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DAMPING AND OSCILLATION BEHAVIOR OF RLC CIRCUITS RESPONSES THROUGH RUNGE KUTTA METHOD OF ORDER -4

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Abstract:

Mathematical modeling of RLC circuit and general solutions of second order differential equation by finding the homogeneous and particular solution by means of complex exponential waveform are discussed in this paper. Further critically damped, over damped and under damped systems along with the oscillatory behavior are analyzed. The optimal solution of the RLC circuit is estimated through simulation results. Further to tune the circuit to a definite frequency, it is necessary to attenuate the amplitudes of all different frequencies and increase the amplitude with the required frequency. Therefore it is assumed that by maximizing the amplitude of the output current at a definite frequency which successfully tuned the circuit based on Runge Kutta method of order 4. Finally Critical damped, over damped and under damped responses are arrived through the simulation results.

Keywords:Parallel RLC circuit, Transient analysis, Runge Kutta method of order 4, damped responses.

I. INTRODUCTION

Application of various sources causes time-varying currents and voltages that adjust over different time intervals in the circuit, referred to as transients. This development usually happens throughout switching. In general an electrical circuit consists of a resistor (R), an inductor (L), and capacitor (C), connected in series or in parallel is called RLC circuit. This circuit is modeled by a second-order ordinary differential equation along with the time component in the general computational science. It's a numerical procedure for resolution ordinary differential equations with a given initial value. This method is taken into account in an extension of Euler's method within the two-stages of second order Runge Kutta methods [4].

The currents and voltages that fluctuate over time because of the abrupt application of sources induce a transient which fluctuates at various intervals of the circuit [2, 7]. This abrupt change of energy in circuits isn't provide a good atmosphere in parallel or series RLC Circuits, transistors, resistor and diodes which are embedded between integrated circuits. Transient analysis play vital role throughout this states, as a result its analyzes the response of electrical components in stable as well as unstable conditions [1, 12].

In general transient analysis methodology consists in measuring the performance of electrical circuits. The general structure of the variables are not modified over the period of time, then the state of a system is stable [9, 13]. The current within the LCR circuit (inductor, capacitance and resistance) in parallel depends not solely on the entity of the applied physical phenomenon [14]. However, the transient analysis at the nano-scale between a semiconductor unit in the hard drive, and motherboard is fatal to the system's existence [6, 10]. The system response was exposed when the conditions are modified from a steady-state value to a distinct value against the RLC circuit [3, 8]. This article reveals the effectiveness of the Runge Kutta methods for locating the second order differential equations.

This paper is organized as follows. Section II provides the necessary basic concepts related to RLC circuit and Runge Kutta methods. Mathematical modeling of RLC circuit and general solutions by means of complex exponential waveform are discussed in section III. Further critically damped, over damped and under damped responses along with the oscillatory behavior are explained. Section IV provides the optimal solution of the RLC circuit. Finally section V concludes the paper.

II Preliminaries

In this section, the fundamental concepts are discussed to derive the specific RLC circuit. In contrast to the Heun method, there are Runge Kutta methods of various orders. These methods are derived as

$$y_{n+1} = y_n + hy_n' + \frac{1}{2}h^2 y_n'' + \cdots + \frac{1}{p}h^p y_n^{(p)} + O(h^{p+1})$$

where h is the step function. The Runge Kutta method is an effective and optimal method for solving first-order ordinary differential equations [11]. Various differential equations along with explicit Euler method, Runge Kutta method of order three (RK3) and conjointly the Butcher's fifth order Runge Kutta method (BRK5) are to approximate the initial value problems (IVP) to solve for the voltage values based on circuit's design [5]. Since the resistor is one of the component of the circuit that resists the flow of electrical charge, and obeys the Ohm's law as follows:

$$V = IR$$

Generally V is the voltage which is applied in the resistor, I referred as the circuit current and R be the resistance. Here the resistance is assumed as constant. Capacitor is a circuit component which obeys the following equation,

$$V = \frac{Q}{C}$$

where Q is the charge, C be the capacitance (constant) of the capacitor, and V be the resulting voltage and denoted as:

$$IR = \frac{Q}{C}$$

The current I at which charge flows through the resistor is equal to the rate at which charge flows out of the capacitor. Hence

$$I = -\frac{dQ}{dt}$$

Hence,

$$\left(-\frac{dQ}{dt}\right)R = \frac{Q}{C}$$

$$\left(\frac{dQ}{dt}\right) = -\frac{1}{RC}Q$$

Finally, the relationship for resistor is noted as

$$V_r = L\frac{dI}{dt}$$

III Mathematical modeling of RLC circuit

By Kirchhoff's Voltage law, resistor, inductor and capacitor are defined as the sum of the voltage drop across the resistor V_r, inductor V_L and the capacitor V_C is equal to the time varying voltage V_t and is defined as :

$$V_r + V_L + V_C = V(t) \tag{1}$$

The modified resistance equation according to its voltage as:

$$RI(t) = V_L$$

where $I(t)$ be a time varied current. The integral component of the current based on the time component and defined

$$\frac{1}{C}\int_0^t I(\tau)d\tau + V(0) = V_C$$

$V(0)$ is the initial value of the voltage [4]. This should be a sine or cosine function with respect to time. Thus, assume $V(t) = V_0 \sin \omega t$. Substituting these values into equation (1) and we get:

$$L\frac{dI}{dt} + RI(t) + \frac{1}{C}\int_0^t I(\tau)d\tau + V(0) = V_0 \sin \omega t \tag{2}$$

Differentiate equation (2) with respect to t

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I(t) = V_0\, \omega \cos \omega t \tag{3}$$

Now find the particular solution by means of complex exponential waveform as

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I(t) = 0 \tag{4}$$

By Leibniz notation

$$I'' + \frac{R}{L}I' + \frac{1}{LC}I = 0 \tag{5}$$

Taking $I = e^{\lambda t}$, provide the characteristic equation

$$\lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC} = 0$$

Solving this equation for λ yields,

$$\lambda = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

By simplifying we get,

$$\lambda = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \tag{6}$$

By damping rate, let $\alpha = \frac{R}{2L}$ and the natural frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Then we get $\lambda_1 = -\alpha \pm \sqrt{\alpha^2 - {\omega_0}^2}$. Therefore, the roots of the characteristic equation are

$$\lambda_1 = -\alpha + \sqrt{\alpha^2 - {\omega_0}^2}$$

$$\lambda_2 = -\alpha - \sqrt{\alpha^2 - {\omega_0}^2}$$

and the homogeneous solution becomes

$$I_h = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}.$$

The parameters c_1 and c_2 are constant and might be determined by applying the initial conditions, we get $I(t = 0)\ and\ I'(t = 0)$.

The value of $\sqrt{\alpha^2 - {\omega_0}^2}$ determines the behavior of the responses. In general there are three kinds of responses are possible:

i. The system is critically damped when $\alpha = \omega_0$ then λ_1 and λ_2 are equal and are real numbers. Obviously no oscillatory behavior.

ii. The system is over damped when $\alpha > \omega_0$ then λ_1 and λ_2 are real numbers but are unequal and there is no oscillatory behavior.

iii. The system is under damped when $\alpha < \omega_0$ then$\sqrt{\alpha^2 - {\omega_0}^2} = j\sqrt{{\omega_0}^2 - \alpha^2}$. In this case λ_1 and λ_2 are complex numbers where $\lambda_1 = -\alpha + j\sqrt{{\omega_0}^2 - \alpha^2}$ and $\lambda_2 = -\alpha - j\sqrt{{\omega_0}^2 - \alpha^2}$ and the system exhibits oscillatory behavior.

The response of the series RLC circuit for these three totally different values are comparable to various input cases with these damping's. Critical damped response is shown in Figure 1, against the assumption is $I_n(0) = 0$. Over damped response is shown in Figure 2 and the assumption is $I_n(0) = 0$. Under damped response is shown in Figure 3 and the assumption is $I_n(0) = 0$.

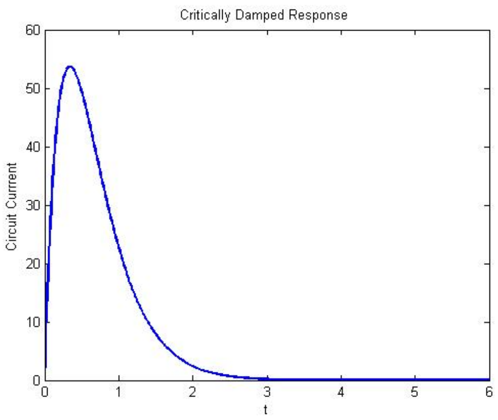

Figure 1 : Critically damped response

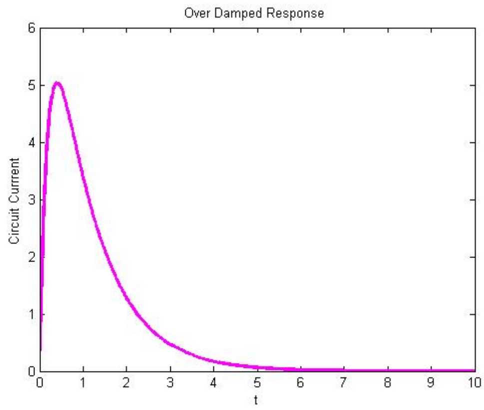

Figure 2: Over damped response

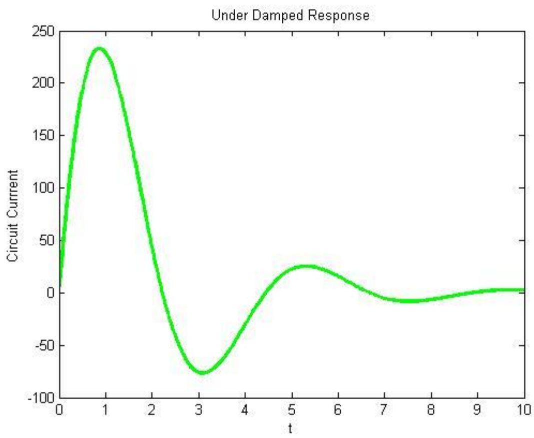

Figure 3 : Under damped response

IV Finding the optimal solution of the RLC circuit

To find the optimal solution of the RLC circuit, the particular solution for the system is found and victimization the strategy of indeterminate coefficients. The complex exponential waveform is widely accustomed to alter (simplify) the RLC circuit analysis. When we use the complex exponential function to express the voltage and current waveforms such as $e^{j\omega t}$, where j is the imaginary value. When we analyze the nature of the function, we come across that the function is still holding sinusoidal nature. In equation (3) $V_0\omega\cos(\omega t)$ is replaced with $V_0\omega j e^{j\omega t}$ such that

$$LI'' + RI' + \frac{1}{C}I = V_0\, j\omega e^{j\omega t}$$

Therefore, the particular solution is $I_p = a\, e^{j\omega t}$, where a is constant, the corresponding derivatives are

$$I_p{}' = a\omega j e^{j\omega t} \tag{7}$$

$$I_p{}'' = -a\omega^2 j e^{j\omega t} \tag{8}$$

Hence, we get

$$-La\omega^2 e^{j\omega t} I'' + aR\omega j e^{j\omega t} + \frac{1}{C} a e^{j\omega t} = V_0\, j\omega e^{j\omega t}$$

By dividing the whole equation by $e^{j\omega t}$, we get

$$-La\omega^2 + aR\omega j + \frac{1}{C}a = V_0\, j\omega$$

$$a\left(R + j\left(\omega L - \frac{1}{\omega C}\right)\right) = V_0$$

Solving for a and for the particular solution, we get

$$a = \frac{V_0}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$I_p = \frac{V_0}{R + j\left(\omega L - \frac{1}{\omega C}\right)} e^{j\omega t} \tag{9}$$

Further to evaluate the steady-state response for the circuit with the particular solution of the current, after applied the voltage. Hence the general solution becomes

$$I(t) = I_h + I_p$$

$$I(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + \frac{V_0}{R + j\left(\omega L - \frac{1}{\omega C}\right)} e^{j\omega t}$$

To identify the magnitude of equation (9):

$$|I_p| = \frac{|V_0 e^{j\omega t}|}{\left|R + j\left(\omega L - \frac{1}{\omega C}\right)\right|}$$

$$|I_p| = \frac{V_0}{\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)^{\frac{1}{2}}}$$

Now the circuit is tuned by the dynamical inductance and satisfy the following subsequent equation

$$\omega L - \frac{1}{\omega C} = 0$$

Solving for the frequency ω in terms of L and C gives

$$\omega^2 = \frac{1}{LC}$$

This maximizes the gain, as well as the quantitative relation of the amplitude and the steady state amplitude. By maximizing the gain for a definite frequency, the amplitude is effectively amplified by tuning the circuit to the frequency.

V Conclusion

It is concluded that by carryout the transient analysis, we are able to verify and identify the stable response of the system even the system undergoes the dynamical changes. Our examination obviously reveals that under damped decay is periodic, oscillatory and exponential. The other two critically damped and over damped systems are leads to non oscillatory exponential decay. Decomposition within the critically damped case is experimentally faster than over damped system. From the simulation results, it is concluded that numerical solutions are suitable in over damped systems than under damped and critically damped systems

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