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GENERALIZED HOMOMORPHISM AND ANTI HOMOMORPHISM OF FUZZY IDEAL OF A RING

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ABSTRACT

In this paper we discussed about the multi fuzzy set and multi dimensional membership, fuzzy multi ideal, fuzzy multi ring and apply to the theory of ring and in their notations.

INTRODUCTION

The innovative works of Zadeh[16] and Rosenfield[12] led to the fuzzification of algebraic structures. The idea of anti fuzzy subgroup was introduced by Biswas[3] which was extended by many researches F.A Azam, A.A Manum and F.Nasrin[2] apply the idea of Biswas to the theorey of ring. they introduced a notion of anti fuzzy ideal A of a ring X.

We introduce the theorey of multi-fuzzy set in terms of multi-dimensional membership functions and investigated some properties of multi level fuzziness. After introducing multi-fuzzy subsets of a crisp set.

We also introduce some elementary properties of multi-fuzzy subgroups. The concept of multi-anti fuzzy subgroup and discussed some of its properties.

We also extended the concept of multi-anti fuzzy subgroup to multi-anti fuzzy ideal of a ring and introduce a notion of multi-anti fuzzy ideal A of a ring X and some of its properties.

In this paper we discuss the properties of image of multi-fuzzy ideal of a ring under homomorphism and anti homomorphism and the properties of image multi-anti fuzzy ideal of a ring under homomorphism and anti-homomorphism.

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CHAPTER-1

PRELIMINARY

DEFINITION: 1.1

Let R and $R^{|}$ be rings. A function f: $R \rightarrow R^{|}$ is called a **homomorphism**. If

(i) f(a+b) = f(a) + f(b)

(ii) f(ab) = f(a)f(b) for all $a,b \in R$

If f is 1-1.then f is called a **monomorphism.**

If f is onto .then f is called an **epimorphism.** A homomorphism of a ring onto itself is called an **endomorphism**.

DEFINITION: 1.2

A Mapping f from a ring R to a ring S (both R and S not necessarily commutative) is called an **anti-homomorphism**. If for all x, $y \in R$

(i) f(x + y) = f(y) + f(x) and (ii) f(xy) = f(yx)

A surjective anti-homomorphism is called an anti-epimorphism.

DEFINITION: 1.3

A non empty subset S of a ring R is said to be an Ideal of R if it is both left and right Ideal of

(i.e.) an additive subgroup S of R is an ideal of R.if

 $a \in S, r \in R \Rightarrow ra \in S, ar \in S$

DEFINITION: 1.4

An additive subgroup S of a ring R is said to be a left ideal of R.if

 $a \in S, r \in R \Rightarrow ra \in S.$

DEFINITION: 1.5

An additive subgroup S of a R is said to be a right ideal of R.if

 $a \in S, r \in R \Rightarrow ar \in S$

DEFINITION: 1.6

A Proper ideal S of a ring R is said to be maximal ideal of R.if it is not strictly contained by any other **proper ideal** of R.

Eg:

DEFINITION: 1.7

A non zero ideal is called minimal if it contains no other **non zero ideal**.

DEFINITION: 1.8

If R be a commutative ring. Then an ideal $S \neq R$ is said to be **prime ideal.**

 $ab \in S \Rightarrow a \in S (or)b \in S$ For all $a, b \in R$.

DEFINITION: 1.9

If R is a commutative ring. Then aR = Ra is an ideal.

This is called the **principal ideal** generated by 'a' and is denoted by (a).

DEFINITION: 1.10

A multi-fuzzy ring A on a ring R is called a **multi-fuzzy ideal** if it is both a multi-fuzzy left ideal and a multi-fuzzy right ideal.

In other words,

a multi-fuzzy set A on R is a multi-fuzzy ideal of a ring.if

(i) $A(X - Y) \ge \min\{A(X), A(Y)\}$ and (ii) $A(XY) \ge \max\{A(X), A(Y) \text{ for all } x, y \in \mathbb{R}$

DEFINITION: 1.11

A non empty set R with binary operation '+' and '.'. If the following axioms hold.

- (i) (R, +)is an abelian group
- (ii) For all $a,b,c \in R$

Such that,

(R,.) is semigroup

Then, (R, +, .) is called a **Ring.**

DEFINITION: 1.12

A non empty subset S of a ring (R,+,.) is called a **subring.** If S itself is a ring under the same operations as in R.

Eg:

Z is a subring of Q.

Q is a subring of R.

DEFINITION: 1.13

 $(x, y \in R)$.

A multi-fuzzy set A on a ring R is said to be a **multi-fuzzy ring** on R.if for every

(i)
$$A(X - Y) \ge \min\{A(X), A(Y)\}$$

(ii) $A(YY) \ge \min(A(X), A(Y))$

(11) $A(XY) \geq \min\{A(X), A(Y)\}$

DEFINITION: 1.14

Let R be a ring a fuzzy ring A of R is called a ring with operator (read as M-fuzzy ring) iff for any $t \in [0,1]$, A_t is a ring with operator of R

(i.e). M-subring of R), when $A_t \neq \phi$

Where,

 $A_t = \{x \in R : A(x) \ge t\}$

DEFINITION: 1.15

Let A be M-fuzzy ideal of R is an M-fuzzy subring of R.

Such that,

 $(i) A(X - Y) \ge \min(A(X), A(y))$ for all X,Y inR $A(XY) \ge \min(A(X), A(Y))$ for all X,Y in R (ii) $A(Y + X - Y) \ge A(X)$ (iii) $A(XY) \ge A(Y)$ (iv) $A((X+Z)Y - XY) \ge A(Z)$ (v)

For all X, Y, $Z \in R$

Note that,

A is a M-fuzzy left ideal of R if it satisfies (i),(ii),(iii) and (iv),and A is said to be a M-fuzzy right ideal of R.if it satisfies (i),(ii),(iii) and (v)

DEFINITION: 1.19

Let A be M-fuzzy anti ideal of R.if A is an anti M-fuzzy subring of R.

Such that,

The following conditions are satisfied`

(i) $A(X - Y) \le \max\{A(X), A(Y)\}$ (ii) $A(XY) \le \max\{A(X), A(Y)\}$

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(iii) $A(Y + X - Y) \le A(X)$ (iv) $A(XY) \le A(Y)$ (v) $A((X + Z)Y - XY) \le A(Z)$ For all X, Y, Z $\in R$.

Note that, A is an anti M-fuzzy left ideal of R if it satisfies (AF_1) , (AF_2) , (AF_3) and (AF_4) and A is called an anti M-fuzzy right ideal of R if it satisfies (AF_1) , (AF_2) , (AF_3) and (AF_5)

CHAPTER-2

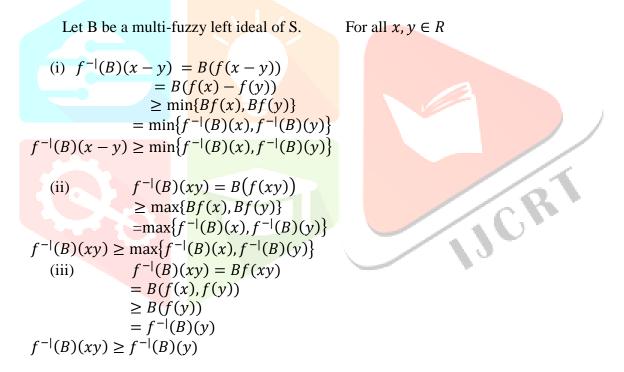
MULTI- FUZZY IDEAL

THEOREM: 2.1

Let f be a homomorphism from a ring R into a ring S and Let B be a multi-fuzzy left ideal of S. Then the pre-image, $f^{-1}(B)$ is a multi-fuzzy left ideal on R.

PROOF:

Consider a ring homomorphism $f: R \to S$.



Therefore,

 $f^{-|}(B)$ is a multi-fuzzy left ideal of R.

THEOREM: 2.2

Let f be a homomorphism from a ring R into a ring S and Let B be a multi-fuzzy right ideal of S. Then the pre-image, $f^{-|}(B)$ is a multi-fuzzy right ideal on R.

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PROOF:

Consider a ring homomorphism f: $R \rightarrow S$.

Let B be a multi-fuzzy right ideal of s. For all $x, y \in R$.

(i)
$$f^{-1}(B)(x - y) = Bf(x - y)$$

 $= B(f(x) - f(y))$
 $\geq \min\{Bf(x), Bf(y)\}$
 $= \min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$
(ii) $f^{-1}(B)(xy) = B(f(xy))$
 $\geq \max\{Bf(x), Bf(y)\}$
 $= \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$
(iii) $f^{-1}(B)(xy) \ge \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$
(iii) $f^{-1}(B)(xy) = Bf(xy)$
 $= B(f(x)f(y))$
 $\geq B(f(x))$
 $= f^{-1}(B)(x)$

Therefore,

f

 $f^{-1}(B)$ is a multi-fuzzy right ideal of R.

THEOREM: 2.3

Let f be a homomorphism from a ring R into a ring S and let A be a multi-fuzzy left ideal of a ring R with sub property. Then the image f (A) is a multi-fuzzy left ideal of a ring S.

PROOF:

Consider a ring homomorphism f: $R \rightarrow S$.

Let A be a multi-fuzzy left ideal of R. For all $x, y \in R$.

(i)
$$f(A)(f(x) - f(y)) = f(A)f(x - y)$$
$$= A(x - y)$$
$$\geq \min\{A(x), A(y)\}$$
$$= \min\{f(A)(x), f(A)(y)\}$$
$$f(A)(f(x) - f(y)) \geq \min\{f(A)(x), f(A)(y)\}$$
(ii)
$$f(A)(f(x)f(y)) = f(A)f(xy)$$
$$= A(xy)$$
$$\geq \max\{A(x), A(y)\}$$
$$= \max\{f(A)(x), f(A)(y)\}$$
(iii)
$$f(A)(f(x)f(y)) \geq \max\{f(A)(x), f(A)(y)\}$$
(iii)
$$f(A)(f(x)f(y)) = f(A)f(xy)$$
$$= A(xy)$$
$$\geq A(y)$$
$$= \Box(\Box)(\Box(\Box))$$

Therefore,

f (A) is a multi-fuzzy left ideal of S.

THEOREM: 2.4

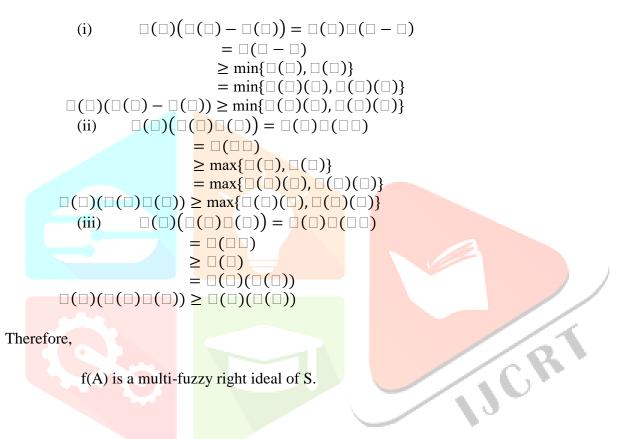
Let f be a homomorphism from a ring R into a ring S and let A be a multi-fuzzy right ideal of a ring R with sub property. Then the image f(A) is a multi-fuzzy right ideal of a ring S.

PROOF:

Consider a ring homomorphism $f: \Box \rightarrow \Box$.

Let A be a multi-fuzzy ideal of R.

For all x, $y \in \Box$



THEOREM: 2.5

Let f be an anti-homomorphism from a ring R into a ring S and let B be a multi-fuzzy left ideal of S.Then the pre-image $\Box^{-1}(\Box)$ is a multi-fuzzy right ideal of R.

PROOF:

Consider a ring anti-homomorphism $f: \Box \rightarrow \Box$.

Let B be a multi-fuzzy left ideal of S.

For all $x, y \in \Box$

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(i) \Box^{-1}(\Box)(\Box - \Box) = \Box(\Box - \Box)
	= \Box(\Box(\Box) - \Box(\Box))
	\geq \min\{\Box(\Box), \Box(\Box)\}
	= \min\{\Box^{-1}(\Box)(\Box), \Box^{-1}(\Box)(\Box)\}
	= \min\{\Box^{-1}(\Box)(\Box), \Box^{-1}(\Box)(\Box)\}
	\Box^{-1}(\Box)(\Box - \Box) \geq \min\{\Box^{-1}(\Box)(\Box), \Box^{-1}(\Box)(\Box)\}
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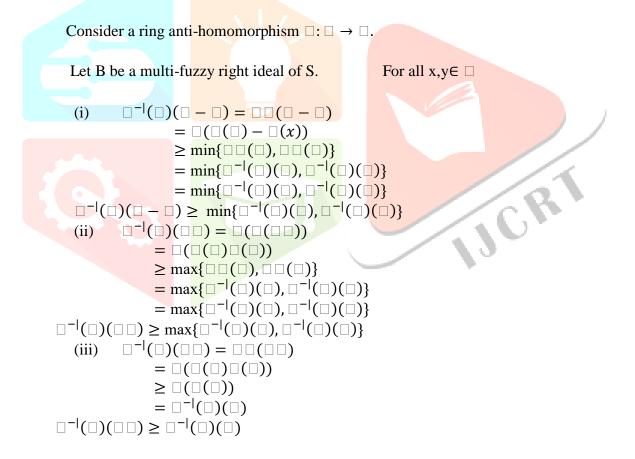
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(ii) \Box^{-1}(\Box)(\Box\Box) = \Box(\Box(\Box\Box)) 
= \Box(\Box(\Box))(\Box)) 
\geq max\{\Box(\Box), \Box\Box(\Box)\} 
= max\{\Box^{-1}(\Box)(\Box), \Box^{-1}(\Box)(\Box)\} 
= max\{\Box^{-1}(\Box)(\Box), \Box^{-1}(\Box)(\Box)\} 
(iii) \Box^{-1}(\Box)(\Box) = \Box(\Box\Box) 
= \Box(\Box(\Box))(\Box)) 
= \Box(\Box(\Box)) 
= \Box^{-1}(\Box)(\Box) 
= \Box^{-1}(\Box)(\Box) 
= \Box^{-1}(\Box)(\Box)
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Therefore, $\Box^{-|}(\Box)$ is a multi-fuzzy right ideal of R.

THEOREM: 2.6

Let f be an anti-homomorphism from a ring R into a ring S and let B be a multi-fuzzy right ideal of S. Then the pre-image, $\Box^{-1}(\Box)$ is a multi-fuzzy left ideal of R.

PROOF:



Therefore, $\Box^{-|}(\Box)$ is a multi-fuzzy left ideal of R.

THEOREM: 2.7

Let f be an anti-homomorphism from a ring R into a ring S and let A be a multi-fuzzy left ideal of a ring R with sub property. Then the image f(A) is a multi-fuzzy right ideal of a ring S.

PROOF:

Consider a ring anti-homomorphism $\Box : \Box \rightarrow \Box$.

Let A be a multi-fuzzy left ideal of R. For all $x,y \in \Box$

 $\Box(\Box)(\Box(\Box) - \Box(\Box)) = \Box(\Box)\Box(\Box - \Box)$ (i) $= \Box (\Box - \Box)$ $\geq \min\{\Box(\Box)\Box(\Box)\}$ $= \min\{\Box(\Box)(\Box), \Box(\Box)(\Box)\}$ $= \min\{\Box(\Box)(\Box), \Box(\Box)(\Box)\}$ $\square(\square)(\square(\square) - \square(\square)) \ge \min\{\square(\square)(\square), \square(\square)(\square)\}$ (ii) $\Box(\Box)(\Box(\Box)\Box(\Box)) = \Box(\Box)\Box(\Box\Box)$ $= \Box(\Box\Box)$ $\geq \max\{\Box(\Box), \Box(\Box)\}$ $= \max\{\Box(\Box)(\Box), \Box(\Box)(\Box)\}$ $= \max\{\Box(\Box)(\Box), \Box(\Box)(\Box)\}$ $\Box(\Box)(\Box(\Box)\Box(\Box)) \ge \max\{\Box(\Box)(\Box), \Box(\Box)(\Box)\}$ $\Box(\Box)(\Box(\Box)) = \Box(\Box) \Box(\Box)$ (iii) $= \Box (\Box \Box)$ $\geq \Box(\Box)$ $= \Box(\Box)(\Box(\Box))$ $(\Box(\Box)\Box(\Box)) \ge \Box(\Box)(\Box(\Box))$ Therefore. f (A) is a multi-fuzzy right ideal of S.

THEOREM: 2.8

Let f be an anti-homomorphism from a ring R into a ring S and let A be a multi-fuzzy right ideal of a ring R with sup property. Then the image, f (A) is a multi-fuzzy left ideal of a ring S.

PROOF:

Consider a ring anti-homomorphism $\Box : \Box \rightarrow \Box$.

Let A be a multi-fuzzy right ideal of R.

For all $x, y \in \Box$.

 $\Box(\Box)(\Box(\Box) - \Box(\Box)) = \Box(\Box)\Box(\Box - \Box)$ (i) $= \Box (\Box - \Box)$ $\geq \min\{\Box(\Box), \Box(\Box)\}$ $= \min\{\Box(\Box)(\Box), \Box(\Box)(\Box)\}$ $\square(\square)(\square(\square) - \square(\square)) \ge \min\{\square(\square)(\square), \square(\square)(\square)\}$ $\Box(\Box)(\Box(\Box)\Box(\Box)) = \Box(\Box)\Box(\Box\Box)$ (ii) $= \Box(\Box\Box)$ $\geq \max\{\Box(\Box), \Box(\Box)\}$ $= \max\{\Box(\Box)(\Box), \Box(\Box)(\Box)\}$ $\square(\square)(\square(\square)\square(\square)) \ge \max\{\square(\square)(\square), \square(\square)(\square)\}$ $\Box(\Box)(\Box(\Box)\Box(\Box)) = \Box(\Box)\Box(\Box-\Box)$ (iii) $= \Box (\Box \Box)$ $\geq \Box(\Box)$

 $\square(\square)(\square(\square)\square(\square)) \ge \square(\square)(\square(\square))$

Therefore,

f(A) is a multi-fuzzy left ideal of S.

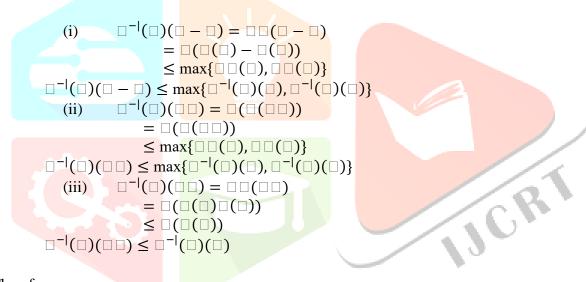
THEOREM: 2.9

Let f be a homomorphism from a ring R into a ring S and let B be a multi-fuzzy left ideal of S.Then the anti pre-image, $\Box^{-1}(\Box)$ is a multi- anti fuzzy left ideal of R.

PROOF:

Consider a ring homomorphism \Box : $\Box \rightarrow \Box$.

Let B be a multi anti-fuzzy left ideal of S. For all $x, y \in \Box$.



Therefore,

 $\Box^{-|}(\Box)$ is a multi-anti fuzzy left ideal of R.

THEOREM: 2.10

Let f be a homomorphism from a ring R into a ring S and let B be a multi-anti fuzzy right ideal of S.Then the anti pre-image, $\Box^{-1}(\Box)$ is a multi-anti fuzzy right ideal of R.

PROOF:

Consider a ring homomorphism $\Box : \Box \rightarrow \Box$.

Let B be a multi anti-fuzzy right ideal of S. For all $x,y \in \Box$.

Therefore,

 $\Box^{-1}(\Box)$ is a multi-anti fuzzy right ideal of R.

CONCLUSION

In this paper we discussed the properties of image of a multi-fuzzy ideal of a ring under homomorphism and anti-homomorphism and the properties of image of multi-anti fuzzy ideal of a ring under homomorphism and anti-homomorphism.

Hence, we also extended the concept of multi-anti fuzzy subgroup to multi-anti fuzzy ideal of a Ring.we verified that every function of homomorphism from a ring R into a ring S. If the multi-fuzzy set be a multi-fuzzy left & Right ideal of a ring with sub property.

Then,

The image of multi-fuzzy set is a multi-fuzzy left & Right ideal of a Ring S.

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