developed for a system having three servers wherein a customer may revisit to any of the servers. The visit of the customer is limited to maximum twice. Customer may require the services of one or all the servers. If he/she requires the service of more than one server, then first of all he/she will visit to first customer and then may go to any of the other two servers in the system. After getting the service from any of the servers, the customer may revisit to any other server or may leave the system at any stage depending upon his/her satisfaction. Whenever, a customer revisits, the probability of leaving the server does not remain same as that was for leaving that server on his/her previous visit. The waiting time of the customer in the system is derived from the mean queue length of the customer in the system and the graphical analysis of the model is done thereafter.

Key Words: Queuing system, Three servers, Feedback, Chances of one Revisit, Waiting time.

Introduction

When a customer is not satisfied with the service of the servers then he/she may revisit the serving system again and again. So there may be queues at the system. Such kind of cyclic queues are known as the feedback queues. We can observe such kind of queues in real life in manufacturing concerns, offices, in hospitals etc. where the customers after getting the service by a server may not be satisfied with the service or service of some other server may be required to complete his/her task. Many researcher including "Jackson (1957), Coffman(1968), Chan(1970), Yashkov (1985), Reiman (1988), Kumar (1990), Berg et al. (1991), Brandon and Yechiali (1991), Epema (1991), Zipkin (1995), Garg and Kumari (1998), Dupuis and Ramanan(2000), Atencia and Moreno (2004), Krieger et al. (2005), Arivudainambi and Krishanmoorthy (2006), Garg and Srivastava (2006),Kumar and Raja (2006), Meyn(2006), Tang and Zhao (2008), Ke and Chang (2009), Luo et al. (2009), Salehirad and Badamchizadeh (2009), Kusum et al. (2010), Luo and Tang (2011), Suzer et al. (2012), Andrian (2013), Kumar et al. (2013), Ekbatani et al. (2014), Gao and Liu (2014), Kalidass and Kasturi (2014), Huang et al. (2015), Melikov et al. (2015), Gupta et al. (2016), Latouche et al. (2016), Peng (2016), Raheja et al. (2016), Sreekumari (2016), Bouchentouf and Yahiaoui (2017), Ayyappan and Udayageetha (2018), Kotb and Akhdar (2018) have been worked on various aspects such as bulk service, impatient customers, cyclic queues, batch arrivals, reneging, balking etc. Kusum et al. (2010) worked on feedback queues having three service channels wherein a customer may go forward/back to any service channel. But there is no restriction on the number of such movements. Also, they considered the same probability on every revisit. However, in practical situations, there may be the possibility of moving backward/forward to the preceding/succeeding channel limited number of times and the probability of each movement may be different. Keeping this in view, Kumar and Taneja (2019), worked on the feedback queuing system comprising of three channels such that first is cenrally linked with the other two. After getting service from the first server a customer may either move to second server or to third server or may leave the system depending upon the need of customer. However, they did not discuss about the waiting time and other queuing characteristics.

Here, in the present study we have discussed numerically and graphically the variation of waiting time of customer in the system with respect to the other queuing parameters.

Notations

"λ: constant arrival rate.

μ_1: constant service rate of 1st server.

μ_2: constant service rate of 2nd server.

μ_3: constant service rate of 3rd server.

n_1, n_2, n_3: the number of customers at 1st, 2nd, 3rd server respectively at any time t.

a: the probability of customer leaving 1st server 1st time.

a': the probability of customer leaving 1st server 2nd time.

b: the probability of customer leaving 2nd server 1st time.

b': the probability of customer leaving 2nd server 2nd time.

c: the probability of customer leaving 3rd server 1st time.

c': the probability of customer leaving 3rd server 2nd time.

p_{12}: the probability of customer going from 1st to 2nd server 1st time.

p_{13}: the probability of customer going from 1st to 3rd server 1st time.

p'_{12}: the probability of customer going from 1st to 2nd server 2nd time.

p'_{13}: the probability of customer going from 1st to 3rd server 2nd time.

p_2: the probability of exit of customer from2nd server 1st time.

p_{23}: the probability of customer going from 2nd to 3rd server 1st time.

p_{21}: the probability of customer going from 2nd to 1st server 1st time.

$p_2^{'}$: the probability of exit of customer from 2nd server 2nd time.

$p_{23}^{'}$: the probability of customer going from 2nd to 3rd server 2nd time.

p_3 : the probability of exit of customer from 3rd server 1st time.

p_{31} : the probability of customer going from 3rd to 1st server 1st time.

p_{32} : the probability of customer going from 3rd to2nd server 1st time.

$p_{32}^{'}$: the probability of customer going from 3rd to 2nd server 2nd time.

$p_3^{'}$: the probability of exit of customer from3rd server 2nd time.

Thus

$a + a' = 1,\ b + b' = 1,\ c + c' = 1;$

$ap_{12} + ap_{13} + a^{'}p_{12}^{'} + a^{'}p_{13}^{'} = 1,$

$bp_2 + bp_{23} + bp_{21} + b^{'}p_2^{'} + b^{'}p_{23}^{'} = 1,$

$cp_3 + cp_{31} + cp_{32} + c^{'}p_{32}^{'} + c^{'}p_3^{'} = 1$"

By kumar and Taneja (2019), if the marginal mean queue lengths at the first server, the second server and the third server are denoted by (Lq_1), (Lq_2) and (Lq_3) respectively and

$$A = \left\{1 - \left(bp_{23} + b^{'}p_{23}^{'}\right)\left(cp_{32} + c^{'}p_{32}^{'}\right)\right\},$$

$$A^{'} = \left\{1 - \left(bp_{23} + b^{'}p_{23}^{'}\right)\left(cp_{32} + c^{'}p_{32}^{'}\right) - cp_{31}\left(ap_{12} + a^{'}p_{12}^{'}\right)\left(bp_{23} + b^{'}p_{23}^{'}\right) - bp_{21}\left(ap_{12} + a^{'}p_{12}^{'}\right) - cp_{31}\left(ap_{13} + a^{'}p_{13}^{'}\right) - bp_{21}\left(ap_{13} + a^{'}p_{13}^{'}\right)\left(cp_{32} + c^{'}p_{32}^{'}\right)\right\}$$

then:

$$Lq_1 = \frac{\lambda A}{A^{'}(-\lambda + \mu_1 - bp_{21}\mu_2 - cp_{31}\mu_3)}$$

$$Lq_2 = \frac{\lambda\left\{\left(ap_{12} + a^{'}p_{12}^{'}\right) + \left(ap_{13} + a^{'}p_{13}^{'}\right)\left(cp_{32} + c^{'}p_{32}^{'}\right)\right\}}{A^{'}\left[\mu_2 - \mu_1\left(ap_{12} + a^{'}p_{12}^{'}\right) - \mu_3\left(cp_{32} + cp_{32}'\right)\right]}$$

$$Lq_3 = \frac{\lambda\left[\left(ap_{12} + a^{'}p_{12}^{'}\right)\left(bp_{23} + b^{'}p_{23}^{'}\right) + \left(ap_{13} + a^{'}p_{13}^{'}\right)\right]}{A^{'}\left[\mu_3 - \mu_2\left(bp_{23} + b^{'}p_{23}^{'}\right) - \mu_1\left(ap_{13} + a^{'}p_{13}^{'}\right)\right]}$$

And Lq as the mean queue length of the queuing system is given by:

$$Lq = Lq_1 + Lq_2 + Lq_3$$

$$Lq = \frac{\lambda}{A'}\left[\frac{A}{(-\lambda + \mu_1 - bp_{21}\mu_2 - cp_{31}\mu_3)} + \frac{\left(ap_{12} + a^{'}p_{12}^{'}\right) + \left(ap_{13} + a^{'}p_{13}^{'}\right)\left(cp_{32} + c^{'}p_{32}^{'}\right)}{\left[\mu_2 - \mu_1\left(ap_{12} + a^{'}p_{12}^{'}\right) - \mu_3\left(cp_{32} + cp_{32}'\right)\right]}\right.$$

$$+\frac{\left(ap_{12}+a^{'}p_{12}^{'}\right)\left(bp_{23}+b^{'}p_{23}^{'}\right)+\left(ap_{13}+a^{'}p_{13}^{'}\right)}{\left[\mu_3-\mu_2\left(bp_{23}+b^{'}p_{23}^{'}\right)-\mu_1\left(ap_{13}+a^{'}p_{13}^{'}\right)\right]}\Bigg]$$

Numerical Results and Discussion

1. **Behaviour of waiting time (W) of customer in the system with respect to arrival rate (λ) for different values of probability of leaving the first server first time (a) is depicted in Table 1 and in Fig. 1 keeping the values of other parameters as fixed.**

Table 1

$\mu_1 = 4$, $\mu_2 = 5$, $\mu_3 = 4$, b= 0.7, b'= 0.3, c = 0.8, c' = 0.2, p_{13} =0.7, p_{12} = 0.3, p_{13}' =0.3, p_{12}'=0.7, p_2=0.7, p_{23}= 0.2, p_{21}=0.1, p_2'= 0.3, p_{23}'=0.7, p_3=0.6, p_{31}=0.2, p_{32}=0.2, p_3'=0.4, p_{32}'=0.6						
	a=0.3, a'=0.7		a=0.4, a'=0.6		a=0.5, a'=0.5	
λ	Lq	W	Lq	W	Lq	W
1.5	4.255236	2.836824	5.129039	3.419359	7.283322	4.855548
2	6.45858	3.22929	7.62589	3.812945	10.50052	5.25026
2.1	7.055027	3.359537	8.281484	3.943564	11.30063	5.381252
2.2	7.748276	3.521944	9.034154	4.106434	12.19809	5.544589
2.3	8.579227	3.730099	9.92492	4.315183	13.23405	5.753935
2.4	9.615603	4.006501	11.0217	4.592374	14.47661	6.031919
2.5	10.97824	4.391297	12.44567	4.978268	16.04729	6.418917
2.6	12.90587	4.963798	14.43625	5.552402	18.18621	6.994695
2.7	15.94527	5.905654	17.54175	6.496946	21.44324	7.941942
2.8	21.68465	7.744516	23.35496	8.341056	27.41572	9.791329
2.9	37.48762	12.92676	39.26047	13.53809	43.50939	15.00324

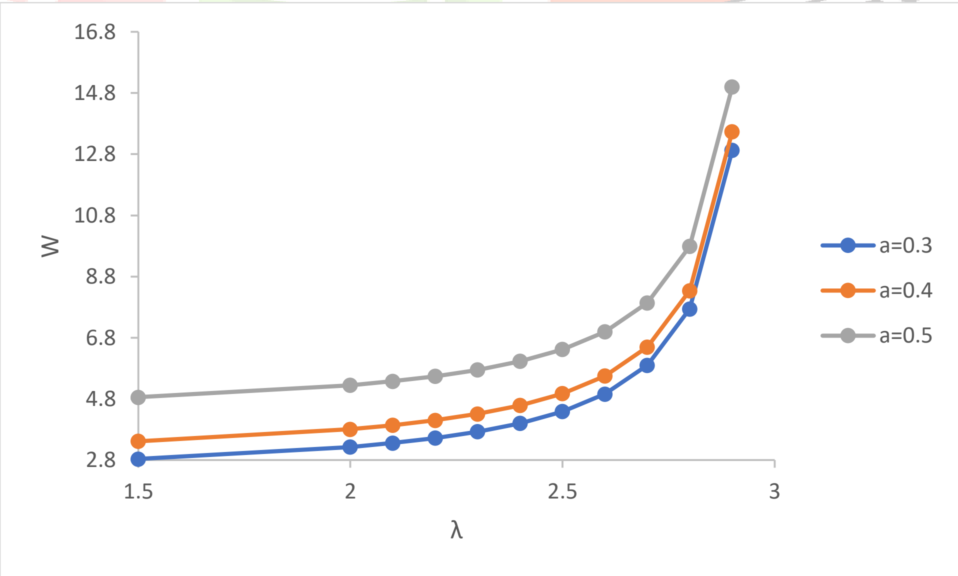

Fig. 1

Following can be interpreted from **Table 1** and **Fig. 1**:

(i) Waiting time get increased with the increase in λ.

(ii) Waiting time increases with respect to probability (a).

(iii) Gradual increase in waiting time for λ up to 2.7 but rapid increase beyond 2.7 can be observed.

2. Behaviour of the waiting time (W) of customer in the system w.r.t. μ_1 for different values of μ_2 is depicted in Table 2 and Fig. 2 keeping the values of other parameters fixed shown therein.

Table 2

μ_3= 4, λ= 1.5, a = 0.3, a' =0.7, b= 0.7, b'= 0.3, c = 0.8, c' = 0.2, p_{13} =0.7, p_{12} = 0.3, p_{13}' =0.3, p_{12}'=0.7, p_2=0.7, p_{23}= 0.2, p_{21}=0.1, p_2'= 0.3, p_{23}'=0.7, p_3=0.6, p_{31}=0.2,p_{32}=0.2, p_3'=4,p_{32}'=0.6						
	μ_2=4		μ_2=5		μ_2=6	
μ_1	Lq	W	Lq	W	Lq	W
3	4.852959	3.235306	4.841703	3.227802	5.361545	3.574364
3.2	4.237115	2.824743	3.948065	2.632043	4.193384	2.795589
3.4	3.977541	2.651694	3.493968	2.329312	3.635897	2.423932
3.6	3.92757	2.61838	3.249655	2.166437	3.347634	2.231756
3.8	4.044861	2.696574	3.127626	2.085084	3.210891	2.140594
4	4.342512	2.895008	3.089094	2.059396	3.177935	2.118624
4.2	4.896818	3.264545	3.116399	2.077599	3.230798	2.153865
4.4	5.924263	3.949509	3.203203	2.135469	3.369127	2.246085
4.6	8.14026	5.42684	3.350982	2.233988	3.60878	2.405853
4.65	9.137983	6.091989	3.398308	2.265539	3.688384	2.458923
4.8	15.74426	10.49617	3.568531	2.379021	3.988958	2.659305

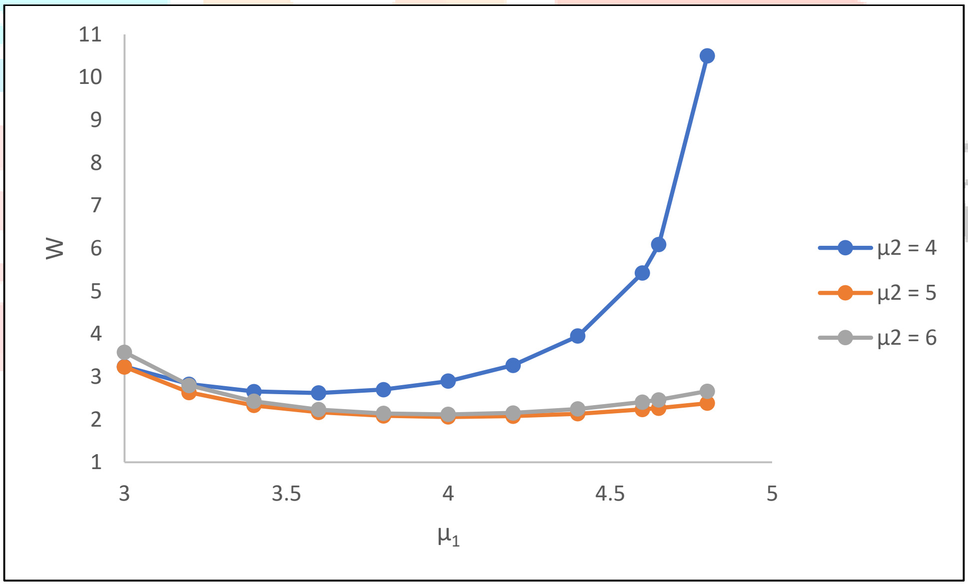

Fig. 2

From the above, it may be observed that initially the waiting time W decreases with the increase of service rate of first server (μ_1). However, after attaining the lowest value, it starts increasing i.e. points of minima are obtained for different value of μ_2. The values of minimum waiting time with respect to μ_1 and μ_2 are as under:

μ_2	μ_1	Minimum Waiting Time (W)
4	3.6	2.61838
5	4	2.059396
6	4	2.118624

3. Nature of the waiting time (W) of customer in the system with respect to probability of leaving the third server first time (p_3) and for μ_3 is depicted in Table 3 and Fig. 3.

Table 3

$\mu_1=3,\mu_2=2,\lambda=1.5,a=0.3,a'=0.7,b=0.7,b'=0.3,c=0.8,c'=0.2,p_{13}=0.7,p_{12}=0.3,p_{12}'=0.7,p_{13}'=0.3,p_2'=0.3,p_{23}'=0.7,p_3'=0.4,p_{32}'=0.6,p_2=0.7,p_{23}=0.2,p_{21}=0.1$					
			$\mu_3=4$	$\mu_3=5$	$\mu_3=6$
p_3	p_{31}	p_{32}	W	W	W
0.1	0.1	0.8	0.870763	0.901867	0.974615
0.15	0.12	0.63	0.861962	0.937663	1.055686
0.2	0.14	0.56	0.893982	1.013096	1.185332
0.25	0.16	0.49	0.92586	1.101104	1.34978
0.3	0.18	0.5	1.025675	1.257612	1.6086
0.35	0.2	0.45	1.089766	1.411784	1.92844
0.4	0.22	0.28	0.987124	1.477412	2.286693
0.45	0.24	0.22	0.989602	1.67745	2.977
0.5	0.26	0.2	1.097493	2.041552	4.319178
0.55	0.28	0.18	1.232056	2.575216	7.433599
0.6	0.283	0.117	0.94812	2.46273	8.107859

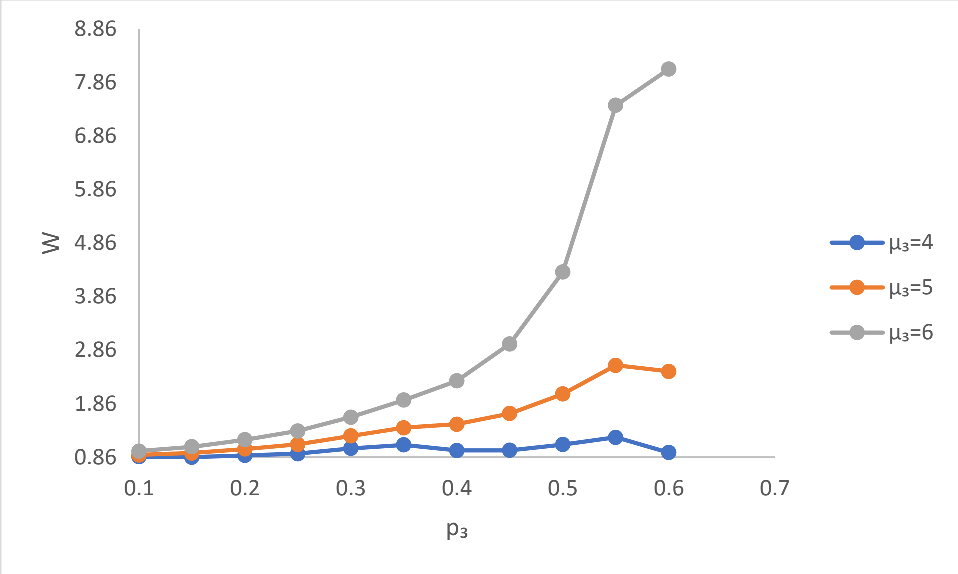

Fig. 3

Following can be interpreted from **Table 3** and **Fig. 3**:

(i) W get increased gradually with the increase in the values of p_3 for μ_3=5 and μ_3=6 but there are ups and down for μ_3=4.

(ii) W continuously increases with the increase of μ_3=4 for any particular value of p_3.

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