IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)
An International Open Access, Peer-reviewed, Refereed Journal

USING TAKASAKI QUANDLES TO DETECT CAUSALITY

[1]Siddhant Sukhani

[1]Student Researcher
[1]Mathematics Research
[1]Jayshree Periwal International School, Jaipur, India

Abstract: This paper aims to verify the ability of the Takasaki quandles to distinguish between the Allen-Swenberg link and the connected sum of two Hopf links. The results obtained are hence suggestive of the extra information required for the Alexander Conway polynomial to distinguish causality in 2+1 globally hyperbolic space-times for all events when the Cauchy surface has R2 for the universal cover. The given quandle was tested for all primes under 50 to check whether the Takasaki quandle modular prime is able to provide a solution that may suggest causality. As the number of homomorphisms given by the Takasaki quandle for all primes for the Allen-Swenberg Link is equal to the number of homomorphisms of the connected sum of two Hopf links. As the links are found to not be distinguishable from one another, the Takasaki Quandle hence provides the information that it can not be utilised to obtain the extra information required for the Alexander Conway polynomial to distinguish causally related events in this particular example.

***Index Terms* - Knot Theory, Causality, Globally Hyperbolic Spacetimes**

I. INTRODUCTION

This paper attempts to utilise the relation between knot theory and causality in order to determine whether the Takasaki quandle computed modular all primes under 47 are able to distinguish between the Allen-Swenberg Link and the connected sum of two Hopf links. The significance of this research lies in its ability to provide the additional information required in addition to the Alexander Conway polynomial in order to detect causality. As stated by Allen-Swenberg [2], the Jones polynomial is able to distinguish between the two aforementioned links as well as a series of other more complicated links that the Alexander Conway polynomial fails to distinguish. Chernov Nemirovski theorem says that if two links are different from the connected sum of the two Hopf links then the evens are causally related. However, if the chosen quandle is able to distinguish between the two, it provides additional information that enables it to determine the causal nature of the two events in the cases where the Alexander-Conway polynomial can not solve the problem.

A. GLOBALLY HYPERBOLIC SPACE-TIMES

Globally hyperbolic space-times are defined by the existence of a Cauchy surface - defined as being a submanifold of the Lorentzian manifold and being representative of an instant of time - such that every causal curve does intersect this Cauchy surface exactly once.A global hyperbolic is, technically, a space time that possesses the intersections of a past and a future point as a compact intersection and it is causal. Completely naked singularities and the various breaks in the causal relation can lead to a possible lack of uniqueness of solutions and do not appear in such space-times In order to obtain a causal relation between two points x,y there must be a curve γ connecting these points with $\gamma'(t)$. $\gamma'(t) < 0$. This is synonymous with saying that one can get from x to y without exceeding the speed of light. The total space ST* Σ of a spherical cotangent bundle of a Cauchy surface Σ is utilised in order to identify the space N_x, of unparameterized future-directed light rays in X. As the light rays pass through the 'x' point, they are then identified with the sphere $S_x \subset N_x$ which is known as the sky of x.

B. CAUSALITY WITH KNOTS

As given in the paper by Chernov and Rudyak[1], their discussion of the universal cover of $\tilde{x}$ of a globally hyperbolic (2+1) dimensional space-time X with a Cauchy surface Σ is a globally hyperbolic space-time whose Cauchy surface $\tilde{\Sigma}$ is the universal cover of Σ. In this paper, the research would focus on two points $x,y \in X$ which are two causally related points with a path γ such that $\gamma'(t)\gamma''(t) \leq 0$ connecting them. In the case explored by them, the universal covering of Σ is not S^2 and hence it is proven that it must be homeomorphic to $\mathbb{R}^2$ and by utilising the same concept of covering, it is shown by Chernov, Martin and Petkova that the Khovanov homology detects causality for all globally (2+1) dimensional space-times whose Cauchy surfaces are not homeomorphic to S^2 or $\mathbb{R}P^2$?

The Allen-Swenberg analysis to detect causality utilises a series of 3-component links and attempts to distinguish them from the connected sum of two Hopf links - which is the link representation of two causally unrelated events as mentioned in section 4.1 of this paper. The Allen - Swenberg link that is utilised within this research is a part of a series of links that are non-distinguishable as compared to the connected sum of two Hopf links by the Alexander - Conway polynomial.

II. TOOLS UTILISED

A. QUANDLES

Quandles see for example the textbook of Nelson et al [7] is a set that satisfies the following algebraic relations. The quandle function is hence non associative and non commutative, allowing it to be used for computing various homomorphisms/colourings for a given quandle. The operation of a quandle must satisfy the following axioms where the quandle is in the set X:

1. $x \triangleright x = x$
2. There is a unique element $x \in X$ for elements y,z $\in X$ that satisfies $z = x \triangleright y$
3. The operation is self-distributive in the following form $(x \triangleright y) \triangleright z = (x \triangleright z) \triangleright (y \triangleright z)$The quandle

operation further extends by having a 'right inverse' which is indicated by $\triangleright^{-1}$.

A relation between knots and Quandles is established by using the quandle colourings via finding a positive or negative crossings in the following manner:

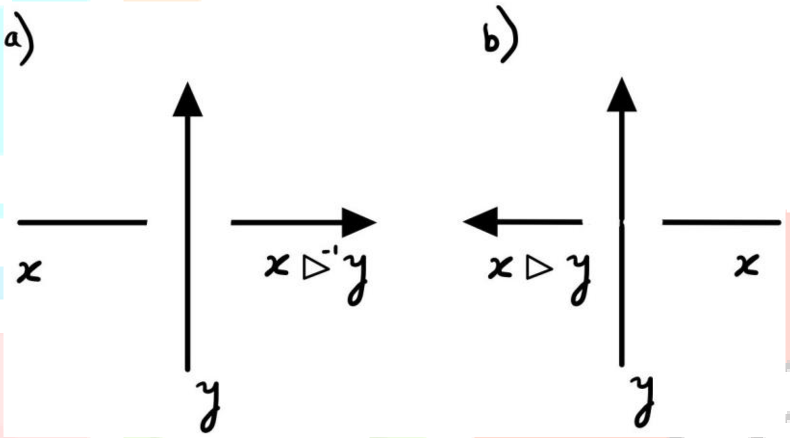

Figure 1 : Positive(b) and Negative(a) crossings with Quandles

The axioms above are related with knots in the following manner:

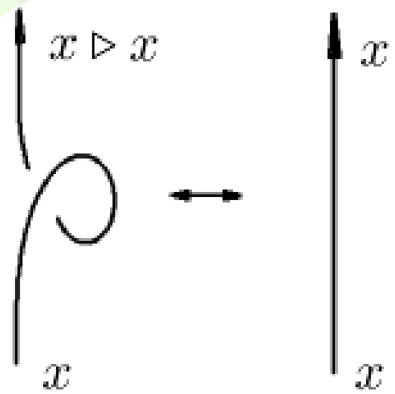

Figure 2 : First Reidmeister move using Quandles[5]

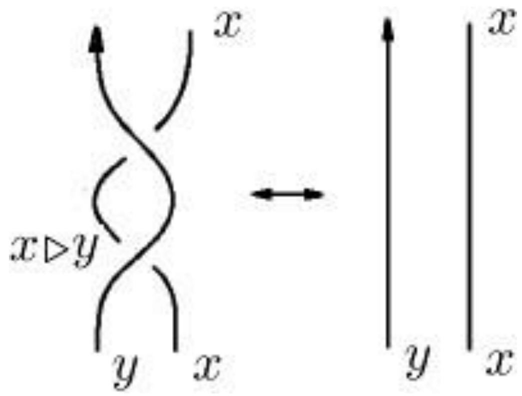

Figure 3 : Second Reidmeister move using Quandles[5]

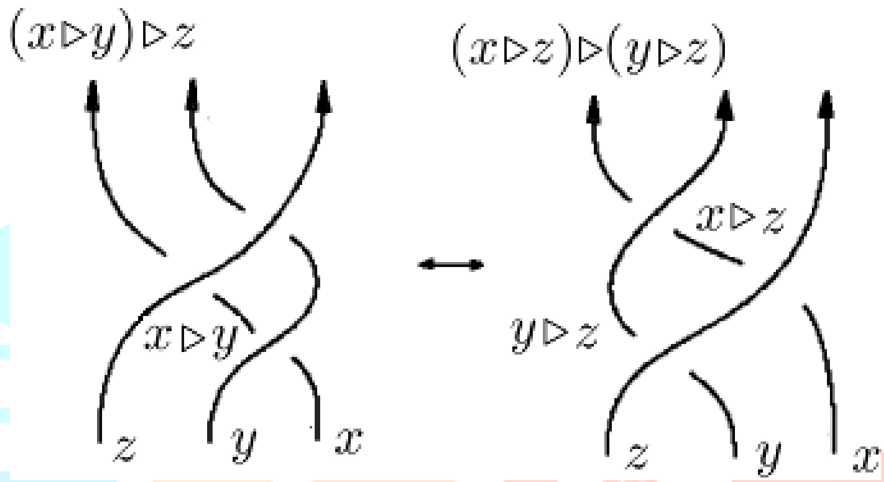

Figure 4 : Third Reidmeister move using Quandles[5]

For any given quandle $\mathbb{Z}_n$, the result obtained must be taken to the n^{th} modular as this would prevent the total number of solutions from blowing up.

1)ALEXANDER AND TAKASAKI QUANDLES

The Alexander quandle is used in order to find equations for various knots and use the quandle operations to find the homomorphisms of the so called fundamental quandle into a given quandle or which is the same quandle colourings of the given knot. The Alexander quandle structure over integers modulo n, i.e. $\mathbb{Z}_n$, can be defined by equation 1, once α is provided to be invertible in $\mathbb{Z}_n$.

$$x \triangleright y = \alpha x + (1 - \alpha)y \qquad (1)$$

The values of n and α must be co-prime integers. This infamous quandle discovered is useful to find the total number of quandle homomorphisms into it. These homomorphisms are synonymous to the colouring of the knots under the Alexander Quandle.

The Takasaki Quandle is an example of the Alexander quandle but it is also extremely powerful in detecting causality. For the Takasaki Quandle, the value of α that is taken is -1. When this is plugged into equation 2 the following result is obtained for any $\mathbb{Z}_n$ where n and α are co-prime:

$$x \triangleright y = -x + 2y \qquad (2)$$

equation (2) represents the Takasaki quandle and the homomorphisms obtained by the equation must be put under a mod of the value of n.

B.HOMOMORPHISMS AND COLOURINGS

When any quandle or rack is applied to any given knot, a set of equations containing the various variables is formed. A set of solutions is then formed from solving the set of equations mod n. The number of solutions given within the set is known as the number of homomorphisms of the fundamental quandle into the quandle of our choice. It hence represents the different manners in which the knot can be coloured in for it to still represent the information which it conveys. Homomorphisms are hence structures of a particular shape within algebraic structure preserving maps. On the other hand, quandle homomorphisms are classified by their property of being a self preserving map of $f : x \rightarrow y$ which respects the operation

$$f(x \triangleright y) = f(x) \triangleright f(y).$$

In the case of the Allen-Swenberg link [2] and Connected sum of two Hopf Links, if the number of homomorphisms is not the same, there the quandle provides extra information which together with the Alexander-Conway polynomial is plausibly enough to capture causality.

III. LITERATURE REVIEW

An introduction to knot theory is provided by J.S. Carter [6] as he defines the various initial concepts such as the positive and negative crossings; his study further extends by describing the handle decomposition of classical knots complements such as the trefoil and the unknot. He further analyses the Alexander-Briggs presentation of knots in order to find the 3 dimensional handle decomposition of knot complements.

The relation amongst knot/link theory and causality as established by Chernov and Nemirovski [3] in their analysis of the Low conjecture, Legendrian links and Causality. By their definition, space-time is known to be causal - if in it there are no closed future pointing causal curves.

On the other hand, D. Joyce [10] commences the era of Quandles and racks by defining the fundamental quandle.

This analysis of Quandles is then further extended by Nelson [7] as he describes the Alexander quandle using the same equation as equation(1). Using this equation as a base, Nelson describes two connected quandles as isomorphic only if they are isomorphic under the Λ module. This result holds specifically true for a finite a pair of Alexander quandles - M,N.

The colouration of these Quandles is further classified in Bae [4] as he classifies trivial and non trivial colourings through the Alexander polynomial while working with Alexander Quandles. This perspective utilises a reduced Alexander Polynomial in order to find the colourability of link diagrams. Using this methodology, he is able to define a non trivial colouring table for various knots using an Alexander quandle.

IV. TAKASAKI QUANDLES IN $\mathbb{Z}_7$

The Takasaki Quandle as previously mentioned in equation (2) will be applied to both the Allen Swenberg link and the connected sum of two Hopf Links in order to compare the total number of homomorphisms that can form under the modulo of all primes from 2 to 47. The answers for this set of equations would then be applied under the mod 7 for this specific example. The total number of solutions would then represent the number of colorings that could form.

The research for the Takasaki quandles would first begin by labelling each strand of the knot as well as the crossings. Then these crossings would either be classified as positive or negative crossings in order to determine the configuration of the quandle that would be utilised. After this is done, a system of equations would be derived by plugging in the labeling of the strands into the Takasaki quandle.

A. CONNECTED SUM OF TWO HOPF LINKS

1) ORIGIN OF THE CONNECTED SUM OF TWO HOPF LINKS

The knots within the space N_x of our class of (2 + 1)-dimensional globally hyperbolic space times is the solid torus. When the Cauchy surface of X is homeomorphic to $\mathbb{R}^2$, the skies of two causally unrelated events as mentioned in section 1.1 of this paper are circles in the solid torus $N_x = S^1 \times \mathbb{R}^2$, the skies are isotopic to the longitude of the solid torus. This is further extended by allow the two knots to be the unlinked longitudes that eventually can be taken under a set of Reidmeister moves to form the connected sum of two Hopf links.

2) Labelling the connected sum of two Hopf links

Using the same method as section 4.1.2, a labeling of the link with colourings for each segment of the knot with specific labelling for the crossings is completed as seen below.

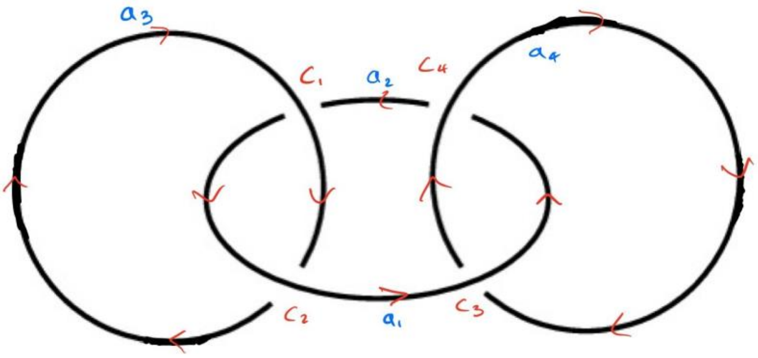

figure 5 : Labeled connected sum of 2 Hopf links

As there are 4 crossings, there can be a variety of homomorphisms. The segment labeling and the orientation of the knot is shown in blue.

3) Takasaki computations for the connected sum of two Hopf links

The table 3 below shows the Takasaki quandle equations at each crossing. These would then be solved manually under modulo 7 and then the number of homomorphisms would be compared with that of the Allen-Swenberg Link.

Crossing No.	Quandle Computation	Takasaki Quandle
1	$A_2 = A_1 \triangleright A_3$	$A_2 = -A_1 + 2A_3$
2	$A_3 = A_3 \triangleright A_1$	$A_3 = -A_3 + 2A_1$
3	$A_4 = A_4 \triangleright A_1$	$A_4 = -A_4 + 2A_1$
4	$A_2 = A_1 \triangleright A_4$	$A_2 = -A_1 + 2A_4$

Table 1: Takasaki Quandle for the connected sum of two Hopf Links

Solving the system of linear equations in the following manner under modulus 7, the following results are obtained:

$$2A_3 = 2A_1$$

$$2A_4 = 2A_1$$

As both sides can be divided by 2 under the modulus of 7, the following results is obtained:

$$A_3 = A_1 \quad (4)$$

$$A_4 = A_1 \quad (5)$$

Plugging in equation (6) into crossing 1, the following result is found:

$A_2 = -A_1 + 2A_1$

$$A_2 = A_1 \quad (6)$$

Hence, all the variables are equal to one another and the total number of homomorphisms for this computation is equal to 7

as
seen
in the

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

following solution set:

B.ALLEN SWENBERG LINK

1)LABELLING ALLEN SWENBERG LINK

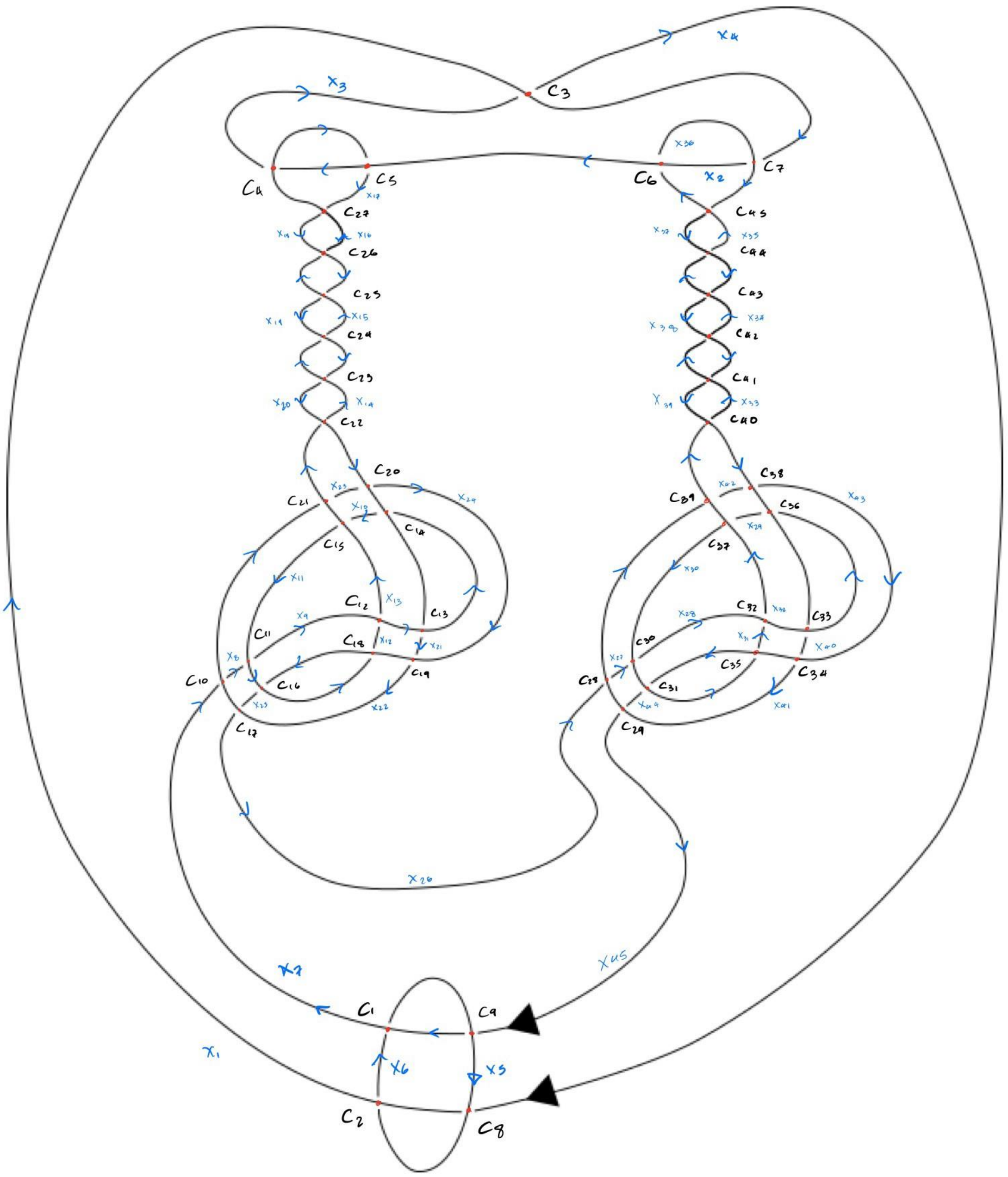

figure 6 : Labeling of the Allen Swenberg link

As seen above, the segments of the links are labelled in blue, starting from X_1 to X_{45}. On the other hand, the points of crossing are labelled in black, starting from C_1 to C_{45}. The direction is also labeled in blue with the segments of the links to help find the orientation of the knot, which is fundamental for the quandle computations.

2)Takasaki computations for the Allen Swenberg Link

The set of equations obtained from the figure 5 above are placed in the table below

Crossing No.	Quandle Computation	Takasaki quandle
1	$X_6 = X_5 \triangleright X_7$	$X_6 = -X_5 + 2X_7$
2	$X_5 = X_6 \triangleright X_1$	$X_5 = -X_6 + 2X_1$
3	$X_4 = X_3 \triangleright X_1$	$X_4 = -X_3 + 2X_1$
4	$X_3 = X_2 \triangleright X_{16}$	$X_3 = -X_2 + 2X_{16}$
5	$X_{17} = X_{16} \triangleright X_2$	$X_{17} = -X_{16} + 2X_2$
6	$X_5 = X_6 \triangleright X_2$	$X_5 = -X_6 + 2X_2$
7	$X_1 = X_2 \triangleright X_{37}$	$X_1 = -X_2 + 2X_{37}$
8	$X_4 = X_1 \triangleright X_5$	$X_4 = -X_1 + 2X_5$
9	$X_{45} = X_7 \triangleright X_5$	$X_{45} = -X_7 + 2X_5$
10	$X_7 = X_8 \triangleright X_{22}$	$X_7 = -X_8 + 2X_{22}$
11	$X_8 = X_9 \triangleright X_{11}$	$X_8 = -X_9 + 2X_{11}$
12	$X_{18} = X_{13} \triangleright X_9$	$X_{18} = -X_{13} + 2X_9$
13	$X_{20} = X_{21} \triangleright X_9$	$X_{20} = -X_{21} + 2X_9$

14	$X_9 = X_{10} \triangleright X_{20}$	$X_9 = -X_{10} + 2X_{20}$
15	$X_{11} = X_{10} \triangleright X_{13}$	$X_{11} = -X_{10} + 2X_{13}$
16	$X_{24} = X_{25} \triangleright X_{11}$	$X_{24} = -X_{25} + 2X_{11}$
17	$X_{26} = X_{25} \triangleright X_{22}$	$X_{26} = -X_{25} + 2X_{22}$
18	$X_{11} = X_{12} \triangleright X_{24}$	$X_{11} = -X_{12} + 2X_{24}$
19	$X_{22} = X_{21} \triangleright X_{24}$	$X_{22} = -X_{21} + 2X_{24}$
20	$X_{24} = X_{23} \triangleright X_{20}$	$X_{24} = -X_{23} + 2X_{20}$
21	$X_{22} = X_{23} \triangleright X_{13}$	$X_{22} = -X_{23} + 2X_{13}$
22	$X_{14} = X_{13} \triangleright X_{20}$	$X_{14} = -X_{13} + 2X_{20}$
23	$X_{20} = X_{19} \triangleright X_{14}$	$X_{20} = -X_{19} + 2X_{14}$
24	$X_{15} = X_{14} \triangleright X_{19}$	$X_{15} = -X_{14} + 2X_{19}$
25	$X_{19} = X_{18} \triangleright X_{15}$	$X_{19} = -X_{18} + 2X_{15}$
26	$X_{16} = X_{15} \triangleright X_{18}$	$X_{16} = -X_{15} + 2X_{18}$
27	$X_{18} = X_{17} \triangleright X_{16}$	$X_{18} = -X_{17} + 2X_{16}$

28	$X_{26} = X_{17} \triangleright X_{41}$	$X_{26} = -X_{17} + 2X_{41}$
29	$X_{45} = X_{44} \triangleright X_{41}$	$X_{45} = -X_{44} + 2X_{41}$
30	$X_{28} = X_{27} \triangleright X_{30}$	$X_{28} = -X_{27} + 2X_{30}$
31	$X_{43} = X_{44} \triangleright X_{30}$	$X_{43} = -X_{44} + 2X_{30}$
32	$X_{32} = X_{31} \triangleright X_{28}$	$X_{32} = -X_{31} + 2X_{28}$
33	$X_{39} = X_{40} \triangleright X_{28}$	$X_{39} = -X_{40} + 2X_{28}$
34	$X_{41} = X_{40} \triangleright X_{43}$	$X_{41} = -X_{40} + 2X_{43}$
35	$X_{30} = X_{31} \triangleright X_{43}$	$X_{30} = -X_{31} + 2X_{43}$
36	$X_{28} = X_{29} \triangleright X_{39}$	$X_{28} = -X_{29} + 2X_{39}$
37	$X_{30} = X_{29} \triangleright X_{32}$	$X_{30} = -X_{29} + 2X_{32}$
38	$X_{43} = X_{42} \triangleright X_{39}$	$X_{43} = -X_{42} + 2X_{39}$
39	$X_{41} = X_{42} \triangleright X_{32}$	$X_{41} = -X_{42} + 2X_{32}$
40	$X_{33} = X_{32} \triangleright X_{39}$	$X_{33} = -X_{32} + 2X_{39}$
41	$X_{39} = X_{38} \triangleright X_{33}$	$X_{39} = -X_{38} + 2X_{33}$

42	$X_{34} = X_{33} \triangleright X_{38}$	$X_{34} = -X_{33} + 2X_{38}$
43	$X_{38} = X_{37} \triangleright X_{34}$	$X_{38} = -X_{37} + 2X_{34}$
44	$X_{35} = X_{34} \triangleright X_{37}$	$X_{35} = -X_{34} + 2X_{37}$
45	$X_{37} = X_{36} \triangleright X_{35}$	$X_{37} = -X_{36} + 2X_{35}$

Table 2: Takasaki Quandle for the Allen-Swenberg Link

As table 1 has a high number of equations, solving such a set of equations manually would be futile and more prone to errors. Hence the following code would be solved on on Mathematica in order to determine the possible value for all the variables. This code is shown below

```
Solve[-X6 -X5+2X7== 0, -X5 -X6+2X1== 0, -X4 -X3+2X1== 0, -X3 -X2+2X16== 0, -X17
-X16+2X2== 0, -X5 -X6+2X2== 0, -X1 -X2+2X37== 0, -X4 -X1+2X5== 0, -X45 -X7+2X5== 0,
-X7 -X8+2X22== 0, -X8 -X9+2X11== 0, -X18 -X13+2X9== 0, -X20 -X21+2X9== 0, -X9
-X10+2X20== 0, -X11 -X10+2X13== 0, -X24 -X25+2X11== 0, -X26 -X25+2X22== 0, -X11
-X12+2X24== 0, -X22 -X21+2X24== 0, -X24 -X23+2X20== 0, -X22 -X23+2X13== 0, -X14
-X13+2X20== 0, -X20 -X19+2X14== 0, -X15 -X14+2X19== 0, -X19 -X18+2X15== 0, -X16
-X15+2X18== 0, -X18 -X17+2X16== 0, -X26 -X17+2X41== 0, -X45 -X44+2X41== 0, -X28
-X27+2X30== 0, -X43 -X44+2X30== 0, -X32 -X31+2X28== 0, -X39 -X40+2X28== 0, -X41
-X40+2X43== 0, -X30 -X31+2X43== 0, -X28 -X29+2X39== 0, -X30 -X29+2X32== 0, -X43
-X42+2X39== 0, -X41 -X42+2X32== 0, -X33 -X32+2X39== 0, -X39 -X38+2X33== 0, -X34
    -X33+2X38== 0, -X38 -X37+2X34== 0, -X35 -X34+2X37== 0, -X37 -X36+2X35== 0
    X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,X13,X14,X15,X16,X17,X18,X19,X20,
    X21,X22,X23,X24, X25,X26,X27,X28,X29,X30,X31,X32,X33,X34,X35,X36,X37,X38,X39,X40,
    X41,X42,X43,X44,X45, Modulus − > 7]
```

Figure 7:*Code in Mathematica input for the solving of the Takasaki set of equations*

The solution set to this code give multiple homomorphism where all values are equal to a solution set where X1=X2=X3=X4=X5=X6=X7=X8=X9=X10=X11=X12=X13=X14=X15=X16=X17=X18=X19=X20=X21=22 =X23=X24=X25=X26=X27=X28=X29=X30=X31=X32=X33=X34=X35=X36=X37=X38=X39=X40=X41 =X42=X43=X44=X45 as seen in the following figure and solution matrix.

```
In[ ]:= Solve[{-X6 - X5 + 2 X7 == 0, -X5 - X6 + 2 X1 == 0, -X4 - X3 + 2 X1 == 0, -X3 - X2 + 2 X16 == 0, -X17 - X16 + 2 X2 == 0, -X5 - X6 + 2 X2 ==
    0, -X1 - X2 + 2 X37 == 0, -X4 - X1 + 2 X5 == 0, -X45 - X7 + 2 X5 == 0, -X7 - X8 + 2 X22 == 0, -X8 - X9 + 2 X11 == 0, -X18 - X13 + 2
    X9 == 0, -X20 - X21 + 2 X9 == 0, -X9 - X10 + 2 X20 == 0, -X11 - X10 + 2 X13 == 0, -X24 - X25 + 2 X11 == 0, -X26 - X25 + 2 X22 == 0,
    -X11 - X12 + 2 X24 == 0, -X22 - X21 + 2 X24 == 0, -X24 - X23 + 2 X20 == 0, -X22 - X23 + 2 X13 == 0, -X14 - X13 + 2 X20 == 0, -X20 -
    X19 + 2 X14 == 0, -X15 - X14 + 2 X19 == 0, -X19 - X18 + 2 X15 == 0, -X16 - X15 + 2 X18 == 0, -X18 - X17 + 2 X16 == 0, -X26 - X17 + 2
    X41 == 0, -X45 - X44 + 2 X41 == 0, -X28 - X27 + 2 X30 == 0, -X43 - X44 + 2 X30 == 0, -X32 - X31 + 2 X28 == 0, -X39 - X40 + 2 X28 ==
    0, -X41 - X40 + 2 X43 == 0, -X30 - X31 + 2 X43 == 0, -X28 - X29 + 2 X39 == 0, -X30 - X29 + 2 X32 == 0, -X43 - X42 + 2 X39 == 0, -X41
    - X42 + 2 X32 == 0, -X33 - X32 + 2 X39 == 0, -X39 - X38 + 2 X33 == 0, -X34 - X33 + 2 X38 == 0, -X38 - X37 + 2 X34 == 0, -X35 - X34 + 2
    X37 == 0, -X37 - X36 + 2 X35 == 0}, {X1, X2, X3, X4, X5, X6, X7, X8, X9, X10, X11, X12, X13, X14, X15, X16, X17, X18, X19, X20, X21,
    X22, X23, X24, X25, X26, X27, X28, X29, X30, X31, X32, X33, X34, X35, X36, X37, X38, X39, X40, X41, X42, X43, X44, X45}, Modulus →
   7]
    ··· Solve: Equations may not give solutions for all "solve" variables.
Out[ ]= {{X36 → X5, X27 → X5, X12 → X5, X45 → X5, X44 → X5, X42 → X5, X40 → X5, X35 → X5, X31 → X5, X29 → X5, X26 → X5, X25 → X5, X23 → X5, X21 →
    X5, X10 → X5, X8 → X5, X4 → X5, X3 → X5, X38 → X5, X34 → X5, X33 → X5, X19 → X5, X17 → X5, X15 → X5, X14 → X5, X7 → X5, X6 → X5, X43 →
    X5, X41 → X5, X37 → X5, X32 → X5, X30 → X5, X28 → X5, X24 → X5, X22 → X5, X18 → X5, X16 → X5, X13 → X5, X11 → X5, X9 → X5, X2 → X5, X1
    → X5, X39 → X5, X20 → X5}}
```

$$\begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_{45} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

figure 8 : Set of solutions given by Mathematica

V.Takasaki Quandle for all primes under 47

Using the same method as the one seen above, the total number of homomorphisms for each modular prime - P - under 47 was computed for the Takasaki quandle on Mathematica and the overall result obtained has been represented in the following table:

P	Homomorphisms for Allen Swenberg link	Homomorphisms for the connected sum of two Hopf links
2	2	2
3	3	3
5	5	5
7	7	7
11	11	11
13	13	13
17	17	17
19	19	19
23	23	23
29	29	29
31	31	31
37	37	37

41	41	41
43	41	43
47	47	47

Table 3: Takasaki Quandle for all primes under 47

VI. Conclusion

This study in knot theory has found that the Takasaki quandle under the modulus of primes below 47 is not able to distinguish between the Allen-Swenberg Link and the connected sum of two Hopf links. Hence, at each given prime under 47, it shows that it is plausible that the Alexander - Conway polynomial together with the Takasaki quandles is not enough to capture causality.

VII. Acknowledgement

These projects were completed in Summer 2021 as a part of the Horizon Academic Research Program organized by the Sunrise International Education Inc. The projects were created by Vladimir Chernov and the program was supervised by Vladimir Chernov and Emanuele Zappala.

VIII. References

[1] V. Chernov, G. Martin, and I. Petkova *"Khovanov homology and causality in spacetimes", J. Math. Phys. 61, 022503 (2020) https://doi.org/10.1063/5.0002297*

***[2]** Samantha Allen and Jacob H. Swenberg* "Do link polynomials detect causality in globally hyperbolicspacetimes?", J. Math. Phys. 62, 032503 (2021) https://doi.org/10.1063/5.0040956

[3] Chernov, V., Nemirovski S. *Legendrian Links, Causality, and the Low Conjecture Geom. Funct. Anal.19, 1320–1333 (2010). https://doi.org/10.1007/s00039-009-0039-x*

***[4]** Bae, Yongju.* "Coloring link diagrams by Alexander Quandle" Journal of Knot Theory and Its Ram- ifications 21. 10(2012): 1250094.

[5] Elhamdadi, Mohamed *"A Survey of Racks and Quandles: Some Recent Developments". AlgebraColloquium 27. 03(2020): 509-522.*

***[6]** Carter, J..* "Classical Knot Theory". Symmetry 4. (2012).

[7] Nelson, Sam. (2002). *Classification of Finite Alexander Quandles. Topology Proc.. 27.*

***[8]** Kim, Jieon et al.* "Quandle coloring quivers of surface-links". Journal of Knot Theory and Its Ram-ifications 30. 01(2021): 2150002.

[9]Carter, J.. (2010). A Survey of Quandle Ideas. Ser. Knots Everything. 46.10.1142/9789814313001 0002.

[10] D. Joyce, A classifying invariant of knots, the knot quandle. J. Pure Appl. Alg. 23 (1982) 37–65