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Certain Notions of Neutroscopic Pythogorean *K*-Subalgebras

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Abstract

We apply the notion of neutrosophic Pythagorean sets to *K*-algebras. We develop the concept of neutrosophic pythogorean *K*-sub algebras, and present some of their properties. Moreover, we study the behavior of valued neutrosophic pythogorean *K*-sub algebras under homomorphism.

Keywords: neutrosophic pythogorean sets, *K*-sub algebras, homomorphism.

Introduction

A new kind of logical algebra, known as *K*-algebra, was introduced by Dar and Akram [9]. A *K*-algebra was built on a group *G* by adjoining the induced binary operation on *G*. The group *G* is particularly of the type in which each non-identity element is not of order 2. This algebraic structure is, in general, non-commutative and non-associative with right identity element [5, 10, 11]. Akram et.al [2–4] introduced fuzzy *K*-algebras. They then developed fuzzy *K*-algebras with other researchers worldwide. The concepts and results of *K*-algebras have been broadened to the fuzzy setting frames by applying Zadeh's fuzzy set theory and its generalizations, namely, interval- valued fuzzy sets, intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets, bipolar fuzzy sets and vague sets.

In handling information regarding various aspects of uncertainty, non-classical logic (a great extension and development of classical logic) is considered to be a more powerful technique than the classical logic. The non- classical logic has nowadays become a useful tool in computer science. Moreover, non-classical logic deals with fuzzy information and uncertainty. In 1998, Smarandache [15] introduced neutrosophic sets as a generalization of fuzzy sets [19] and intuitionistic fuzzy sets [6]. A neutrosophic set is identified by three functions called truth- membership (T), indeterminacy-membership (I) and falsity-membership (F) whose values are real standard or non-standard subset of unit interval $]^{-}0, 1^{+}[$, where $^{-}0 = 0 \quad \epsilon$, $1^{+} = 1 + \epsilon$, ϵ is an infinitesimal number. To apply neutrosophic set in real-life problems more conveniently, Smarandache [15] and Wang et al. [16] defined single-valued neutrosophic sets which takes the value from the subset of [0, 1]. Thus, a single-valued neutrosophic set is an instance of neutrosophic set, and can be used expediently to deal with real-world problems, especially in decision support. Algebraic structures have a vital place with vast applications in various disciplines. Neutrosophic set theory has been applied to algebraic structures [1, 8, 13]. In this research article, we introduce the notion of neutrosophic pythogorean *K*-subalgebra and investigate some of their properties. We discuss *K*-sub algebra in terms of level sets using neutrosophic pythogorean environment. We study the homomorphisms between the neutrosophic pythogorean *K*-sub algebras. We discuss characteristic *K*-sub algebras and fully invariant -sub algebras.

Neutrosophic pythogorean K-algebras

The concept of K-algebra was developed by Dar and Akram in [14].

Definition 2.1. Let $(G, \cdot, e)$ be a group in which each non-identity element is not of order 2. Then a K- *algebra* is a structure $\mathrm{K} = (G, \cdot, \odot, e)$ on a group G in which induced binary operation $\odot : G \times G \rightarrow G$ is defined by $\odot(x, y) = x \odot y = x.y^{-1}$ and satisfies the following axioms:

(i) $(x \odot y) \odot (x \odot z) = (x \odot ((e \odot z) \odot (e \odot y))) \odot x$,

(ii) $x \odot (x \odot y) = (x \odot (e \odot y)) \odot x$,

(iii)$(x \odot x) = e$,

(iv)$(x \odot e) = x$,

(v) $(e \odot x) = x^{-1}$, for all $x, y, z \in G$.

Definition 2.2. [16] Let Z be a space of objects with a general element $z \in Z$. A neutrosophic pythogorean set A in Z is characterized by three membership functions, T_A-truth membership function, I_A-indeterminacy membership function and F_A-falsity membership function, where $T_A(z), I_A(z), F_A(z) \in [0, 1]$, for all $z \in Z$.

That is $T_A : Z \rightarrow [0, 1], I_A : Z \rightarrow [0, 1], F_A : Z \rightarrow [0, 1]$ with no restriction on the sum of these three components.

A can also be written as $A = \{< z, T_A(z), I_A(z), F_A(z) > | z \in Z\}$.

Definition 2.3. A neutrosophic pythogorean set A = (T_A, I_A, F_A) in a K-algebra K is called a neutrosophic pythogorean K-sub algebra of K if it satisfy the following conditions:

$T_A(s \odot t) \geq \min\{T_A(s), T_A(t)\}$,

$I_A(s \odot t) \geq \min\{I_A(s), I_A(t)\}$,

$F_A(s \odot t) \leq \max\{F_A(s), F_A(t)\}$, for all $s, t \in G$.

Note that $T_A(e) \geq T_A(s)$, $I_A(e) \geq I_A(s)$, $F_A(e) \leq F_A(s)$, for all $s \in G$.

Example 2.1. Consider $\mathrm{K} = (G, \cdot, \odot, e)$ be a K-algebra, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$ is the cyclic group of order 9 and Caley's table for $\odot$ is given as:

$\odot$	e	x	x^2	x^3	x^4	x^5	x^6	x^7	x^8
e	e	x^8	x^7	x^6	x^5	x^4	x^3	x^2	x
x	x	e	x^6	x^7	x^6	x^5	x^4	x^3	x^2
x^2	x^2	x^2	e	x^8	x^7	x^6	x^5	x^4	x^3
x^3	x^3	x^3	x	e	x^8	x^7	x^6	x^5	x^4
x^4	x^4	x^4	x^2	x	e	x^8	x^7	x^6	x^5
x^5	x^5	x^5	x^3	x^2	x	e	x^8	x^7	x^6
x^6	x^6	x^6	x^4	x^3	x^2	x	e	x^8	x^7
x^7	x^7	x^7	x^5	x^4	x^3	x^2	x	e	x^8
x^8	x^8	x^2	x^6	x^5	x^4	x^3	x^2	x	e

We define a neutrosophic pythogorean set $A = (T_A, I_A, F_A)$ in K-algebra as follows:
$T_A(e) = 0.7, I_A(e) = 0.6, F_A(e) = 0.3$,

$T_A(s) = 0.1, I_A(s) = 0.2, F_A(s) = 0.5$, for all $s \neq e \in G$.
Clearly, $A = (T_A, I_A, F_A)$ is a neutrosophic pythogorean K-sub algebra of K.

Example 2.2. Consider $K = (G, \cdot, \odot, e)$ be a K-algebra on dihedral group $D4$ given as $G = \{e, a, b, c, x, y, u, v\}$, where $c = ab, x = a^2, y = a^3, u = a^2b, v = a^3b$ and Caley's table for $\odot$ is given as:

$\odot$	e	a	b	c	x	y	u	v
e	e	y	b	c	x	a	u	v
a	a	e	c	u	y	x	v	b
b	b	c	e	y	u	v	x	a
c	c	u	a	e	v	b	y	x
x	x	a	u	v	e	y	b	c
y	y	x	v	b	a	e	c	u
u	u	v	x	a	b	c	e	y
v	v	b	y	x	c	u	a	e

We define a neutrosophic pythogorean set $A = (T_A, I_A, F_A)$ in K-algebra as follows:
$T_A(e) = 0.8, I_A(e) = 0.2, F_A(e) = 0.2$,

$T_A(s) = 0.5, I_A(s) = 0.1, F_A(s) = 0.3$, for all $s \neq e \in G$.
By routine calculations, it can be verified that A is a neutrosophic pythogorean K-sub algebra ok K.

Proposition 2.1. If $A = (T_A, I_A, F_A)$ is a neutrosophic pythogorean K-sub algebra of K, then

1. $(\forall s, t \in G), (T_A(s \odot t) = T_A(t) \Rightarrow T_A(s) = T_A(e))$. $(\forall s, t \in G)(T_A(s) = T_A(e)$
   $\Rightarrow T_A(s \odot t) \geq T_A(t))$.
2. $(\forall s, t \in G), (\mathbf{I}_A(s \odot t) = \mathbf{I}_A(t) \Rightarrow \mathbf{I}_A(s) = \mathbf{I}_A(e))$. $(\forall s, t \in G)(\mathbf{I}_A(s) = \mathbf{I}_A(e)$
   $\Rightarrow \mathbf{I}_A(s \odot t) \geq \mathbf{I}_A(t))$.
3. $(\forall s, t \in G), (F_A(s \odot t) = F_A(t) \Rightarrow F_A(s) = F_A(e))$. $(\forall s, t \in G)(F_A(s) = F_A(e)$
   $\Rightarrow F_A(s \odot t) \leq F_A(t))$.

Proof. 1. Assume that $T_A(s \odot t) = T_A(t)$, for all $s, t \in G$. Taking $t = e$ and using (iii) of Definition 2.1, wehave $T_A(s) = T_A(s \odot e) = T_A(e)$. Let for $s, t \in G$ be such that $T_A(s) = T_A(e)$.
Then $T_A(s \odot t) \geq \min\{T_A(s), T_A(t)\} = \min\{T_A(e), T_A(t)\} = T_A(t)$.
Again assume that $I_A(s \odot t) = I_A(t)$, for all $s, t \in G$. Taking $t = e$ and by (iii) of Definition 2.1, we have $I_A(s) = I_A(s \odot e) = I_A(e)$. Also let $s, t \in G$ be such that $I_A(s) = I_A(e)$. Then $I_A(s \odot t) \geq \min\{I_A(s), I_(t)\} = \min\{I_A(e), I_A(t)\} = I_A(t)$.
Consider that $F_A(s \odot t) = F_A(t)$, for all $s, t \in G$. Taking $t = e$ and again by (iii) of Definition 2.1, wehave $F_A(s) = F_A(s \odot e) = F_A(e)$. Let $s, t \in G$ be such that $F_A(s) = F_(e)$.
Then $F_A(s \odot t) \leq \max\{F_A(s), F_A(t)\} = \max\{F_A(e), F_A(t)\} = F_A(t)$.
This completes the proof.

Definition 2.4. Let $A = (T_A, I_A, F_A)$ be a neutrosophic pythogorean set in a K-algebra K and let $(\alpha, \beta, \gamma) \in [0, 1] \times [0, 1] \times [0, 1]$ with $\alpha + \beta + \gamma \leq 3$. Then level subsets of A are defined as:
$A_{(\alpha,\beta,\gamma)} = \{s \in G \mid T_A(s) \geq \alpha, I_A(s) \geq \beta, F_A(s) \leq \gamma\}$
$A_{(\alpha,\beta,\gamma)} = \{s \in G \mid T_A(s) \geq \alpha\} \cap \{s \in G \mid I_A(s) \geq \beta\} \cap \{s \in G \mid F_A(s) \leq \gamma\}$
$A_{(\alpha,\beta,\gamma)} = \cup(T_A, \alpha) \cap \cup (I_A, \beta) \cap L(F_A, \gamma)$. are called (α, β, γ) -level subsets of neutrosophic pythogorean set A.

The set of all $(\alpha, \beta, \gamma) \in Im(T_A) \times Im(I_A) \times Im(I_A)$ is known as image of $A = (T_A, I_A, F_A)$.
The set $A_{(\alpha,\beta,\gamma)} = \{s \in G \mid T_A(s) > \alpha, I_A(s) > \beta, F_A(s) < \gamma\}$ is known as strong (α, β, γ)- level subset of A.

Proposition 2.2. If $A = (T_A, I_A, F_A)$ is a neutrosophic pythogorean K-sub algebra of K, then the level subsets $\cup(T_A, \alpha) = \{s \in G \mid T_A(s) \geq \alpha\}$, $\cup'(I_A, \beta) = \{s \in G \mid I_A(s) \geq \beta\}$ and $L(F_A, \gamma) = \{s \in G \mid F_A(s) \leq \gamma\}$ are k-sub algebras of K, for every $(\alpha, \beta, \gamma) \in Im(T_A) \times Im(I_A) \times Im(F_A) \subseteq [0, 1]$, where $Im(T_A)$, $Im(I_A)$ and $Im(F_A)$ are sets of values of $T_(A)$, $I_(A)$ and $F_(A)$, respectively.
Proof. Assume that $A = (T_A, I_A, F_A)$ is a neutrosophic pythogorean K-sub algebra of K and let $(\alpha, \beta, \gamma) \in$
$Im(T_A) \times Im(I_A) \times Im(F_A)$ be such that $\cup(T_A, \alpha) \neq \emptyset, \cup'(I_A, \beta) \neq \emptyset$ and $L(F_A, \gamma) \neq \emptyset$. Now to prove that $\cup, \cup'$ and L are level K-sub algebras. Let for $s, t \in \cup(T_A, \alpha)$, $T_A(s) \geq \alpha$ and $T_A(t) \geq \alpha$. It follows from Definition 3.1 that $T_A(s \odot t) \geq \min\{T_A(s), T_A(t)\} \geq \alpha$. It implies that $s \odot t \in \cup(T_A, \alpha)$. Hence $\cup(T_A, \alpha)$ is a level K-sub algebra of K. Similar result can be proved for $\cup'(I_A, \beta)$ and $L(F_A, \gamma)$.

Theorem 2.1. Let $A = (T_A, I_A, F_A)$ be a neutrosophic pythogorean set in K-algebra K. Then $A = (T_A, I_A, F_A)$ is a neutrosophic pythogorean K-sub algebra of K if and only if $A_{(\alpha,\beta,\gamma)}$ is a K-sub algebra of K, for every $(\alpha, \beta, \gamma) \in Im(T_A) \times Im(I_A) \times Im(F_A)$ with $\alpha + \beta + \gamma \leq 3$.

Proof. Let $A = (T_A, I_A, F_A)$ be a pythogorean set in a K-algebra K. Assume that $A = (T_A, I_A, F_A)$ be a neutrosophic pythogorean K-sub algebra of K. i.e., the following three conditions of Definition 3.1 hold.
$T_A(s \odot t) \geq \min\{T_A(s), T_A(t)\}$,
$I_A(s \odot t) \geq \min\{I_A(s), I_A(t)\}$,
$F_A(s \odot t) \leq \max\{F_A(s), F_A(t)\}$, for all $s, t \in G$.
$T_A(e) \geq T_A(s)$, $I_A(e) \geq I_A(s)$, $F_A(e) \leq F_A(s)$, for all $s \in G$.
Let for $(\alpha, \beta, \gamma) \in Im(T_A) \times Im(I_A) \times Im(F_A)$ with $\alpha + \beta + \gamma \leq 3$ be such that $A_{(\alpha,\beta,\gamma)} \neq \emptyset$. Let $s, t \in A_{(\alpha,\beta,\gamma)}$
be such that

$T_A(s) \geq \alpha, T_A(t) \geq \alpha'$,
$I_A(s) \geq \beta, I_A(t) \geq \beta'$,
$F_A(s) \leq \gamma, F_A(t) \leq \gamma'$.

Without loss of generality we can assume that $\alpha \leq \alpha'$, $\beta \leq \beta'$ and $\gamma \geq \gamma'$. It follows from Definition 3.1 that
$T_A(s \odot t) \geq \alpha = \min\{T_A(s), T_A(t)\}$,
$I_A(s \odot t) \geq \beta = \min\{I_A(s), I_A(t)\}$,
$F_A(s \odot t) \leq \gamma = \max\{F_A(s), F_A(t)\}$.

It implies that $s \odot t \in A_{(\alpha,\beta,\gamma)}$. So, $A_{(\alpha,\beta,\gamma)}$ is a K-sub algebra of K.
Conversely, we suppose that $A_{(\alpha,\beta,\gamma)}$ is a K-sub algebra of K. If the condition of the Definition 3.1 is not true, then there exist $u, v \in G$ such that

$T_A(u \odot v) < \min\{T_A(u), T_A(v)\}$,
$I_A(u \odot v) < \min\{I_A(u), I_A(v)\}$,
$F_A(u \odot v) > \max\{F_A(u), F_A(v)\}$.

Taking
$\alpha_1 = \frac{1}{2}(T_A(u \odot v) + \min\{T_A(u), T_A(v)\})$,
$\beta_1 = \frac{1}{2}(I_A(u \odot v) + \min\{I_A(u), I_A(v)\})$,
$\gamma_1 = \frac{1}{2}(F_A(u \odot v) + \min\{F_A(u), F_A(v)\})$.
We have $T_A(u \odot v) < \alpha_1 < \min\{T_A(u), T_A(v)\}$, $I_A(u \odot v) < \beta_1 < \min\{I_A(u), I_A(v)\}$ and $F_A(u \odot v) > \gamma_1 > \max\{F_A(u), F_A(v)\}$. It implies that $u, v \in A_{(\alpha,\beta,\gamma)}$ and $u \odot v \notin A_{(\alpha,\beta,\gamma)}$, a contradiction. Therefore, the condition of Definition 3.1 is true. Hence $A = (T_A, I_A, F_A)$ is a neutrosophic pythogorean k-sub algebra of K.

Theorem 2.2. Let $A = (T_A, I_A, F_A)$ be a neutrosophic pythogorean k-sub algebra and $(\alpha_1, \beta_1, \gamma_1)$, $(\alpha_2, \beta_2, \gamma_2) \in Im(T_A) \times Im(I_A) \times Im(F_A)$ with $\alpha_j + \beta_j + \gamma_j \leq 3$ for $j = 1, 2$. Then $A_{(\alpha 1,\beta 1,\gamma 1)} = A_{(\alpha 2,\beta 2,\gamma 2)}$ if $(\alpha_1, \beta_1, \gamma_1) = (\alpha_2, \beta_2, \gamma_2)$.

Proof. If $(\alpha_1, \beta_1, \gamma_1) = (\alpha_2, \beta_2, \gamma_2)$, then clearly $A_{(\alpha 1,\beta 1,\gamma 1)} = A_{(\alpha 2,\beta 2,\gamma 2)}$.
Assume that $A_{(\alpha 1,\beta 1,\gamma 1)} = A_{(\alpha 2,\beta 2,\gamma 2)}$. Since $(\alpha_1, \beta_1, \gamma_1) \in Im(T_A) \times Im(I_A) \times Im(F_A)$, there exist $s \in G$ such that $T_A(s) = \alpha_1$, $I_A(s) = \beta_1$ and $F_A(s) = \gamma_1$. It follows that $s \in A_{(\alpha 1,\beta 1,\gamma 1)} = A_{(\alpha 2,\beta 2,\gamma 2)}$. So that $\alpha_1 = T_A(s) \geq \alpha_2$, $\beta_1 = I_A(s) \geq \beta_2$ and $\gamma_1 = F_A(s) \leq \gamma_2$.
Also $(\alpha_2, \beta_2, \gamma_2) \in Im(T_A) \times Im(I_A) \times Im(F_A)$, there exist $t \in G$ such that $T_A(t) = \alpha_2$, $I_A(t) = \beta_2$ and $F_A(t) = \gamma_2$. It follows that $t \in A_{(\alpha 2,\beta 2,\gamma 2)} = A_{(\alpha 1,\beta 1,\gamma 1)}$.
So that $\alpha_2 = T_A(t) \geq \alpha_1$, $\beta_2 = I_A(t) \geq \beta_1$ and $\gamma_2 = F_A(t) \leq \gamma_1$. Hence $(\alpha_1, \beta_1, \gamma_1) = (\alpha_2, \beta_2, \gamma_2)$.

Theorem 2.3. Let H be a *K*-sub algebra of *K*-algebra K. Then there exist neutrosophic pythogorean *K*- sub algebra $A = (T_A, I_A, F_A)$ of *K*-algebra K such that $A = (T_A, I_A, F_A) = H$, for some $\alpha, \beta \in (0, 1]$, $\gamma \in [0, 1)$.

Proof. Let $A = (T_A, I_A, F_A)$ be a neutrosophic pythogorean set in *K*-algebra K given by

$$T_A(s) = \begin{cases} \alpha\epsilon(0,1] & if\ s\epsilon H. \\ 0 & otherwise \end{cases}$$

$$I_A(s) = \begin{cases} \beta\epsilon(0,1] & if\ s\epsilon H. \\ 0 & otherwise \end{cases}$$

$$F_A(s) = \begin{cases} \gamma\epsilon(0,1] & if\ s\epsilon H. \\ 0 & otherwise \end{cases}$$

Let $s, t \in G$. If $s, t \in H$, then $s \odot t \in H$ and so
$T_A(s \odot t) \geq \min\{T_A(s), T_A(t)\}$,
$I_A(s \odot t) \geq \min\{I_A(s), I_A(t)\}$,
$F_A(s \odot t) \leq \max\{F_A(s), F_A(t)\}$.
But if $s \notin H$ or $t \notin H$, then $T_A(s) = 0$ or $T_A(t)$, $I_A(s) = 0$ or $I_A(t)$ and $F_A(s) = 0$ or $F_A(t)$. It follows that
$T_A(s \odot t) \geq \min\{T_A(s), T_A(t)\}$, $I_A(s \odot t) \geq \min\{I_A(s), I_A(t)\}$, $F_A(s \odot t) \leq \max\{F_A(s), F_A(t)\}$.
Hence $A = (T_A, I_A, F_A)$ is a SVN *K*-sub algebra of K. Consequently $A_{(\alpha,\beta,\gamma)} = H$.
The above Theorem shows that any *K*-sub algebra of K can be perceived as a level *K*-sub algebra of some neutrosophic pythogorean *K*-sub algebras of K.

Theorem 2.4.

Let K be a K-algebra. Given a chain of K-sub algebras: $A_0 \subset A_1 \subset A_2 \subset \ldots \subset A_n = G$. Then there exist a neutrosophic pythogorean K-sub algebra whose level K-sub algebras are exactly the K-sub algebras in this chain.

Proof. Let $\{\alpha_k \mid k = 0, 1, \ldots, n\}$, $\{\beta_k \mid k = 0, 1, \ldots, n\}$ be finite decreasing sequences and $\{\gamma_k \mid k = 0, 1, \ldots, n\}$ be finite increasing sequence in [0, 1] such that $\alpha_i + \beta_i + \gamma_i \leq 3$, for $i = 0, 1, 2, \ldots, n$. Let $A = (T_A, I_A, F_A)$ bea neutrosophic pythogorean set in K defined by $T_A(A_0) = \alpha_0$, $I_A(A_0) = \beta_0$, $F_A(A_0) = \gamma_0$, $T_A(A_k \setminus A_{k-1}) = \alpha_k$, $I_A(A_k \setminus A_{k-1}) = \beta_k$ and $F_A(A_k \setminus A_{k-1}) = \gamma_k$, for $0 < k \leq n$. We claim that $A = (T_A, I_A, F_A)$ is a neutrosophic pythogorean K-sub algebra of K. Let $s, t \in G$. If $s, t \in A_k \setminus A_{k-1}$, then it implies that $T_A(s) = \alpha_k = T_A(t)$, $I_A(s) = \beta_k = I_A(t)$ and $F_A(s) = \gamma_k = F_A(t)$. Since each A_k is a K-sub algebra, it follows that $s \odot t \in A_k$. So that either $s \odot t \in A_k \setminus A_{k-1}$ or $s \odot t \in A_{k-1}$. In any case, we conclude that

$T_A(s \odot t) \geq \alpha_k = \min\{T_A(s), T_A(t)\}$,
$I_A(s \odot t) \geq \beta_k = \min\{I_A(s), I_A(t)\}$,
$F_A(s \odot t) \leq \gamma_k = \max\{F_A(s), F_A(t)\}$.

For $i > j$, if $s \in A_i \setminus A_{i-1}$ and $t \in A_j \setminus A_{j-1}$, then $T_A(s) = \alpha_i$, $T_A(t) = \alpha_j$, $I_A(s) = \beta_i$, $I_A(t) = \beta_j$ and $F_A(s) =$
γ_i, $F_A(t) = \gamma_j$ and $s \odot t \in A_i$ because A_i is a K-sub algebra and $A_j \subset A_i$. It follows that

$T_A(s \odot t) \geq \alpha_i = \min\{T_A(s), T_A(t)\}$,
$I_A(s \odot t) \geq \beta_i = \min\{I_A(s), I_A(t)\}$,
$F_A(s \odot t) \leq \gamma_i = \max\{F_A(s), F_A(t)\}$.

Thus, $A = (T_A, I_A, F_A)$ is a neutrosophic pythogorean K-sub algebra of K and all its non empty level subsetsare level K-sub algebras of K.

Since $\mathrm{Im}(T_A) = \{\alpha_0, \alpha_1, \ldots, \alpha_n\}$, $\mathrm{Im}(I_A) = \{\beta_0, \beta_1, \ldots, \beta_n\}$, $\mathrm{Im}(F_A) = \{\gamma_0, \gamma_1, \ldots, \gamma_n\}$. Therefore, the level K-sub algebras of $A = (T_A, I_A, F_A)$ are given by the chain of K-sub algebras:

$\cup(T_A, \alpha_0) \subset \cup(T_A, \alpha_1) \subset \ldots \subset \cup(T_A, \alpha_n) = G$,
$\cup'(I_A, \beta_0) \subset \cup'(I_A, \beta_1) \subset \ldots \subset \cup'(I_A, \beta_n) = G$,
$L(F_A, \gamma_0) \subset L(F_A, \gamma_1) \subset \ldots \subset L(F_A, \gamma_n) = G$,
respectively. Indeed,

$\cup(T_A, \alpha_0) = \{s \in G \mid T_A(s) \geq \alpha_0\} = A_0$,
$\cup(I_A, \beta_0) = \{s \in G \mid I_A(s) \geq \beta_0\} = A_0$,
$L(F_A, \gamma_0) = \{s \in G \mid F_A(s) \leq \gamma_0\} = A_0$.

Now we prove that $\cup(T_A, \alpha_k) = A_k$, $\cup'(I_A, \beta_k) = A_k$ and $L(F_A, \gamma_k) = A_k$, for $0 < k \leq n$. Clearly, $A_k \subseteq \cup(T_A, \alpha_k)$, $A_k \subseteq \cup'(I_A, \beta_k)$ and $A_k \subseteq L(F_A, \gamma_k)$. If $s \in \cup(T_A, \alpha_k)$, then $T_A(s) \geq \alpha_k$ and so $s \notin A_i$, for
$i > k$.

Hence $T_A(s) \in \{\alpha_0, \alpha_1, \ldots, \alpha_k\}$ which implies that $s \in A_i$, for some $i \leq k$ since $A_i \subseteq A_k$. It follows that $s \in A_k$. Consequently, $\cup(T_A, \alpha_k) = A_k$ for some $0 < k \leq n$. Similar case can be proved for $\cup'(I_A, \beta_k) = A_k$. Now if $t \in L(F_A, \gamma_k)$, then $F_A(s) \leq \gamma_k$ and so $t \notin A_i$, for some $j \leq k$.Thus, $F_A(s) \in \{\gamma_0, \gamma_1, \ldots, \gamma_k\}$ which implies that $s \in A_j$, for some $j \leq k$. Since $A_j \subseteq A_k$. It follows that $t \in A_k$. Consequently, $L(F_A, \gamma_k) = A_k$, for some $0 < k \leq n$. Hence the proof. □

2.1 Homomorphism of neutrosophic pythogorean K-algebras

Definition 2.5. Let $K_1 = (G_1, \cdot, \odot, e_1)$ and $K_2 = (G_2, \cdot, \odot, e_2)$ be two K-algebras and let ϕ be a function from K_1 into K_2. If $B = (T_B, I_B, F_B)$ is a neutrosophic pythogorean K-sub algebra of K_2, then the *preimage* of $B = (T_B, \mathbf{I}_B, F_B)$ under ϕ is a neutrosophic pythogorean K-sub algebra of K_1 defined by $\phi^{-1}(T_B)(s) = T_B(\phi(s))$, $\phi^{-1}(\mathbf{I}_B)(s) = \mathbf{I}_B(\phi(s))$ and $\phi^{-1}(F_B)(s) = F_B(\phi(s))$, for all $s \in G_1$.

Theorem 2.5. Let $\phi : K_1 \to K_2$ be an epimorphism of K-algebras. If $B = (T_B, I_B, F_B)$ be a neutrosophic pythogorean K-sub algebra of K_2, then $\phi^{-1}(B)$ be a neutrosophic pythogorean K-sub algebra of K_1.

Proof. It is easy to see that $\phi^{-1}(T_B)(e) \geq \phi^{-1}(T_B)(s)$, $\phi^{-1}(\mathbf{I}_B)(e) \geq \phi^{-1}(\mathbf{I}_B)(s)$ and $\phi^{-1}(F_B)(e) \leq \phi^{-1}(F_B)(s)$ for all $s \in G_1$. Let $s, t \in G_1$, then

$\phi^{-1}(T_B)(s \odot t) = T_B(\phi(s \odot t))$
$\phi^{-1}(T_B)(s \odot t) = T_B(\phi(s) \odot \phi(t))$
$\phi^{-1}(T_B)(s \odot t) \geq \min\{T_B(\phi(s)), T_B(\phi(t))\}$
$\phi^{-1}(T_B)(s \odot t) \geq \min\{\phi^{-1}(T_B)(s), \phi^{-1}(T_B)(t)\}$,
$\phi^{-1}(\mathbf{I}_B)(s \odot t) = \mathbf{I}_B(\phi(s \odot t))$
$\phi^{-1}(\mathbf{I}_B)(s \odot t) = \mathbf{I}_B(\phi(s) \odot \phi(t))$
$\phi^{-1}(\mathbf{I}_B)(s \odot t) \geq \min\{\mathbf{I}_B(\phi(s)), \mathbf{I}_B(\phi(t))\}$
$\phi^{-1}(\mathbf{I}_B)(s \odot t) \geq \min\{\phi^{-1}(\mathbf{I}_B)(s), \phi^{-1}(\mathbf{I}_B)(t)\}$,
$\phi^{-1}(F_B)(s \odot t) = F_B(\phi(s \odot t))$
$\phi^{-1}(F_B)(s \odot t) = F_B(\phi(s) \odot \phi(t))$
$\phi^{-1}(F_B)(s \odot t) \leq \max\{F_B(\phi(s)), F_B(\phi(t))\}$
$\phi^{-1}(F_B)(s \odot t) \leq \max\{\phi^{-1}(F_B)(s), \phi^{-1}(F_B)(t)\}$.
Hence $\phi^{-1}(B)$ is a neutrosophic pythogorean K-sub algebra of K_1.

Theorem 2.6. $\phi : K_1 \to K_2$ be an epimorphism of K-algebras. If $B = (T_B, I_B, F_B)$ is a neutrosophic pythogorean K-sub algebra of K_2 and $A = (T_A, I_A, F_A)$ is the *preimage* of B under ϕ. Then A is a neutrosophic pythogorean K-sub algebra of K_1.
Proof. It is easy to see that $T_A(e) \geq T_A(s)$, $I_A(e) \geq I_A(s)$ and $F_A(e) \leq F_A(s)$, for all $s \in G_1$. Now for any $s, t \in G_1$,

$T_A(s \odot t) = T_B(\phi(s \odot t))$
$T_A(s \odot t) = T_B(\phi(s) \odot \phi(t))$
$T_A(s \odot t) \geq \min\{T_B(\phi(s)), T_B(\phi(t))\}$
$T_A(s \odot t) \geq \min\{T_A(s), T_A(t)\}$,
$\mathbf{I}_A(s \odot t) = \mathbf{I}_B(\phi(s \odot t))$
$\mathbf{I}_A(s \odot t) = \mathbf{I}_B(\phi(s) \odot \phi(t))$
$\mathbf{I}_A(s \odot t) \geq \min\{\mathbf{I}_B(\phi(s)), \mathbf{I}_B(\phi(t))\}$
$I_A(s \odot t) \geq \min\{I_A(s), I_A(t)\}$,
$F_A(s \odot t) = F_B(\phi(s \odot t))$
$F_A(s \odot t) = F_B(\phi(s) \odot \phi(t))$
$F_A(s \odot t) \leq \max\{F_B(\phi(s)), F_B(\phi(t))\}$
$F_A(s \odot t) \leq \max\{F_A(s), F_A(t)\}$.
Hence A is a neutrosophic pythogorean K-sub algebra of K_1.□

Definition 2.6. Let a mapping $\phi : K_1 \rightarrow K_2$ from K_1 into K_2 of K-algebras and let $A = (T_A, I_A, F_A)$ be a neutrosophic pythogorean set of K_2. The map $A = (T_A, I_A, F_A)$ is called the *preimage* of A under ϕ, if $T_A^{\phi}(s) = T_A(\phi(s))$, $I_A^{\phi}(s) = I_A(\phi(s))$ and $F_A^{\phi} = F_A(\phi(s))$ for all $s \in G_1$.

Proposition 2.3. Let $\phi : K_1 \rightarrow K_2$ be an epimorphism of K-algebras. If $A = (T_A, I_A, F_A)$ be a neutrosophic pythogorean K-sub algebra of K_2, then $A^{\phi} = (T_A^{\phi}, I_A^{\phi}, F_A^{\phi})$ be a neutrosophic pythogorean K-sub algebra of K_1.

Proof. For any $s \in G_1$, we have

$T_A^{\phi}(e_1) = T_A^{\phi}(\phi(e_1)) = T_A(e_2) \geq T_A(\phi(s)) = T_A(s)$,

$I_A^{\phi}(e_1) = I_A^{\phi}(\phi(e_1)) = I_A(e_2) \geq I_A(\phi(s)) = I_A(s)$,

$F_A^{\phi}(e_1) = F_A^{\phi}(\phi(e_1)) = F_A(e_2) \leq F_A(\phi(s)) = F_A(s)$.

For any $s, t \in G_1$, since A is a neutrosophic pythogorean K-sub algebra of K_2

$T_A^{\phi}(s \odot t) = T_A(\phi(s \odot t))$

$T_A^{\phi}(s \odot t) = T_A(\phi(s) \odot \phi(t))$

$T_A^{\phi}(s \odot t) \geq \min\{T_A(\phi(s)), T_A(\phi(t))\}$

$T_A^{\phi}(s \odot t) \geq \min\{T_A(s), T_A(s)\}$,

$I_A^{\phi}(s \odot t) = I_A(\phi(s \odot t))$

$I_A^{\phi}(s \odot t) = I_A(\phi(s) \odot \phi(t))$

$I_A^{\phi}(s \odot t) \geq \min\{I_A(\phi(s)), I_A(\phi(t))\}$

$I_A^{\phi}(s \odot t) \geq \min\{I_A(s), I_A(s)\}$,

$F_A^{\phi}(s \odot t) = F_A(\phi(s \odot t))$

$F_A^{\phi}(s \odot t) = F_A(\phi(s) \odot \phi(t))$

$F_A^{\phi}(s \odot t) \leq \max\{F_A(\phi(s)), F_A(\phi(t))\}$

$F_A^{\phi}(s \odot t) \leq \max\{F_A(s), F_A(s)\}$.

Hence $A^{\phi} = (T_A, I_A, F_A)$ is a neutrosophic pythogorean K-sub algebra of K_1. □

Proposition 2.4. Let $\phi : K_1 \rightarrow K_2$ be an epimorphism of K-algebras. If $A^{\phi} = (T_A^{\phi}, I_A^{\phi}, F_A^{\phi})$ be a neutrosophic pythogorean K-sub algebra of K_2, then $A = (T_A, I_A, F_A)$ is neutrosophic K-sub algebra of K_1.

Proof. Since there exist $s \in G_1$ such that $t = \phi(s)$, for any $t \in G_2$

$T_A(t) = T_A(\phi(s)) = T\ \phi(s)_A \leq T\ \phi(e_1)_A = T_A(\phi(e_1)) = T_A(e_2)$,
$I_A(t) = I_A(\phi(s)) = I^{\phi(s)}{}_A \leq I^{\phi(e1)}{}_A = I_A(\phi(e_1)) = I_A(e_2)$,
$F_A(t) = F_A(\phi(s)) = F^{\phi(s)}{}_A \geq F^{\phi(e1)}{}_A = F_A(\phi(e_1)) = F_A(e_2)$.

for any $s, t \in G_2$, $u, v \in G_1$ such that $s = \phi(u)$ and $t = \phi(v)$. It follows that

$T_A(s \odot t) = T_A(\phi(u \odot v))$
$T_A(s \odot t) = T_A(u \odot v)$
$T_A(s \odot t) \geq \min\{T_A^{\phi}(u), T_A^{\phi}(v)\}$
$T_A(s \odot t) \geq \min\{T_A(\phi(u)), T_A(\phi(v))\}$
$T_A(s \odot t) \geq \min\{T_A(s), T_A(t)\}$,

$I_A(s \odot t) = I_A(\phi(u \odot v))$
$I_A(s \odot t) = I_A(u \odot v)$

$I_A(s \odot t) \geq \min\{I_A^{\phi}(u), I_A^{\phi}(v)\}$
$I_A(s \odot t) \geq \min\{I_A(\phi(u)), I_A(\phi(v))\}$
$I_A(s \odot t) \geq \min\{I_A(s), I_A(t)\}$,

$F_A(s \odot t) = F_A(\phi(u \odot v))$
$F_A(s \odot t) = F_A(u \odot v)$
$F_A(s \odot t) \leq \max\{F_A^{\phi}(u), F_A^{\phi}(v)\}$
$F_A(s \odot t) \leq \max\{F_A(\phi(u)), F_A(\phi(v))\}$
$F_A(s \odot t) \leq \max\{F_A(s), F_A(t)\}$.

Hence $A = (T_A, I_A, F_A)$ is a neutrosophic pythogorean K-sub algebra of K_2. From above two propositions we obtain the following theorem. □

Theorem 2.7. Let $\phi : K_1 \to K_2$ be an epimorphism of K-algebras. Then $A^{\phi} = (T_A^{\phi}, I_A^{\phi}, F_A^{\phi})$ is a neutrosophic pythogorean K-sub algebra of K_1 if and only if $A = (T_A, I_A, F_A)$ is neutrosophic pythogorean K-sub algebra of K_2.

Definition 2.7. A neutrosophic pythogorean K-sub algebra $A = (T_A, I_A, F_A)$ of a K-algebra K is called *characteristic* if $T_A(\phi(s)) = T_A(s)$, $I_A(\phi(s)) = I_A(s)$ and $F_A(\phi(s)) = F_A(s)$, for all $s \in G$ and $\phi \in Aut(K)$.

Definition 2.8. A K-sub algebra S of a K-algebra K is said to be *fully invariant* if $\phi(S) \subseteq S$, for all $\phi \in End(K)$, where $End(K)$ is the set of all endomorphisms of a K-algebra K. A neutrosophic pythogorean K-sub algebra $A = (T_A, I_A, F_A)$ of a K-algebra K is called *fully invariant* if $T_A(\phi(s)) \leq T_A(s)$, $I_A(\phi(s)) \leq I_A(s)$ and $F_A(\phi(s)) \leq F_A(s)$, for all $s \in G$ and $\phi \in End(K)$.

Definition 2.9. Let $A_1 = (T_{A1}, I_{A1}, F_{A1})$ and $A_2 = (T_{A2}, I_{A2}, F_{A2})$ be neutrosophic pythogorean K-sub algebras of K. Then $A_1 = (T_{A1}, I_{A1}, {}_{A1})$ is said to be the same type of $A_2 = (T_{A2}, I_{A2}, F_{A2})$ if there exist $\phi \in Aut(K)$ such that $A_1 = A_2 \circ \phi$, i.e., $T_{A1}(s) = T_{A2}(\phi(s))$, $I_{A1}(s) = I_{A2}(\phi(s))$ and $F_{A1}(s) = F_{A2}(\phi(s))$, for all $s \in G$.

Theorem 2.8. Let $A_1 = (T_{A1}, I_{A1}, F_{A1})$ and $A_2 = (T_{A2}, I_{A2}, F_{A2})$ be neutrosophic pythogorean K- sub algebras of K. Then $A_1 = (T_{A1}, I_{A1}, F_{A1})$ is a neutrosophic pythogorean K-sub algebra having the same type of $A_2 = (T_{A2}, I_{A2}, F_{A2})$ if and only if A_1 is isomorphic to A_2.

Proof. Sufficient condition holds trivially so we only need to prove the necessary condition. Let $A_1 = (T_{A1}, I_{A1}, F_{A1})$ be a neutrosophic pythogorean K-sub algebra having same type of $A_2 = (T_{A2}, I_{A2},$

F_{A2}). Then there exist $\phi \in Aut(K)$ such that $T_{A1}(s) = T_{A2}(\phi(s))$, $I_{A1}(s) = I_{A2}(\phi(s))$ and $F_{A1} = F_{A2}(\phi(s))$, for all $s \in G$ .Let $f: A_1(K) \to A_2(K)$ be a mapping defined by $f(A_1(s)) = A_2(\phi(s))$, for all $s \in G$, that is, $f(T_{A1}(s)) = T_{A2}(\phi(s))$, $f(I_{A1}(s)) = I_{A2}(\phi(s))$ and $f(F_{A1}(s)) = F_{A2}(\phi(s))$, for all $s \in G$.

Clearly, f is surjective. Also, f is injective because if $f(T_{A1}(s)) = f(T_{A1}(t))$, for all $s, t \in G$, then $T_{A2}(\phi(s)) = T_{A2}(\phi(t))$ and we have $T_{A1}(s) = T_{A1}(t)$. Similarly, $I_{A1}(s) = I_{A1}(t)$, $F_{A1}(s) = F_{A1}(t)$.

Therefore, f is a homomorphism, for $s, t \in G$

$$f(T_{A1}(s \odot t)) = T_{A2}(\phi(s \odot t)) = T_{A2}(\phi(s) \odot \phi(t)),$$
$$f(I_{A1}(s \odot t)) = I_{A2}(\phi(s \odot t)) = I_{A2}(\phi(s) \odot \phi(t)),$$
$$f(F_{A1}(s \odot t)) = F_{A2}(\phi(s \odot t)) = F_{A2}(\phi(s) \odot \phi(t)).$$

Hence $A_1 = (T_{A1}, I_{A1}, F_{A1})$ is isomorphic to $A_2 = (T_{A2}, I_{A2}, F_{A2})$. Hence the proof.

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