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PROJECTIVE INVARIANT TENSORS

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ABSTRACT -

The main purpose of this paper is to write Weyl tensor and Douglas Tensor in term of quantities of Projective Connection $P\,\Gamma_{(T)}$

1. INTRODUCTION :-

Projective geometry was initialed by E. Cartan and Tracey Thomos. The most basic application of what has come to be known as Bernstein -Gelfond Machinery. As such it is completely Parallel to conformal geometry on the other hand there are direct application within Riemannian, and Finsler differential geometry. Various interesting result on Finsler Space have been found by O. Varga, H. Rund and others. The Theory of Finsler Spaces were develop relative to Riemannian Spaces. An invariant of a tensor is a scalar associated with that tensor. It does not vary under co-ordinate change, an invariant in terms of components A_{ij} will give the same result for all Cartesian base. A Scalar function that depends entitely on Principal invariant of tensor is independent from rotation of co-ordinate system. A tensor in 3D has as many as six independent invariants.

In mathematics Projective geometry is the study of geometric properties that are invariant with respect to Projective transformation, a Projective geometry is the study of geometric properties that are invariant with respect to Projective transformation. It is not possible to refer to angles in Projective geometry as it is in Euclidean geometry because angle is an example of a concept not invariant with respect to Projective transformation. Projective geometry was mainly a development of 19th Century. Another topic that developed from axiomatic studies of Projective geometry is finite geometry. Projective Connection is a kind of Cartan Connection.

The purpose of Present Paper is to write weyl Douglous tensors in terms of Projective Connection.

2. PROJECTIVE INVARIANT TENSORS :-

It is well known [1] that the weyl tensor W^i_{hjk} and Douglas tensor D^i_{hjk} play essential role in theory of Projective Changes. First we get from (1.1)

(1.1) (a) $Q^i = G^i - GY^i$ /(n+1)

(b) $Q^i_j = \partial_i\ Q^i = G^i_j$ - $(G_j\ Y^i + G_j\ \delta^i_j)$/(n+1)

(c) $Q^i_{jk} = \partial_k\ Q^i_j = G^i_{jk}$ - $(G_{jk}Y^i + G_j\ \delta^i_k + G_k\ \delta^i_j)$/(n+1)

a Projective invariant tensor

$$D^i_{hjk} = \partial_h\ \partial_j\ \partial_k\ Q^i = G_{hjk} - \frac{1}{(n+1)} [G^i_{jkh}\ Y^i + S_{(hjk)})\ \{G_{hj}\ \delta^i_k\}]$$

Which are nothing but components of Douglas tensor D [1 (2.10)].

From (2.2) (2.3)

(2.2) P^2 : U^i_{hjk} = [G^i_{hjk} - ($G_{hj.k}$ Y^i + G_{hj} δ^i_k + G_{hk} δ^i_k)/(n+1)

(2.3) U_{ij} = G_{ij} / (n+1) = ∂_i ∂_j G / (n+1)

We get

(2.4) D^i_{hjk} = U^i_{hjk} - δ^i_h U_{jk}

From which we can easily verify the identities [2]

(2.5) (a) D^i_{hjk} = D^i_{jhk} = D^i_{jkh}

(b) D^i_{ijk} = 0

(c) $D^i_{mjk.h}$ = $D^i_{mjh.k}$

(d) D^i_{mjk} = 0

Secondarily we are concerned with Weyl tensor

([3].[1], (2.14)] :

(2.6) W^i_{hjk} = H^i_{hjk} + $\vartheta_{(jk)}$ {Y^i $H_{jk.h}$ + δ^i_h H_{jk} + δ^i_j H_k .h}/(n+1)

and Weyl torsion tensor

(2.7) W^i_{jk} (= W^i_{ojk}) = $H^i_{jk} + \vartheta_{(jk)}$ {Y^i H_{jk} + δ^i_j H_k }/(n+1)

To write W^i_{hjk} in term of quantities of $P\,\Gamma_{(T)}$ we recall first Bianchi identities (1.4)e) and (1.6)a) of [1] which gives

(2.8) $H^h_{hij.k}$ = $G_{ik;j}$ - $G_{jk;i}$ = - (H_{ij} - H_{ji}).k

Next it we put

(2.9) M_{ij} = (nN_{ij} + N_{ji})/ (n^2-1) $-\frac{1}{(nh)^2}$ {$hG_{i;j}$ + $G_{j;i}$ }

Then we have

(2.10) N_{ij} = (nM_{ij} - M_{ji}) + $\frac{(n-1)}{(n+1)}$ $G_{i;j}$

and (2.1)

(2.11) $N_{ij} = H_{ij} - \frac{1}{(n+1)} \; G_{ij;o} \; \frac{(n-1)}{(n+1)^2}$ [(n+1) $G_{i;j} + GG_{ij} + G_i \;\; G_j$]

We get

(2.12) $M_{ij} = \frac{1}{(nh)} [H_{j;i} + GG_{ij}$ /(n+1) + $G_i \;\; G_j$ / (n+1)]

Which implies

(2.13) (n+1) (M_{ij} - M_{ji}) = H_{ij} - H_{ji}

Further (2.2) (2.4) (a)

(2.14) (a) $x^i|_j|_k$ - $x^i|_k|_j$ = $x^h \; N^i_{hjk}$ - $h \; N^h_{jk}$ - $x^i|_h \; T^h_{jk}$

Shows

(2.15) (a) $H_{i.j}$ = (n+1) M_{ji} - GG_{ij} /(n+1) - $G_i \;\; G_j$ /(nH)

(b) H_i = (n+1) M_{oi} - GG_i /(n+1)

Now (1.6) and

(2.16) $R^2 \; : N^i_{hjk} = H^i_{hjk} - \frac{1}{(n+1)} [G_{hj;k} - G_{hk;j}) \, Y^i + G_{hj:k} \; \delta^i_j - G_{h:j} \; \delta^i_k]$

gives

(2.17) $W^i_{hjk} = H^i_{hjk} + \vartheta_{(jk)} \{ Y^i \;\; H_{jk.h} + \delta^i_h \; H_{jk} + \delta^i_j \;\; H_{k.h} \}$/(n+1)

Consequently (1.8), (2.3), (2.5) gives the conclusion

(2.18) $W^i_{hjk} = N^i_{hjk} + \vartheta_{(jk)} \{ \delta^i_h \; M_{jk} + \delta^i_j \; M_{hk} + \delta^i_j \;\; G_{h;k}$ /(n+1)}

The Weyl Torsion Tensor is also written in the form

(2.19) $W^i_{jk} = N^i_{jk} + \vartheta_{(jk)} \{ Y^i_h \; M_{jk} + \delta^i_j \; M_{ok} + \delta^i_j \;\; G_{;k}$ }/(n+1)}

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