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On The Surd Equation

$$\sqrt{2z} = \sqrt{x+iy} + \sqrt{x-iy}$$

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Abstract:

In this short paper, non-zero integer distinct integer solutions to the surd equation with three unknowns given by $\sqrt{2z} = \sqrt{x+iy} + \sqrt{x-iy}$ are obtained through the integer solutions of Pythagorean equation.

Keywords: surd equation, transcendental equation ,integer solutions

Introduction:

Diophantine equations have an unlimited field of research by reason of their variety. Most of the Diophantine problems solved by the researchers are algebraic equations [1,2].

It seems that much work has not been done in finding the integer solutions to transcendental equations involving surds. In this context ,refer [3-18] to the integral solutions of transcendental equations involving surds .This short communication analyses a transcendental equation with three unknowns given by $\sqrt{2z} = \sqrt{x+iy} + \sqrt{x-iy}$. Infinitely many non-zero integer triples (x, y, z) satisfying the above equation are obtained through employing the integer solutions to the well-known Pythagorean equation.

Method of analysis:

The surd equation to be solved is

$$\sqrt{2z} = \sqrt{x + iy} + \sqrt{x - iy} \qquad (1)$$

On squaring both sides of (1) ,it simplifies to

$$z = x + \sqrt{x^2 + y^2} \qquad (2)$$

To eliminate the square-root on the R.H.S. of (2) ,take

$$x^2 + y^2 = \alpha^2 \qquad (3)$$

which is nothing but the well-known Pythagorean equation satisfied by

$$x = r^2 - s^2 \ , y = 2rs \ , r \geq s \geq 0 \qquad (4)$$

and $\alpha = r^2 + s^2$

In view of (2),it is seen that

$$z = 2\, r^2 \qquad (5)$$

Thus,(4) and (5) represent the integer solutions to (1).

A few numerical solutions are presented in Table:1 below

Table:1 Numerical solutions

r	s	x	y	z
2	1	3	4	8
3	2	5	12	18
4	2	12	16	32
5	3	16	30	50

It is worth mentioning that , (3) is also satisfied by

$$x = 2rs, y = r^2 - s^2 \ , r \geq s \geq 0 \qquad (6)$$

and $\alpha = r^2 + s^2$

From (2) ,the value of z is given by

$$z = (r + s)^2 \qquad (7)$$

Thus, (6) and (7) satisfy (1).

A few numerical solutions are presented in Table:2 below

Table:2 Numerical solutions

r	s	x	y	z
2	1	4	3	9
3	2	12	5	25
4	2	16	12	36
5	3	30	16	64

Further , (3) is also satisfied by

$$\left.\begin{aligned} x &= (r^2+s^2)\left[(A^2-B^2)(r^2-s^2)-4rsAB\right] \\ y &= (r^2+s^2)\left[(A^2-B^2)2rs+2AB(r^2-s^2)\right] \end{aligned}\right\} \tag{8}$$

and $\alpha = (r^2+s^2)^2(A^2+B^2)$

From (2) ,the value of z is given by

$$z = 2(r^2+s^2)(Ar-Bs)^2 \tag{9}$$

Thus, (8) and (9) satisfy (1). It should be remembered that the values of r ,s, A and B are chosen so that x and y are non-zero positive integers as they represent the legs of pythagorean triangle.

A few numerical solutions are presented in Table:3 below

Table:3 Numerical solutions

A	B	r	s	x	y	z
2	1	4	1	17*13	17*84	34*49
3	1	4	1	17*72	17*154	34*121
4	2	5	1	26*128	26*504	52*324

Conclusion:

In this paper, we have presented integer solutions to the surd equation with three unknowns given by $\sqrt{2z} = \sqrt{x+iy} + \sqrt{x-iy}$. To conclude one may attempt to find integer solutions to other choices of surd equations .with unknowns three or more than three.

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