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MORE ON FUZZY γ-GENERALIZED CLOSED SETS IN TOPOLOGY

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Abstract: The aim of this paper is in introduce and study a new class of Fuzzy sets called Fuzzy γ-generalized regular weakly closed (briefly, γgrw -closed) set This new class of sets lies between the class of Fuzzy regular weakly closed (briefly, rw –closed) sets and the class of Fuzzy γ-generalized closed (briefly. γg-closed) sets. Also, we study the fundamental properties of this class of fuzzy sets.

***Index Terms* - MSC: 54A05; 54C10; 54D10t**

Keywords: Fuzzy Regular closed sets; Fuzzy γ-regular closed sets; Fuzzy Weakly closed sets; Fuzzy γ -generalized regular weakly closed sets and Fuzzy regular semi-kernal.

1. INTRODUCTION AND PRELIMINARIES

The study of fuzzy sets was initiated with the famous paper of Zadeh [25] and thereafter Chang [7] paved the way for subsequent tremendous growth of the numerous fuzzy topological concepts. Every Fuzzy Topological space can be defined either with the help of fuzzy axioms sets. So. one can imagine that, how important the concept of fuzzy closed sets is in the fuzzy topological spaces. The concept of generalized fuzzy closed set in general fuzzy topological space was instigated by M. Pu , Y. M. Liu[17] , R. K. Saraf, M. Caldas and S. Mishra [19] which has been extensively used as an excel-lent tool for studying different concepts in the said fuzzy space.

The generalized closed set is the most common but important and interesting concepts in topological spaces as well as fuzzy topological spaces. The concept of generalized closed set in general topological space was first instigated by Levine (1970), which has been extensively used as an excel-lent tool for studying different concepts in the said space. In fuzzy setting, the concept of generalized fuzzy closed set was initiated by Balasubramanian et al. (1997). Subsequently, ma-ny authors have devoted their work to the study of various forms of generalized fuzzy closed set, for instance Saraf et al. (2005) and Park et al. (2003). On the other hand, Császár (2002) introduced the notion of generalized neighborhood sys-tems and generalized topological spaces (in short, GTS's). Very recently a number of researchers are attempting to ex-tend the idea of generalized closed set in generalized topology which is a broader framework of general topology. Moreover, Maragathavalli et al. (2010) have studied generalized closed set and its fundamental properties in generalized topological spaces. Before that, Chetty (2008) has extended the concept of generalized topological space in fuzzy environment and named it generalized fuzzy topological space. Furthermore, Császár (2011) defined the concept of weak structure which is a weaker form of generalized topology. Afterwards various researchers have worked on that field namely Ghareeb et al. (2015), Zaharan et al. (2012), and Zakari et al. (2017) and studied various properties of weak structures in various directions

In fuzzy setting, the concept of generalized fuzzy closed set was initiated by Chetty, G. P. (2008) . By definition of a fuzzy subset λ of a fuzzy topological space X is called fuzzy generalized closest (briefly. fg-closed) fuzzy set if Cl($\lambda \leq \mu$), whenever $\lambda \leq \mu$ and if μ is fuzzy open in (X. τ). This notion had been studied extensively in the recent years by many fuzzy topologists since fuzzy generalized closed sets are not only the fuzzy generalization of closed sets. Moreover. they also suggest fuzzy separation axioms weaker than T1and some of them found to be useful in computer science and digital topology.

Furthermore. the study of fuzzy generalized closed sets aIso provides new characicrizations of come known classes of spaces, for example, extremely fuzzy disconected spaces by Cao et al. [5]. In 2007. the notion of fuzzy regular weakly closed set was defined by Munir Abdul khalik Alkhafaji, et al. and they proved that this class lies between the class of all fuzzy w-closed sets given by Talal AL-Hawary in 2017 and the class of all fuzzy regular generalized closed sets defined by Jin Han Park and JK Park et al. [16]. In the present paper, we introduce and study a new class of fuzzy sets called a fuzzy λ-generalized regular weakly closed (briefly, λgrw -closed) set in fuzzy topological spaces which is properly placed between the fuzzy regular weakly closed sets and fuzzy λ-generalized regular closed sets.
Now. we recall the following definitions which are useful in the sequal.

Throughout this paper (X, τ) and (Y, σ) (or simply, X and Y) represent the non-empty fuzzy topological spaces on which no fuzzy separation axioms are assumed, unless other- wise mentioned. For a fuzzy subset λ of X, cI(λ) int(λ) and λ 'or X — λ represent the fuzzy closure of λ the fuzzy interior of λ and the complement of λ. respectively.

Definition 1.1. A subset λ of a space X is ca1led:

(i) Fuzzy regular open [21] if λ = int(cl(λ)) and Fuzzy regular closed [21] if λ = cl(int(λ)),

(ii) Fuzzy α-open [21] if $\lambda \leq$ int(cl(int(λ))) and Fuzzy α-closed if cl(int(cl(λ))$\leq \lambda$,

(iii) Fuzzy α-open preopen [19] if $\lambda \leq$ int(cl(λ)) and Fuzzy α-open preclosed [19] if cl(int(λ))$\leq \lambda$,

(iv) Fuzzy α-open semi-open [15] if $\lambda \leq$ cl(int(λ)) and Fuzzy α-open semi-closed [13] if int(cl(λ))$\leq \lambda$,

(v) Fuzzy α-open , Fuzzy γ-open [21] or Fuzzy b-open [21] or Fuzzy sp-open [8] if $\lambda \leq$ int(cl(λ)) $\vee$ cl(int(λ)) and Fuzzy γ-closed [9] or Fuzzy b-closed [1] or Fuzzy sp-closed [8] if cl(int(λ)) $\wedge$ int(cl(λ))$\leq \lambda$,

(vi) Fuzzy δ-closed [20] if δ-cl(λ) = λ, where δcl(λ) = {x ∈ X : int(cl(η)) $\wedge$ λ =/ φ, η ∈ τ and x ∈ η}.

(vii) Fuzzy regular semi-open (briefly, Fuzzy rs-open) [4] if there is a Fuzzy regular open set η such that $\eta \leq \lambda \leq$ cl(η),

Definition 1.2.

A Fuzzy subset λ of a space X is called:

(i) Fuzzy generalized closed (briefly, g-closed) [13] if cl(λ)$\leq \eta$ whenever $\lambda \leq \eta$ and H is Fuzzy open in (X, τ),
(ii) Fuzzy generalized α-closed (briefly, gα-closed)[14] if αcl(λ)$\leq \eta$ whenever $\lambda \leq \eta$ and η is Fuzzy α-open in (X, τ),
(iii) Fuzzy α-generalized closed (briefly, αg-closed)[14] if αcl(λ)$\leq \eta$ whenever $\lambda \leq \eta$ and η is Fuzzy open in (X, τ),
(iv) Fuzzy regular generalized closed (briefly, rg-closed)[13] if cl(λ)$\leq \eta$ whenever $\lambda \leq \eta$ and η is Fuzzy regular open in (X, τ),
(v) Fuzzy δ-generalized closed (briefly, δg-closed)[20] if δcl(λ)$\leq \eta$ whenever $\lambda \leq \eta$ and η is Fuzzy open in (X, τ),
(vi) Fuzzy weakly closed (briefly, w-closed)[20] if cl(λ)$\leq \eta$ whenever A $\leq \eta$ and η is Fuzzy semi-open in (X, τ),
(vii) Fuzzy regular weakly generalized closed (briefly, rwclosed) [4] if cl(λ)$\leq \eta$ whenever $\lambda \leq \eta$ and η is Fuzzy regular semi-open in (X, τ),
(viii) Fuzzy γ-generalized closed (briefly, γg-closed)[10] if γcl(λ)$\leq \eta$ whenever $\lambda \leq \eta$ and η is Fuzzy open in (X, τ).
(ix) Fuzzy γ-generalized regular closed (briefly, γgr-closed)[10] if γ-cl(λ)$\leq \eta$ whenever $\lambda \leq \eta$ and η is Fuzzy regular open in (X, τ).

The family of all Fuzzy regular semi-open sets in a space X is denoted by fRSO(X).

2. Fuzzy γ-Generalized regular weakly closed sets .

In this section, we introduce and study some basic properties of a new class of sets called fuzzy γ-generalized regular weakly closed (briefly, γgrw-closed) sets.

Definition 2.1.

Let (X, τ) be a fuzzy topological space and λ be a fuzzy subset of X. Then λ is said to be:

(i) a fuzzy γ-generalized regular weakly closed (briefly, γgrw-closed)set if γ-cl(λ)≤ η whenever $\lambda \leq \eta$ and H is fuzzy regular semi-open in (X, τ),

(ii) a fuzzy γ-generalized regular weakly open (briefly, γgrwopen) set if λ^c is γgrw-closed in X. The family of all γgrw-closed sets in a space X is denoted by γGRWC(X).

Theorem 2.1.

For a fuzzy topological space (X, τ), the following statements are hold:

(i) every fuzzy closed (resp. δ-closed, w-closed, rw-closed, γ-closed) set of a fuzzy topological space (X, τ) is γgrwclosed,

(ii) every γgrw-closed set of a fuzzy topological space (X, τ) is γgr-closed.

Proof.

We prove this point for the case of rw-closed. Let λ be an arbitrary rw-closed in (X, τ) such that $\lambda \leq \eta$ and η be regular semi-open. Then by definition of rw-closed, we have cl(λ)≤ η . Since every closed set in a topological space (X, τ) is preclosed, then pcl(λ)≤ cl(λ). So, we have, pcl(λ)≤cl(λ)≤ η . Hence, A is γgrw-closed.

(ii) Let (X, τ) be a topological space and λ be a γgrwclosed subset of X such that $\lambda \leq \eta$, where η is regular open. Since every regular open set is regular semi-open, then by definition of γgrw-closed, we have pcl(λ)≤ η . Hence λ is γgr-closed.

The converse of the above theorem may be not true as is shown by the following examples.

Example 2.1

Let $X = \{a,b,c\}$, $\tau = \{0_X,1_X,\{(a,0.5),(b,0.4),(c,0.3)\},\{(a,0.6),(b,0.3),(c,0.5)\},\{(a,0.6),(b,0.4),(c,0.5)\}\}$ and $\mu = \{(a,0.6),(b,0.7),(c,0.4)\}$. In the γgrw-closed in (X, τ), μ is a is γgrw-closed but it is neither closed nor δ-closed,

Example 2.2

Let $X = \{a,b,c\}, \tau = \{0_X, 1_X ,\{(a,1),(b,0),(c,0)\}\}$, $\lambda_1 = \{(a,1),(b,0.5),(c,0)\}$ is γgrw-closed but not w-closed,

Example 2.3

Let $X = \{a,b,c\}$, $\tau = \{0_X,1_X,\{(a,0.5),(b,0.4), (c,0.3)\},\{(a,0.6),(b,0.3),(c,0.5)\},\{(a,0.6),(b,0.4),(c,0.5)\}$ and $\mu = \{(a,0.6),(b,0.7),(c,0.4)\}$. In the FTS (X, τ), μ is a gf gX- γgrw-closed but not γ-closed.

Example 2.4

Let = $\{a,b,c\}$, $\tau = \{0X,1X,\{(a,0.5),(b,0.4), (c,0.3)\},\{(a,0.6),(b,0.3),(c,0.5)\},\{(a,0.6),(b,0.4),(c,0.5)\}\}$ and $\mu = \{(a,0.5),(b,0.4),(c,0.3)\}$. In the FTS (X, τ), μ is a is γgr-closed but not γgrw-closed.

Example 2.4

Let $X = \{a, b, c\}, \tau = \{0_X, 1_X,\{(a, 0.5), (b, 0.4), (c, 0.3)\},\{(a, 0.6), (b, 0.3), (c, 0.5)\},\{(a, 0.6), (b, 0.4), (c, 0.5)\}\}$ and $\mu = \{(a, 0.6), (b, 0.7), (c, 0.4)\}$. In the FTS (X, τ), μ is a is γgrw-closed but not rw-closed.

Remark 2.1. We can see from the following example that a γgrw-closed set is independent of α-closed (resp. gα-closed, αg-closed, rg-closed, δg-closed, γg-closed).

Lemma 1.1 ([11]).

For a space (X, τ) and λ is a subset of X, then:

every fuzzy regular closed set, fuzzy regular open set and fuzzy clopen set is fuzzy regular semi-open in X, if λ is a fuzzy regular semi-open set, then X − λ is also fuzzy regular semi-open in X.

Lemma 1.2 ([3])

Let X be a fuzzy topological space and Y be an fuzzy open subspace of X such that $\lambda \leq Y \leq X$. Then $\lambda \in$ RSO(Y), if $\lambda \in$ RSO(X).

Lemma 1.3 ([3]).

If Y is fuzzy regular open in X and η is a fuzzy subset of Y, then his fuzzy regular semi-open in X if and only if η is fuzzy regular semi-open in the fuzzy subspace Y.

Definition 1.3

The intersection of all fuzzy regular semi-open subsets of (X, τ) containing A is called the fuzzy regular semi-kernel of λ and is denoted by rs ker(λ).

Lemma 1.4
If x is a point of (X, τ), then {x} is either fuzzy nowhere dense or fuzzy preopen.

Remark 1.1
In the notation of Lemma 1.4, we may consider the following decomposition of a given fuzzy topological space (X, τ), namely $X = X_1 \vee X_2$, where X1 = {x ∈ X : {x} is fuzzy nowhere dense } and X_2 = {x ∈ X : {x} is fuzzy preopen }.

Lemma 1.5
For any fuzzy subset λ of (X, τ), we have $\lambda \leq$ rs ker(λ) .
The union of two γgrw -closed sets need not be a γgrw -closed set in a FTS (*X*, τ).

Example2.5
Let $X = \{a, b, c\}$, τ = {0_X, 1_X,{(*a*, 0.4), (*b*, 0.7), (*c*, 0.5)},{(*a*, 0.6), (*b*, 0.3), (*c*, 0.5)},{(*a*, 0.6), (*b*, 0.7), (*c*, 0.5)}}.
We suppose that
λ_1 = {(*a*, 0.4), (*b*, 0.7), (*c*, 0.5)},
λ_2 = {(*a*, 0.6), (*b*, 0.3), (*c*, 0.5)} and
λ_3 = {(*a*, 0.6), (*b*, 0.7), (*c*, 0.5)}.
We have Cl (λ_1) = $\lambda_1 \leq \lambda_1$ and Cl (λ_2) = $\lambda_2 \leq \lambda_2$. Also $\lambda_1 \vee \lambda_2 = \lambda_3$ but C_l ($\lambda_1 \vee \lambda_2$) = $1_X \nleq \lambda_3$. Therefore, λ_1 and λ_2 are two γgrw -closed sets while $\lambda_1 \vee \lambda_2$ is not a γgrw -closed set.

Remark2.6
The intersection of two γgrw -closed sets may not be a γgrw -closed set in a GFTS (*X*, τ).

Example 2.7
Let $X = \{a, b, c\}$, gX = {0*X*,{(*a*, 1), (*b*, 0), (*c*, 0)}}, λ_1 = {(*a*, 1), (*b*, 0.5), (*c*, 0)} and λ_2 = {(*a*, 1), (*b*, 0), (*c*, 0.6)}. It is easy to verify that both λ_1 and λ_2 are γgrw -closed sets in GFTS (*X*, τ) but the intersection $\lambda_1 \wedge \lambda_2$ = {(*a*, 1), (*b*, 0), (*c*, 0)} is not a γgrw -closed set

Theorem 2.2.
Let (X, τ) be a topological space and λ be a γgrw-closed subset of X. Then γcl(λ) − λ does not contain any non empty regular semi-open set of X.

Proof. Suppose that η be a non empty regular semi-open set of X such that $\eta \leq$ γcl(λ) − λ . Hence $\eta \leq X - \lambda$ or $\lambda \leq X - \eta$, then by Theorem 1.2, X − η is regular semi-open. But, A is a γgrw-closed subset of X, hence γcl(A) ≤ X − η this implies that $\eta \leq$ X − γcl(λ) and we know that $\eta \leq$ γcl(λ). Therefore, $\eta \leq$ [γcl(λ)∩(X − γcl(λ))] = ϕ this shows that η is empty set which is a contradiction. Then γcl(λ) − λ does not contain any non empty regular semi-open set of X. In the following example, we show that the converse of the above theorem is not true.

Proposition 2.1.
If (X, τ) is a topological space and A is a γgrw-closed subset of X, then γcl(A) − A does not contain any non empty regular open set of X.

Proof. The proof follows directly from the fact that every regular open set is regular semi-open.

Proposition 2.2. If λ is a γgrw-closed subset of X and $\lambda \leq \mu \leq$ γ-cl(λ), then μ is a γgrw-closed set of X.

Proof. Let λ be an γgrw-closed subset of X such that $\lambda \leq \mu \leq$ γ-cl(λ) and H be a regular semi-open set of X such that μ ≤H. Then $\lambda \leq \eta$. But A is γgrw-closed, then γ-cl(λ)≤ η , hence, γ-cl(μ)≤ γ-cl(γ-cl(λ)) = γcl(λ)≤ η . Therefore μ is γgrw-closed in X.

Remark 2.3. The converse of Proposition 2.1 is not true.
Let $X = Y = \{a, b, c\}$, τ = {0*X*, 1*X*,{(*a*, 0.5), (*b*, 0.4), (*c*, 0.3)},{(*a*, 0.6), (*b*, 0.3), (*c*, 0.5)},{(*a*, 0.6), (*b*, 0.4), (*c*, 0.5)}} and λ ={(*a*, 0.3), (*b*, 0.3), (*c*, 0.5)} and μ ={(*a*, 0.4), (*b*, 0.3), (*c*, 0.5)} λ , μ are γgrw-closed sets subsets of X and $\lambda \leq \mu$ which is not subset in γ-cl(λ).

Theorem 2.3. Let(X, τ) be a topologicalspace and λ be a γgrw-closed subset of X.
Then the following statements are equivalent:

(i) λ is γ-closed,
(ii) γcl(λ) − λ is regular semi-open.

(i)⇒(ii). Since A is γ-closed, γ-cl(λ) = λ and so γcl(λ) − λ = ϕ which is regular semi-open in X.

(ii)⇒(i). Suppose that γcl(λ) – λ is regular semiopen in X. But λ is a γgrw-closed set of X, then by Proposition 2.1, γcl(λ) − λ does not contain any non empty regular semi-open set of X. Hence, γcl(λ) − λ = ϕ, so, λ is γ-closed.

Theorem 2.4. Let (X, τ) be a topological space. Then for x ∈ X, the set X − {x} is γgrw-closed or regular semiopen.
Proof. If the set X − {x} is γgrw-closed or regular semi-open, then we are done. Now,suppose that X − {x} is notregularsemi-open. Then X isthe only regularsemiopen set containing X − {x} and hence γcl(X − {x}) ≤ X that is the biggest set containing all of its subsets. Therefore, X − {x} is a γgrw-closed set of X.

Proposition 2.3. In a topologicalspace (X, τ), for each x ∈ X, the singleton set {x} is either γgrw-open or regular semi-open.

Proof. Obvious from Theorem 2.4. 3. Some properties of γgrw-closed sets.

Theorem 3.1. Let λ be a regular open and a γgrwclosed set of (X, τ). Then λ is γ-clopen.

Proof. Suppose that λ is a regular open and γgrwclosed set of(X, τ). Since every regular open set isregular semi-open and $\lambda \leq \lambda$, γ-cl(λ)≤ λ. Also, λ ≤ γ-cl(λ). Therefore, λ = γ-cl(λ) this means that λ is γ-closed. But, λ is regular open, hence, λ is γ-open. Therefore, λ is γ-clopen.

Theorem 3.2. Let λ be a regular open and a rg-closed set of (X, τ). Then λ is a γgrw-closed.

Proof. Let H be a regular semi-open set of X such that λ ≤ H. But, A is a regular open and rg-closed set of(X, τ), then by Theorem 3.1, γ-cl(λ)≤ λ. Hence, γ-cl(λ)≤ H. So, λ is a γgrw-closed.

Proposition 3.1. If λ is a regular semi-open and a γgrw-closed set of (X, τ), then λ is a γ-closed.

Proof. Obvious.

Theorem 3.3. Let λ be a regular semi-open and a γgrw-closed set of (X, τ). Suppose that δ is a closed set of X. Then $\lambda \wedge \delta$ is a γgrw-closed set of X.
Proof. Suppose that λ is a regular semi-open and a γgrw-closed set of (X, τ). Then by Proposition 3.1, λ is γ-closed. But, δ is closed set of X, hence, $\lambda \cap \delta$ is a γ-closed and therefore $\lambda \cap \delta$ is a γgrw-closed set of X.

Theorem 3.4. Suppose that λ is both regular open and γgrw-closed set of (X, τ). If $\mu \leq \lambda \leq$ X and B is a γgrw closed set relatives to λ, then B is a γgrw-closed set relative to X.

Proof. Let $\mu \leq \eta$ and η be a regular semi-open set of X. But, $\mu \leq \lambda \leq$X, then $\mu \leq \lambda \wedge \eta$. We need to prove that $\lambda \wedge \eta$ is regular semi-open in λ. Firstly, we prove that $\lambda \wedge \eta$ isregularsemi-open in X. Since λ is open and η is semi-open in X, hence $\lambda \wedge \eta$ issemi-open in X. Also, λ is both regular open and γgrw-closed set of (X, τ), then by Theorem 3.1, λ is semi-closed in X. Since, every regular semi-open set is semi-closed, hence, η is semi-closed in X. Therefore, $\lambda \wedge \eta$ is semi-open in X. Thus, $\lambda \wedge \eta$ is both semi-open and semi-closed in X and hence $\lambda \wedge \eta$ is regular semi-open in X. Further $\lambda \wedge \eta \leq \lambda \leq$ X and λ is open subspace of X, then by Lemma 1.2, $\lambda \wedge \eta$ isregular semi-open in λ. Since, μ is a γgrw-closed set. (ii). Hence, from(i) and (ii), it follows that λ ∩γ-cl(μ) $\leq \lambda \wedge \eta$. Consequently, λ ∧γ-cl(B) ≤ η. Since, λ is both regular open and γgrw-closed sets, hence by Theorem 3.1, γ-cl(λ) = λ and so γ-cl(μ) ≤ λ. We have λ ∧γ-cl(μ) = γ-cl(μ). Thus γ-cl(μ)≤ η and hence μ is a γgrw-closed set relative to X.

Theorem 3.5. If λ is a γgrw-closed set of (X, τ) and A ≤ Y ≤X, then λ is γgrw-closed in Y, if Y is regular open in X.

Proof. Let λ be γgrw-closed in X and Y be regular open subspace of X. If H is any regular semi-open set of Y such that $\lambda \leq \eta$, but λ ≤Y ≤ X, then by Lemma 1.3, η is regular semi-open in X. Since λ is a γgrw-closed set of X, then γ-cl(λ)≤ η. Hence, Y∧γ-cl(λ) ≤ Y∧ η = η. So, γ-cl(λ)≤H. Therefore, λ is γgrw-closed in Y.

Proposition 3.2. If λ is both open and γg-closed sets of X, then λ is γgrw-closed in X.

Proof. Suppose that $\lambda \leq \eta$ and η is regular semi-open in X. Since λ is both open and γg−closed in X and $\lambda \leq \lambda$, then γ-cl(λ) ≤ λ this implies that γ-cl(λ) ≤ η. Hence λ is γgrw-closed in X.

Remark 3.1. If λ is both open and γgrw-closed in X, then λ need not be γg-closed in X. In Example 2.4, the subset {x, y} is both open and γgrw-closed in X but not γg-closed in X.

Theorem 3.6. For a topological space (X, τ), if RSO(X) = {X, ϕ}, then every subset of X is γgrw-closed in X.

Proof. Let λ be any subset of X and RSO(X) = {X, ϕ}. If λ = ϕ, then A is γgrw-closed in X. Assume that $\lambda \neq \phi$. Then X is the only regular semi-open containing λ and so, $\gamma - cl(\lambda) \leq X$. Therefore A is γgrw-closed in X.

Remark 3.2. The converse of the above theorem need not be true in general as shown by the following example.

Example 3.1.
Let $X = Y = \{a, b, c\}$, $\tau = \{0_X, 1_X$ $\{(a, 0.4), (b, 0.7)$ $(c, 0.5)\}, \{(a, 0.6), (b, 0.7), (c, 0.5)\}\}$ and $\mu = \{(a, 0.4), (b, 0.6), (c, 0.5)\}$. then every subset of X is γgrw-closed in X, but RSO(X) = {X, ϕ, {x, y}, {z, u}}.

Theorem 3.7. For a topological space (X, τ), the following statements are equivalent: (i) RSO(X) ⊆ { δ ⊆ X : δ is γ-closed }. (ii) every subset of X is γgrw-closed.

Proof. (i)⇒(ii). Let λ be any subset of X such that $\lambda \subseteq \eta$, η is regular semi-open in X and suppose that RSO(X) $\leq$ { $\delta \leq$ X : δ is γ-closed }. Then $\eta \in$ { $\delta \leq$ X : δ is γ-closed }, thus η is γ-closed, that is γ-cl(η) = η and hence γcl(λ) $\leq \eta$. Therefore λ is γgrw-closed.
(ii)⇒(i). Suppose that every subset of X is γgrwclosed and δ ∈ RSO(X). Since $\delta \leq \delta$ and η is γgrw-closed, then γ-cl(δ) $\leq \delta$ and hence $\delta \in$ { $\delta \leq$ X : δ is γ-closed }. Therefore RSO(X) $\leq$ { $\delta \leq$ X : δ is γ-closed }.

4. Applications

In this section,
we introduce and study some applications on the concept of γgrw-closed sets of a space X.

Theorem 4.1. Let (X, τ) be a regular space in which every regular semi-open subset of X is open and λ be a compact subset of X. Then λ is γgrw-closed in X.
Proof. Assume that $\lambda \leq \eta$ and η is regular semi-open in X. Then by hypothesis, η is open. But, λ is a compact subset in the regular space (X, τ), hence there exists an open set G such that $\lambda \leq \chi \leq cl(\chi) \leq \eta$. Then γ-cl($\lambda$) $\leq$ γ-cl(χ) $\leq \eta$, that is, γ-cl(λ) $\leq \eta$. Thus λ is γgrw-closed in X.

Lemma 4.1. For any subset λ of (X, τ), we have $X_2 \wedge$ γcl(λ) $\leq$ rs ker(λ).

Theorem 4.2. If λ is a subset of a topological space(X, τ), then the following statements are equivalent:
λ is γgrw-closed in X, (ii) γ-cl(λ) ≤ rs ker(λ).

Proof. (i)⇒(ii). Since λ is γgrw-closed in X, γ-cl(λ) $\leq \eta$, whenever $\lambda \leq \eta$ and η is regular semiopen in X. If x ∈ γ-cl(λ) and suppose that x ∈/∉ rs ker(λ), then there is a regularsemi-open set η containing λ such that x ∈/ η. But, λ is γgrw-closed in X, then γ-cl(λ) ≤ η. We have x γ-cl(λ) which is a contradiction, hence x ∈ rs ker(A) and so γ-cl(λ) ≤ rs ker(λ). (ii)⇒(i). Suppose that γ-cl(λ) ≤ rs ker(λ). If η is any regular semi-open set containing λ, hence rs ker(λ) ≤ η, that is, γ-cl(λ) ≤ rs ker(λ) ≤ η. Therefore λ is γgrw-closed in X.

Theorem 4.3. For any subset λ of a topological space (X, τ) and $X_1 \wedge$ γ-cl(λ) $\leq \lambda$, then λ is γgrw-closed in X.

Proof. Suppose that $X_1 \wedge$ γ-cl(λ) ≤ λ and λ any subset of (X, τ). We need to prove that λ is γgrw-closed in X, then γ-cl(λ) ≤ rs ker(λ). But by Lemma 1.5, λ ≤ rs ker(λ) hence γcl(λ) = X ∧ γ-cl(λ) = (X1 ∨ X2) ∧ γ-cl(λ), that is, γ-cl(A) = (X_1 ∨ γ-cl(A)) ∨ (X_2 ∧ γ-cl(A)) ≤ rs ker(A). Since X_1 ∧ γ-cl(A) ≤ rs ker(A) and by Lemma 4.1, γ-cl(λ) ≤ rs ker(λ). Therefore by Theorem 4.2, λ is γgrw-closed in X.

Remark 4.1. The converse of the above theorem need not be true in general asshown by the following example.

Example 4.1. In

Example 2.1, if $X_1 = \{z, u\}$, $X_2 = \{x, y\}$ and $A = \{x, y, z\}$, hence λ is γgrw-closed in X. But $X_1 \wedge \gamma\text{-cl}(\lambda) = \{z, u\} \wedge X = \{z, u\}$ which is not a subset of λ.

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