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CUBIC CONVOLUTION AND OSCULATORY INTERPOLATION FOR IMAGE ANALYSIS

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Abstract: *This paper establishes a connection involving classical osculatory interpolation and convolution (cubic and higher order) based interpolation. These well-structured cubic convolution formally equivalent to oscillatory interpolation and modern convolution- schemes. Further it is discussed about the computational difference among the sample images of cubic interpolation. Furthermore separable bi-cubic convolution strategies are applied for image interpolation. This examines the theoretical and sensible problems with non-separable two-dimensional cubic and higher order convolution. This article expands two non-separable cubic convolution kernels. The primary kernel, has 3 parameters with constraints and focuss about biaxal symmetry (diagonal symmetry), continuity and smoothness of the sample image. The second kernel, is processed with higher order to arrive biaxial symmetry, diagonal symmetry, continuity and smoothness.*

Key Words: *Cubic spline and Osculatory interpolation, convolution, cubic convolution kernel*

I. *Introduction*

In the past five decades polynomial interpolation ways are studied and demonstrated broadly in the digital and medical image processing literature [9]. In the recent days it is extended in the field of natural language processing, which is understood to provide interpolants that are incessantly differentiable [3, 12]. Lot of uniform interpolants, as could also be necessary for a few applications; many various interpolation ways are projected [7].

Interpolation concepts plays a vital role in digital and bio medical image processing , notably in operations that need image re-sampling like resizing, registration, warping, and geometric distortion and correction [2 , 10]. Interpolation is often enforced with a image with kernel calculated according to its coefficient function [16]. Well known convolution interpolation ways embrace the nearest neighbor interpolation techniques. Cubic convolution provides an honest contribution in the quality of digital images from the sampled images [1, 15]. However, images are sometimes not statistically divisible. The primary kernel, with 3 parameters, relaxes the kernel value. Most general piecewise two-dimensional cubic interpolators are classified with symmetrical constraints [4, 6]. The second kernel, with higher parameters relaxes the diagonal symmetry constraint, supported the observation of input images and its rotation [14].

The structure of the paper is as follows. Section II formulates the interpolation which supports the convolution. Section III dealt with osculatory interpolation which has been represented as the type of interpolation within the corresponding sequence of interpolation curves which are employed and forming a compound curve adjacent to particular range (cubic spline, fourth order cubic spline and so on) of its derivatives, on various interpolation intervals. Section provides the details about the derivations of for image interpolation of two parameters to five parameters. Finally section V concludes the paper.

II Interpolation supported convolution

The convolution-based uniformly sampled information involves the kernel as $\varphi: \mathbb{R} \rightarrow \mathbb{R}$, which determines the weights. Consider the samples as $f_k = f\,(kT)$ where $f: \mathbb{R} \rightarrow \mathbb{R}$, however without loss of generality, use $T = 1$ and $k \in \mathbb{Z}$. If so, this method is often described as

$$\tilde{f}(x) = \sum_{k \in \mathbb{Z}} f_k \, \varphi(x - k) \qquad (1)$$

$\tilde{f}$ is described as interpolator and the kernel φ proves the factors $\varphi(0) = 1$ and $\varphi(k) = 0, \forall\, k \neq 0$. The familiar case is the theory related to ideal synchronization which operates the digital images and the process and the purpose of this digital image processing is highly attracted by the researchers in the recent years. Different examples are computationally fascinating, however in theory establishes the perfect line from interpolation kernel and nearest neighbor [5].

The cubic convolution kernel family provides significantly higher compromise between the process of accurate data and noisy data. These creates the piecewise interrogatory polynomials and are continuously differentiable [11]. The primary approximate order is 3, which means that the ensuing interpolator converges to the initial operate as quick as possible between the samples. It conjointly implies that the kernel is ready to sort out the second degree polynomials [8]. This hypothetical concepts are outlined as follows:

$$\varphi_{CC3}(x) = \begin{cases} \frac{3}{2}|x|^3 - \frac{5}{2}|x|^2 + 1 & if\ 0 \leq |x| \leq 1 \\ -\frac{1}{2}|x|^3 + \frac{5}{2}|x|^2 - 4|x| + 2 & if\ 1 \leq |x| \leq 2 \\ 0 & if\ 2 \leq |x| \end{cases} \qquad (2)$$

To maintain the best degree of polynomial as $n = 3$, further we can extend a cubic convolution kernel with approximation order 4 and is defined as

$$\varphi_{CC4}(x) = \begin{cases} \frac{4}{3}|x|^3 - \frac{7}{3}|x|^2 + 1 & if\ 0 \leq |x| \leq 1 \\ -\frac{7}{12}|x|^3 + 3|x|^2 - \frac{59}{12}|x| + \frac{5}{2} & if\ 1 \leq |x| \leq 2 \\ \frac{1}{12}|x|^3 - \frac{2}{3}|x|^2 + \frac{7}{4}|x| - \frac{3}{2} & if\ 2 \leq |x| \leq 3 \\ 0 & if\ 3 \leq |x| \end{cases} \qquad (3)$$

III Recursive Osculatory Interpolation

This section dealt with osculatory interpolation which has been represented as the type of interpolation within the corresponding sequence of interpolation curves which are employed and forming a compound curve adjacent to particular range (cubic spline, fourth order cubic spline and so on) of its derivatives, on various interpolation intervals.

Exclusively this osculatory interpolation involves when central variations of the given multiple samples is very high. Obviously these are defined as:

$$\tilde{f}(x) = \tilde{f}(k+\xi) + F(\xi,\delta)f_{k+1} + F(1-\xi,\delta)f_k \quad (4)$$

$$F(x,\delta) = F_{KK}(x,\delta) = x + \frac{1}{2}x^2(x-1)\delta^{2i} \quad (5)$$

With $k = |x|$, $0 \le \xi \le 1$, and $F(x,\delta) = \sum_{i=0}^{imax} F_i(x)\,\delta^{2i}$ for some i_{max} , wherever F_i are polynomial functions appropriately chosen in x and its ensuing interpolator which satisfies the specified criteria like smoothness of the images [13]. Here, the central differences of the p-th order δp of any operation g is outlined as

$$\delta^p g(x) = \delta^{p-1}g\left(x+\frac{1}{2}\right) - \delta^{p-1}g\left(x-\frac{1}{2}\right),$$

To extend and simplified (5), we get

$$F(x,\delta) = F_H(x,\delta) = x + \frac{1}{6}x(x^2-1)\delta^2 - \frac{1}{12}x^2(x-1)\delta^4 \quad (6)$$

Proceeding in an identical approach we acquired the osculatory interpolation as

$$\delta^{2i}f_k = \sum_{m=0}^{2i}\binom{2i}{m}(-1)^m f_{k-m+i} \quad (7)$$

which is valid for all $i \ge 0$ integers. further, we get

$$\tilde{f}(x) = \tilde{f}(k+\xi) = \sum_{i=0}^{i_{max}}\sum_{m=0}^{2i}\binom{2i}{m}(-1)^m[F_i(\xi)f_{k-m+i+1} + F_i(1-\xi)f_{k-m+i}] \quad (8)$$

In general $\beta^1(x)$ is the interpolation kernel, or B-spline of degree and the facts that $\beta^1(-x) = \beta^1(x), \forall x \in \mathbb{R}$. It is further written like $F_i(\xi)f_{k-m+i+1} + F_i(1-\xi)f_{k-m+i}$ and are often combined with $\sum_{i\in\mathbb{Z}} F_i\left(\beta^1(x-k-l)\right) f_{k-m+i+l}$, hence we get

$$\varphi(x) = \sum_{i=0}^{i_{max}}\sum_{m=0}^{2i}\binom{2i}{m}(-1)^m F_i\left(\beta^1(x-m+i)\right) \quad (9)$$

Taking $i_{max} = 1$, and $F_1(x) = \frac{1}{2}x^2(x-1)$, and simplifying (9) we get the kernel of osculating interpolation which is exactly same as in (2). Simultaneously, considering $i_{max} = 2$, $F_1(x) = \frac{1}{6}x(x^2 - 1)$ and $F_2(x) = -\frac{1}{12}x^2(x-1)$, and hence we discover the higher order core style of osculatory interpolation is exactly as in (3).

Therefore, it seems that the osculating convolution-based interpolation schemes provide quicker algorithms, however needs a lot of memory.

Using (9) we determine it's kernel as

$$\varphi(x) = \begin{cases} \frac{7}{9}|x|^3 - \frac{3}{2}|x|^2 - \frac{5}{18}|x| + 1 & if\ 0 \le |x| \le 1 \\ -\frac{11}{36}|x|^3 + \frac{7}{4}|x|^2 - \frac{28}{9}|x| + \frac{5}{3} & if\ 1 \le |x| \le 2 \\ \frac{1}{36}|x|^3 - \frac{1}{4}|x|^2 + \frac{13}{18}|x| - \frac{2}{3} & if\ 2 \le |x| \le 3 \\ 0 & if\ 3 \le |x| \end{cases} \quad (10)$$

This kernel's approximation order is 4. The central three-dimensional Lagrange interpolation of the kernel is given by $F_1(x) = x(x-1)\left(\left(2\alpha + \frac{1}{2}\right)x - \alpha\right)$, and $F_2(x) = \frac{1}{2}\alpha x^2(x-1)$. Due to the parameter, α, it constitutes an entire family of three-dimensional interpolation whose general type follows from (9) as

$$\varphi(x) = \begin{cases} \left(\alpha + \frac{3}{2}\right)|x|^3 - \left(\alpha + \frac{5}{2}\right)|x|^2 + 1 & if\ 0 \le |x| \le 1 \\ \frac{1}{2}(\alpha - 1)|x|^3 - \left(3\alpha - \frac{5}{2}\right)|x|^2 + \left(\frac{11}{2}\alpha - 4\right)|x| - (3\alpha - 2) & if\ 1 \le |x| \le 2 \\ -\frac{1}{2}\alpha|x|^3 + 4\alpha|x|^2 - \frac{21}{2}\alpha|x| + 9\alpha & if\ 2 \le |x| \le 3 \\ 0 & if\ 3 \le |x| \end{cases} \quad (11)$$

Analyzing (11), when $\alpha = 0$ and $\alpha = -\frac{1}{6}$, for every $\alpha \in \mathbb{R}$, its ensuing that the kernel has a minimum regularity and approximation order is 3. Finally, assume that

$$F_1(x) = x(x-1)\left(\left(2\alpha + \frac{1}{2}\right)x - \alpha\right)$$

$$F_2(x) = x\left(\left(\frac{1}{2}\alpha + 2\beta\right)x^2 - \left(\frac{1}{2}\alpha + 3\beta\right)x + \beta\right), \quad \text{and}$$

$$F_3(x) = \frac{1}{2}\beta x^2(x-1).$$

The overall pattern is expressed as

$$\varphi(x) = \begin{cases} \left(\alpha - \frac{5}{2}\beta + \frac{3}{2}\right)|x|^3 - \left(\alpha - \frac{5}{2}\beta + \frac{5}{2}\right)|x|^2 + 1 & 0 \le |x| \le 1 \\ \frac{1}{2}(\alpha - \beta - 1)|x|^3 - \left(3\alpha - \frac{9}{2}\beta - \frac{5}{2}\right)|x|^2 + \left(\frac{11}{2}\alpha - 10\beta - 4\right)|x| - (3\alpha - 6\beta - 2) & 1 \le |x| \le 2 \\ -\frac{1}{2}(\alpha - 3\beta)|x|^3 + \left(4\alpha - \frac{25}{2}\beta\right)|x|^2 - \left(\frac{21}{2}\alpha - 34\beta\right)|x| - (9\alpha - 30\beta) & 2 \le |x| \le 3 \\ -\frac{1}{2}\beta|x|^3 + \frac{11}{2}\beta|x|^2 - 20\beta|x| + 24\beta & 3 \le |x| \le 4 \\ 0 & 4 \le |x| \end{cases} \quad (12)$$

IV Cubic interpolation parameters formulation

This section outlines the derivations of for image interpolation of two parameters to five parameters. Image interpolation makes an attempt to recreate the sample images as $s(x,y), (x,y \in R)$ is a sample image consisting of uniformly spaced from the normalized spatial coordinates. Without loss of generality, the interpolation is usually enforced by convolving the sample image as $s[m,n], m,n \in R$ with a kernel $f(x,y)$ where $(x,y \in R)$ and are denoted as follows:

$$r(x,y) = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} s[m,n] f(x-m, y-n) \quad (13)$$

In the general observation, the spatial image is convolved, and process through Fourier frequency domain as

$$\tilde{r}(u,v) = \tilde{f}(u,v) \sum_{u=-\infty}^{+\infty} \sum_{v=-\infty}^{+\infty} \tilde{s}(u-\mu, v-\gamma) \tag{14}$$

where $\tilde{r}(u,v)$, $\tilde{f}(u,v)$ and $\tilde{s}(u,v)$ are the Fourier transforms of the sample image $r(x,y)$, and this leads to the optimal image $s(x,y)$. Several well-known image interpolation strategies are outlined in this manner, together with nearest neighbor interpolation.

Consider the region $[-2,2] \times [-2,2]$ has sixteen unit-sized items, every bit with sixteen parameters, for example,

$$f(x,y) = \sum_{j=0}^{3} \sum_{k=0}^{3} a_{jk} x^j y^k \tag{15}$$

$0 \leq x \leq 1, 0 \leq y \leq .$ This permits $16 \times 16 = 256$ parameters. The massive varieties of parameters are often reduced by constrictive kernel. These parameters developed subject to image interpolation which includes symmetry, continuity, and smoothness.

These constraints, observes the cubic-convolution kernel of 3 coefficients say (a_{33}, a_{32}, a_{30}). Further it is extended to the alternative quadrants outlined by line symmetry as $f(-x,y) = f(x,y)$ and $f(x,-y) = f(x,y)$. Piecewise cubic interpolator are also developed in terms of the consequent parameter β are derived as

$$a_0 = \alpha + 2$$

$$a_1 = \beta + (\alpha + 2)^2$$

$$a_2 = \gamma - (\alpha + 2)(\alpha + 3) - \beta \tag{16}$$

With these parameters, we have the following functions:

$$f(x,y) = \big(f_0(x) + \alpha f_1(x)\big)\big(f_0(y) + \alpha f_1(y)\big) + \beta f_1(x) f_1(y) + \gamma f_2(x,y) \tag{17}$$

where f_0 and f_1 are the convolution functions and f_2 is additional non separable function parameterized by γ .

The higher order parameter comes with constraints of symmetry and smoothness through interpolation. Cubic convolution is often needs additional details on these constraints and the non-separable symmetric higher order parameter's kernel. This is reduced to

$$f(x,y) = a_{33} f_{33}(x,y) + a_{32} f_{32}(x,y) + a_{23} f_{23}(x,y) + a_{30} f_{30}(x,y) + a_{03} f_{03}(x,y) + f_{00}(x,y) \tag{18}$$

where $(a_{33}, a_{32}, a_{23}, a_{30}, a_{03})$ are the 5 parameters.

V Conclusion

In this article we derived the general expression for the implicit classical osculatory interpolation schemes. In addition its variations, from cubic to higher order osculatory versions are computationally done. The interpolated images are to limit the consequences and the visual variations for many interpolated results are aliasing two parameters to five parameters. Our results progresses in various analysis like extending non-separable cubic form convolution to image restoration and exploitation of alternative fields.

Furthermore separable bi-cubic convolution strategies are applied for image interpolation. This examines the theoretical and sensible problems with non-separable cubic and higher order convolution. This article expands two non-separable two-dimensional cubic convolution kernels. The primary kernel, has 3 parameters with constraints and focuss about biaxal symmetry (diagonal symmetry), continuity and smoothness of the sample image. The second kernel, is processed with higher order to arrive biaxal symmetry, diagonal symmetry, continuity and smoothness.

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