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A NOTE ON FUZZY PRE-γ-GSG-CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

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Abstract : In this paper, we introduce and study the notion of fuzzy pre-γ-gsg-closed sets in fuzzy topological spaces and their essential characteristics and operations. Key concepts such as fuzzy pre-γ-gsg neighborhoods, fuzzy pre-γ-gsg closure, and fuzzy pre-γ-gsg interior are introduced and analyzed, providing a powerful tool for handling uncertainty in fuzzy topological spaces.

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1. INTRODUCTION

The investigation of fuzzy sets theory has developed over the past ten years, spurring the development of innovative applications in mathematics, computer science, engineering, and decision-making processes, among others. The introduction of pre-γ-fuzzy sets, that extends fundamental concept of fuzzy sets through introducing graded membership and non-membership values, is a significant development in artificial intelligence, is a notable extension of this theory. These pre-γ-fuzzy sets sets provide a more flexible and expressive framework for modelling uncertainty and imprecision in realistic situations. Concurrently, the study of topological spaces has been enriched by the introduction of fuzzy topological spaces, which permit a more nuanced and adaptable exploration of spatial relationships. The combination of pre-γ-fuzzy sets and fuzzy topological spaces provides an efficient method for addressing problems involving vagueness, ambiguity, and ambiguous spatial relationships.

The notion of fuzzy sets was introduced by Zadeh [9] in 1965. Fuzzy sets play a crucial role in all fields of mathematics. In 1968, Chang [1] introduced the idea of fuzzy topological space. Kasahara [5] defined an operation in topological spaces. A fuzzy operation γ on fuzzy topological space was introduced by Kalitha and Das [4]. Hariwan Z. Ibrahim [2] introduced the notion of pre- γ-open sets in topological spaces. C. Sivashanmugaraja [8] studied the operation approaches on fuzzy pre-γ-open and fuzzy pre-γ-closed mappings in Fuzzy Topological Spaces The concept generalized sg-closed sets (gsg closed set) was introduced and studied by Lellis et al [6] in classical topology. Kalaiselvi et al [3] extends the notion of generalized sg-closed sets and its characterizations in fuzzy topological spaces. In this paper, we study about the notion of fuzzy pre-γ-gsg-closed sets in fuzzy topological spaces and their characteristics.

2. PRELIMINARIES

Throughout this paper, (X,τ) (or simply X always mean fuzzy topological space on which no separation axioms are assumed unless otherwise mentioned. For a fuzzy set A of $(X,\tau), Cl(A), Int(A)$ and $1 - A$ denote the closure, the interior and complement of A respectively.

Definition: 2.1.[3] A fuzzy set λ of (X,τ) is called a fuzzy generalized sg-closed set (in short, Fgsg-closed) if $Cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is Fsg open in X.

Definition: 2.2.[3] Let λ be any fuzzy set in (X,τ) then we define Fgsg-Closure and Fgsg- Interior as

$$gsg - Cl(\lambda) = \wedge \{ \mu : \mu \text{ is Fgsg-closed and } B \geq \lambda,$$
$$gsg - Int(\lambda) = \vee \{ \mu : \mu \text{ is Fgsg-open and } B \leq \lambda.$$

Definition: 2.3.[3] A fuzzy topological space (X,τ) is called a Fuzzy Tgsg Space if every Fgsg-closed set in it is fuzzy closed.

Definition: 2.4. [5] Let (X,τ) be a fuzzy topological space. A fuzzy operation γ on the topology τ is a mapping from τ into set I^X such that $\lambda \leq \gamma(\lambda)$, for every $\lambda \in \tau_X$, where $\gamma(\lambda)$ denotes the value of γ at λ.

Definition: 2.5 [7] A fuzzy set λ of a fts (X,τ) is called a fuzzy γ-open, if for every $p_x^\lambda \, q \, \lambda, \exists$ a $\mu \in \tau$ and $p_x^\lambda \, q \, \mu$ such that $\gamma(\mu) \leq \lambda. \tau_\gamma$ denotes the set of all γ-open fuzzy sets. Clearly we have $\tau_\gamma \subseteq \tau_X$.

Definition 2.6. [8] A fuzzy set λ of a fuzzy topological space X is called fuzzy pre-γ-open , if $\lambda \leq \tau_\gamma$-$int(cl(\lambda))$.

Definition 2.7. [8] Let λ be a fuzzy set in a fuzzy topological space X. Then the pre-γ-interior of λ is defined as $pint_\gamma(\lambda) = \vee \{\mu: \mu \leq \lambda, \mu \in FP\gamma O(\tau)\}$ and pre-γ-closure of λ is defined as $pcl_\gamma(\lambda) = \wedge \{\mu: \mu \geq \lambda, \mu \in FP\gamma C(\tau)\}$.

Definition 2.8. [8] A fuzzy set μ of a fuzzy topological space X is called
i. a neighborhood of a fuzzy point x_β , iff there exists a such that
ii. a fuzzy pre-γ-neighborhood of a fuzzy point $x_\beta \in X$, if ∃ a pre-γ-open fuzzy set η such that

$$x_\beta \in \eta \leq \lambda.$$

Definition 2.9. [8] A mapping $f: (X,\tau_X) \rightarrow (Y,\tau_Y)$ is called
i. fuzzy continuous, if $f^{-1}(\lambda)$ is an open fuzzy set of X, for each open fuzzy set λ of Y.
ii. fuzzy pre-γ-continuous , if $f^{-1}(\lambda)$ is pre-γ-open fuzzy set in X, for each open fuzzy set λ in Y.

3. FP_γgsg-CLOSED SETS AND FP_γgsg-OPEN SETS

This section extends the concept of fuzzy pre-γ-generalized semi generalized closed set in fuzzy topological space and presents its characterizations.

Definition: 3.1. A fuzzy set λ of fuzzy topological space (X,τ) is called a fuzzy pre-γ-generalized semi generalized closed set (in short, FP_γgsg-closed) if $P_\gamma Cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy pre-γ- semi generalized open set in (X,τ).

The complement of fuzzy pre-γ-generalized semi generalized closed set is fuzzy pre-γ-generalized semi generalized open set (in short, FP_γgsg-open) in (X,τ).

Remark: 3.1. For any fuzzy subset λ of fuzzy topological space (X,τ),

λ is FP_γgsg-open if and only if $1 - \lambda$ is FP_γgsg-closed.
λ is FP_γgsg-closed if and only if $1 - \lambda$ is FP_γgsg-open.

Theorem: 3.1. In a fuzzy topological space (X, τ), every fuzzy pre-γ-closed set is fuzzy pre-γ- gsg closed.
Proof : Let λ be fuzzy pre-γ-closed set and μ be any pre-γ-sg-open set such that $\lambda \leq \mu$. Since λ is fuzzy pre-γ-closed, $P_\gamma Cl(\lambda) = \lambda \leq \mu$. Hence λ is fuzzy pre-γ- gsg closed.

The reverse implication in the above theorem is not true as seen in the following example.

Example: 3.1. Let $X = \{a, b, c\}$ and fuzzy sets $\lambda_1, \lambda_2 \in I^X$ are defined by $\lambda_1(a) = 0.3, \lambda_1(b) = 0.7, \lambda_1(c) = 0.2, \lambda_2(a) = 0.2, \lambda_2(b) = 0.6, \lambda_2(c) = 0.1$. Let $\tau = \{0,1, \lambda_1, \lambda_2\}$. Clearly (X, τ) is fuzzy topological space. Define $\gamma: \tau \to I^X$ by $\gamma(1) = 1, \gamma(0) = 0$, $\gamma(\lambda_1,) = \lambda_1,, \gamma(\lambda_2) = cl(\lambda_2)$. Clearly λ_1 is fuzzy pre-γ- gsg closed but not fuzzy pre-γ-closed.

Theorem : 3.2. In a fuzzy topological space (X, τ), every fuzzy pre-γ-gsg closed set is fuzzy pre-γ-g closed.

Proof: Let λ be any fuzzy pre-γ-gsg closed set and μ be any fuzzy pre-γ- open set such that $\lambda \leq \mu$. Since every fuzzy pre-γ-open set is fuzzy pre-γ-sg-open and λ is fuzzy pre-γ-gsg closed, $P_\gamma Cl(\lambda) \leq \mu$. Hence λ is fuzzy pre-γ-g closed.

The reverse implication in the above theorem is not true as seen in the following example.

Example: 3.2. Let $X = \{a, b, \}$ and fuzzy sets $\lambda_1, \lambda_2 \in I^X$ are defined by $\lambda_1(a) = 0.2, \lambda_1(b) = 0.3, \lambda_2(a) = 0.4, \lambda_2(b) = 0.6$. Let $\tau = \{0,1, \lambda_1\}$. Clearly (X, τ) is fuzzy topological space. Define $\gamma: \tau \to I^X$ by $\gamma(1) = 1, \gamma(0) = 0$, $\gamma(\lambda_1) = Int(Cl(\lambda_1)), \gamma(\lambda_2) = Cl(\lambda_2)$. Clearly λ_1 is fuzzy pre-γ- g- closed but not fuzzy pre-γ-gsg-closed.

4. CHARACTERIZATION OF FP_γgsg-CLOSED SETS

Definition : 4.1. Let λ be any fuzzy set in (X, τ) then we define fuzzy pre-γ-gsg-Closure and fuzzy pre-γ-gsg-Interior as

FP_γgsg-Cl(λ) $= \wedge \{\mu : \mu$ is fuzzy pre-γ-gsg-closed and $\mu \geq \lambda\}$,
FP_γgsg- Int(λ) $= \vee \{\mu : \mu$ is fuzzy pre-γ-gsg -open and $\mu \leq \lambda\}$.

It is evident that

FP_γgsg-Cl(λ) $= \lambda$ if and only if λ is fuzzy pre-γ-gsg-closed.
FP_γgsg-Int(λ) $= \lambda$ if and only if λ is fuzzy pre-γ-gsg -open.
FP_γgsg-Cl(λ) is the smallest fuzzy set containing λ.
FP_γgsg- Int(λ) is the largest fuzzy set contained in λ.

Theorem : 4.1. Let λ be any fuzzy set in a fuzzy topological space (X, τ) .Then

FP_γgsg-Int$(1 - \lambda) = 1 - (FP_\gamma$gsg-Cl$(\lambda))$.
FP_γgsg-Cl$(1 - \lambda) = 1 - (FP_\gamma$gsg- Int$(\lambda))$.

Proof: (i) By definition,

FP_γgsg- Cl(λ) $= \wedge \{\mu : \mu$ is fuzzy pre-γ-gsg-closed and $\mu \geq \lambda\}$
$1 - FP_\gamma$gsg-Cl(λ) $= 1 - \wedge \{\mu : \mu$ is fuzzy pre-γ-gsg-closed and $\mu \geq \lambda\}$
$= \vee \{1 - \mu : \mu$ is fuzzy pre-γ-gsg-closed and $\mu \geq \lambda\}$
$= \vee \{\delta : \delta$ is fuzzy prepre-γ-gsg-open and $\delta \leq 1 - \lambda\}$
$= FP_\gamma$gsg-Int$(1 - \lambda)$

The proof is similar to (i)

Property : 4.1. If λ and μ are fuzzy sets in a fuzzy topological space (X, τ) Then the following are true.

$FP_\gamma gsg\text{-}Cl(0) = 0, FP_\gamma gsg\text{- }Cl(1) = 1$.

$FP_\gamma gsg\text{-}Cl(\lambda)$ is fuzzy pre-γ-closed in (X, τ).

$FP_\gamma gsg\text{-}Cl(\lambda) \leq FP_\gamma gsg\text{-}Cl(\mu)$ when $\lambda \leq \mu$.

$FP_\gamma gsg\text{-}Cl(\lambda) = FP_\gamma gsg\text{- }Cl(FP_\gamma gsg - Cl(\lambda))$.

$FP_\gamma gsg\text{-}Cl(\lambda \wedge \mu) \leq FP_\gamma gsg\text{- }Cl(\lambda) \wedge FP_\gamma\ gsg\text{-}Cl(\mu)$.

$FP_\gamma gsg\text{-}Cl(\lambda \vee \mu) = FP_\gamma gsg\text{-}Cl(\lambda) \vee FP_\gamma gsg\text{-}Cl(\mu)$.

Definition: 4.2. A fuzzy set λ in a fuzzy topological space (X, τ) is called fuzzy pre-γ-gsg-nhd of a fuzzy point x_κ if there exixts a F_{gsg}-open set μ such that $x_\kappa \in \mu \leq \lambda$.

Definition: 4.3. A fuzzy pre-γ-gsg-nhd, λ in a fuzzy topological space (X, τ) is said to be fuzzy pre-γ-gsg-open-nhd (resp. F_{gsg}-closed- nhd) if and only if λ is fuzzy pre-γ-gsg-open(resp. fuzzy pre-γ-gsg-closed).

Definition: 4.4. A fuzzy set λ in in a fuzzy topological space (X, τ) is called fuzzy pre-γ-gsg-q-nhd of a fuzzy point x_κ (resp.fuzzy set μ), if there exists a fuzzy pre-γ-gsg-open set δ in (X, τ) such that $x_\kappa q\delta \leq \lambda$ (resp. $\mu q\delta \leq \lambda$).

Theorem: 4.2. If λ and μ are fuzzy pre-γ-gsg-closed sets in a fuzzy topological space (X, τ) then $\lambda \vee \mu$ is fuzzy pre-γ-gsg-closed.

Proof : Let λ and μ be two fuzzy fuzzy pre-γ-gsg-closed sets in (X, τ) and let δ be any fuzzy pre-γ-sg-open set such that $\lambda \leq \delta$ and $\mu \leq \delta$. Therefore $P_\gamma Cl(\lambda) \leq \delta$ and $P_\gamma Cl(\mu) \leq \delta$.Since $\lambda \leq \delta$ and $\mu \leq \delta$, $\lambda \vee \mu \leq \delta$. Now $P_\gamma Cl(\lambda \vee \mu) = P_\gamma Cl(\lambda) \vee P_\gamma Cl(\mu) \leq \delta$. Hence $\lambda \vee \mu$ is fuzzy pre-γ-gsg-closed.

Theorem: 4.3. If λ and μ are fuzzy pre-γ-gsg-open sets in a fuzzy topological space (X, τ) then $\lambda \wedge \mu$ is fuzzy pre-γ-gsg-open.

Proof : Let λ and μ be two fuzzy fuzzy pre-γ-gsg-open sets in (X, τ). Then $1 - \lambda$ and $1 - \mu$ are fuzzy pre-γ-gsg-closed. By above theorem ,$(1- \lambda) \vee (1- \mu)$ is fuzzy pre-γ-gsg-closed. Since $(1 - \lambda) \vee (1 - \mu) = 1 - (\lambda \wedge \mu)$. Hence $\lambda \wedge \mu$ is fuzzy pre-γ-gsg-open.

Theorem: 4.4. If a fuzzy set λ is fuzzy pre-γ-gsg-closed in a fuzzy topological space (X, τ) and $P_\gamma Cl(\lambda) \wedge (1 - P_\gamma Cl(\lambda)) = 0$ then $P_\gamma Cl(\lambda) - \lambda$ does not contain any non-zero fuzzy pre-γ-sg-closed set in (X, τ).

Proof: Let λ be fuzzy pre-γ-gsg-closed in (X, τ) and $P_\gamma Cl(\lambda) \wedge (1 - P_\gamma Cl(\lambda)) = 0$. The result is proved by contradiction. Let μ be any fuzzy pre-γ-sg-closed set in (X, τ) such that $\mu \leq P_\gamma Cl(\lambda) - \lambda$ and $\mu \neq 0$. This gives $\mu \leq P_\gamma Cl(\lambda)$ and $\mu \leq 1- \lambda$. Therefore $\lambda \leq 1- \mu$, which is fuzzy pre-γ-sg-open. Since λ is fuzzy pre-γ-gsg-closed , $P_\gamma Cl(\lambda) \leq 1- \mu$. This implies $\mu \leq 1- P_\gamma Cl(\lambda)$. Therefore $\mu \leq P_\gamma Cl(\lambda) \wedge 1- P_\gamma Cl(\lambda) = 0$.
That is $\mu = 0$, which is a contradiction. Hence $P_\gamma Cl(\lambda) - \lambda$ does not contain any non- zero fuzzy pre-γ-sg-closed set in (X, τ).

Theorem: 4.5. A fuzzy set λ is fuzzy pre-γ-gsg-open in a fuzzy topological space (X, τ) if and only if $\mu \leq P_\gamma Int(\lambda)$ where μ is fuzzy pre-γ-gsg-closed and $\mu \leq \lambda$ in (X, τ).

Proof : Let $\mu \le P_\gamma Int(\lambda)$ where μ is fuzzy pre-γ-gsg-closed and $\mu \le \lambda$. Then $1-\lambda \le 1-\mu$ and $1-\mu$ is fuzzy pre-γ-sg-open. Now $P_\gamma Cl(1-\lambda) = 1 - P_\gamma Int(\lambda) \le 1-\mu$, by hypothesis. Then $1-\lambda$ is fuzzy pre-γ-gsg-closed. Hence λ is fuzzy pre-γ-gsg-open.

Conversely, let λ is fuzzy pre-γ-gsg-open and μis fuzzy pre-γ-sg-closed and $\mu \le \lambda$. Then $1-\lambda \le 1-\mu$. Since $1-\lambda$ is fuzzy pre-γ-gsg-closed and $1-\mu$ is fuzzy pre-γ-sg-open, $P_\gamma Cl(1-\lambda) \le 1-\mu$. Then $\mu \le P_\gamma Int(\lambda)$.

Theorem: 4.6. Let x_κ and λ be a fuzzy point and fuzzy set respectively in a fuzzy topological space (X,τ). Then $x_\kappa \in FP_\gamma gsg\text{-} Cl(\lambda)$ if and only if every fuzzy pre-γ- gsg-q-nhd of x_κ is q-coincident with λ.

Proof: The result is proved by the method of contradiction. Let $x_\kappa \in$ gsg-Cl(λ). Suppose there exists a fuzzy pre-γ- gsg- q -nhd μ of x_κ such that $\mu_{\bar{q}}\lambda$. Since μ is gsg- q -nhd of x_κ there exists fuzzy pre-γ-gsg-open set δ in (X,τ) such that $x_{\kappa\ q}\delta \le \mu$ which gives that $\delta_{\bar{q}}\lambda$ and hence $\lambda \le 1-\delta$. Then $FP_\gamma gsg\text{-} Cl(\lambda) \le 1-\delta$, as $1-\delta$ is fuzzy pre-γ-gsg-closed. Since $x_\kappa \notin 1-\delta$, we have $x_\kappa \notin FP_\gamma gsg\text{-} Cl(\lambda)$, a contradiction. Hence every fuzzy pre − γ − gsg-q-nhd of x_κ is q-coincident with λ.

Definition: 4.5. A fuzzy topological space (X,τ) is called a fuzzy pre-γ- T_{gsg} space if every fuzzy pre-γ-gsg-closet set in it is fuzzy pre-γ-closed.

Theorem: 4.7. Every fuzzy pre-γ $T_{1/2}$ space is fuzzy pre-γ- T_{gsg} space.
Proof: Let (X,τ) be a fuzzy pre-γ $T_{1/2}$ space and let λ be fuzzy pre-γ-gsg-closed set in (X,τ).Then λ is fuzzy pre-γ-g-closed. Since (X,τ) is $T_{1/2}$ space, λ is fuzzy pre-γ-closed in (X,τ). Hence (X,τ) is fuzzy pre-γ- T_{gsg} space.

Definition: 4.6. Let (X,τ_X) and (Y,τ_Y) be two fuzzy topological spaces and γ be a fuzzy operation on τ_X. A mapping $f: (X,\tau_X) \to (Y,\tau_Y)$ is called

i. fuzzy pre-γ-gsg-open if the image of every fuzzy pre-γ-open set of X is a fuzzy pre-γ-open set of Y.
ii. fuzzy pre-γ-gsg-closed if the image of every fuzzy pre-γ-closed set of X is a fuzzy pre-γ-closed set of Y.

Example: 4.1 Let $X = Y = \{a,b,c\}$ and $\lambda_1,\lambda_2,\lambda_3,\lambda_4 \in I^X$ defined as $\lambda_1(a) = 0.5, \lambda_1(b) = 0.8, \lambda_1(c) = 0.2, \lambda_2(a) = 0.4, \lambda_2(b) = 0{:}2, \lambda_2(c) = 0.3;$ $\lambda_3(a) = 0.4, \lambda_3(b) = 0.5, \lambda_3(c) = 0.6, \lambda_4(a) = 0.2, \lambda_4(b) = 0.5, \lambda_4(c) = 0.7$.Let $\tau_X = \{0,1,\lambda_1,\lambda_2,\lambda_3,\lambda_4\}$ and $\tau_X = \{0,1\}$ .Then (X,τ_X) and $(Y,\tau_Y)$are fuzzy topological spaces . Define an operation $\gamma: X \to I^X$ by $\gamma(1) = 1, \gamma(0) = 0, \gamma(\lambda_1) = \lambda_1, \gamma(\lambda_2) = \lambda_2, \gamma(\lambda_3) = Cl(\lambda_3), \gamma(\lambda_4) = \lambda_4$. $f: (X,\tau_X) \to (Y,\tau_Y)$ is defined as $f(a) = b; f(b) = a; f(c) = c$: Then f is a fuzzy pre-γ-gsg-open mapping.

Definition: 4.7. Let (X,τ_X) and (Y,τ_Y) be two fuzzy topological spaces and γ be a fuzzy operation on τ_X. A mapping $f: (X,\tau_X) \to (Y,\tau_Y)$ is called fuzzy pre-γ-gsg-continuous, if $f^{-1}(\lambda)$ is fuzzy pre-γ-open set in X, for every fuzzy pre-open set λ in Y.

Theorem: 4.8. For a mapping $f: (X,\tau_X) \to (Y,\tau_Y)$ is a fuzzy pre-γ-continuous mapping if and only if $f^{-1}(\lambda)$ is a fuzzy pre-γ-closed fuzzy set in in X, for every fuzzy pre-open set λ in Y.

5. CONCLUSION:

Throughout this paper, we have discussed the essential concepts and properties of fuzzy pre-γ-gsg-closed sets and their operations, shedding light on their adaptability and versatility. We expect that this research will lay the groundwork for future studies, encouraging researchers to explore new avenues and applications for fuzzy pre-γ-gsg-closed sets in a wide variety of domains, thereby contributing to the advancement of knowledge and the creation of innovative solutions to difficult problems.

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