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GENERALIZED HOMOMORPHISM AND ANTI HOMOMORPHISM OF FUZZY IDEAL OF A RING

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ABSTRACT

In this paper we discussed about the multi fuzzy set and multi dimensional membership, fuzzy multi ideal , fuzzy multi ring and apply to the theory of ring and in their notations.

INTRODUCTION

The innovative works of Zadeh[16] and Rosenfield[12] led to the fuzzification of algebraic structures.The idea of anti fuzzy subgroup was introduced by Biswas[3] which was extended by many researches F.A Azam, A.A Manum and F.Nasrin[2] apply the idea of Biswas to the theory of ring.they introduced a notion of anti fuzzy ideal A of a ring X.

We introduce the theory of multi-fuzzy set in terms of multi-dimensional membership functions and investigated some properties of multi level fuzziness.After introducing multi-fuzzy subsets of a crisp set.

We also introduce some elementary properties of multi-fuzzy subgroups.The concept of multi-anti fuzzy subgroup and discussed some of its properties.

We also extended the concept of multi-anti fuzzy subgroup to multi-anti fuzzy ideal of a ring and introduce a notion of multi-anti fuzzy ideal A of a ring X and some of its properties.

In this paper we discuss the properties of image of multi-fuzzy ideal of a ring under homomorphism and anti homomorphism and the properties of image multi-anti fuzzy ideal of a ring under homomorphism and anti-homomorphism.

CHAPTER-1

PRELIMINARY

DEFINITION: 1.1

Let R and $R^{|}$ be rings. A function f: R→ $R^{|}$ is called a **homomorphism**.If

(i) $f(a+b) = f(a) + f(b)$

(ii) $f(ab) = f(a)f(b)$ for all a,b∈ R

If f is 1-1.then f is called a **monomorphism.**

If f is onto .then f is called an **epimorphism.** A homomorphism of a ring onto itself is called an **endomorphism**.

DEFINITION: 1.2

A Mapping f from a ring R to a ring S (both R and S not necessarily commutative) is called an **anti-homomorphism**. If for all x, y ∈ R

(i) $f(x+y) = f(y) + f(x)$ *and*

(ii) $f(xy) = f(yx)$

A **surjective** anti-homomorphism is called an **anti-epimorphism**.

DEFINITION: 1.3

A non empty subset S of a ring R is said to be an Ideal of R if it is both left and right Ideal of

(i.e.) an additive subgroup S of R is an **ideal** of R.if

$a \in S, r \in R \Rightarrow ra \in S, ar \in S$

DEFINITION: 1.4

An additive subgroup S of a ring R is said to be a **left ideal** of R.if

$a \in S, r \in R \Rightarrow ra \in S.$

DEFINITION: 1.5

An additive subgroup S of a R is said to be a **right ideal** of R.if

$a \in S, r \in R \Rightarrow ar \in S$

DEFINITION: 1.6

A Proper ideal S of a ring R is said to be maximal ideal of R.if it is not strictly contained by any other **proper ideal** of R.

Eg:

If R is a ring of even integer's .Then the ideal of R if there exist no proper ideal between S and R.

DEFINITION: 1.7

A non zero ideal is called minimal if it contains no other **non zero ideal**.

DEFINITION: 1.8

If R be a commutative ring.Then an ideal S$\neq R$ is said to be **prime ideal.**

$ab \in S \Rightarrow a \in S\ (or) b \in S$ For all $a, b \in R$.

DEFINITION: 1.9

If R is a commutative ring.Then $aR = Ra$ is an ideal.

This is called the **principal ideal** generated by 'a' and is denoted by (a).

DEFINITION: 1.10

A multi-fuzzy ring A on a ring R is called a **multi-fuzzy ideal** if it is both a multi-fuzzy left ideal and a multi-fuzzy right ideal.

In other words,

a multi-fuzzy set A on R is a multi-fuzzy ideal of a ring.if

(i) $A(X - Y) \geq \min\{A(X), A(Y)\}$ *and*
(ii) A(XY)$\geq$ max{A(X), A(Y) for all $x, y \in R$

DEFINITION: 1.11

A non empty set R with binary operation '+' and '**.**'. If the following axioms hold.

(i) $(R, +)$ *is an* ***abelian group***
(ii) For all a,b,c $\in R$

Such that,

$(R, .)$*is* ***semigroup***

Then, (R, +,.) is called a **Ring.**

DEFINITION: 1.12

A non empty subset S of a ring (R,+,.) is called a **subring.** If S itself is a ring under the same operations as in R.

Eg:

Z is a subring of Q.

Q is a subring of R.

DEFINITION: 1.13

A multi-fuzzy set A on a ring R is said to be a **multi-fuzzy ring** on R.if for every $(x, y \in R)$.

(i) $A(X - Y) \geq \min\{A(X), A(Y)\}$
(ii) $A(XY) \geq \min\{\mathrm{A(X), A(Y)}\}$

DEFINITION: 1.14

Let R be a ring a fuzzy ring A of R is called a ring with operator (read as M-fuzzy ring) iff for any $t \in [0,1]$, A_t is a ring with operator of R

(i.e). M-subring of R), when $A_t \neq \phi$

Where,

$A_t = \{x \in R: A(x) \geq t\}$

DEFINITION: 1.15

Let A be M-fuzzy ideal of R is an M-fuzzy subring of R.

Such that,

(i) $A(X - Y) \geq \min\big(A(X), A(y)\big)$ for all X,Y inR
(ii) $A(XY) \geq \min(A(X), A(Y))$ for all X,Y in R
(iii) $A(Y + X - Y) \geq A(X)$
(iv) $A(XY) \geq A(Y)$
(v) $A((X + Z)Y - XY) \geq A(Z)$

For all X, Y, Z$\in R$

Note that,

A is a M-fuzzy left ideal of R if it satisfies (i),(ii),(iii) and (iv),and A is said to be a M-fuzzy right ideal of R.if it satisfies (i),(ii),(iii) and (v)

DEFINITION: 1.19

Let A be M-fuzzy anti ideal of R.if A is an anti M-fuzzy subring of R.

Such that,

The following conditions are satisfied`

(i) $A(X - Y) \leq \max\{A(X), A(Y)\}$
(ii) $A(XY) \leq \max\{A(X), A(Y)\}$

(iii) $A(Y + X - Y) \leq A(X)$
(iv) $A(XY) \leq A(Y)$
(v) $A((X + Z)Y - XY) \leq A(Z)$
For all X, Y, Z$\in R$.

Note that, A is an anti M-fuzzy left ideal of R if it satisfies (AF_1), (AF_2), (AF_3) and (AF_4) and A is called an anti M-fuzzy right ideal of R if it satisfies (AF_1), (AF_2), (AF_3) and (AF_5)

CHAPTER-2

MULTI- FUZZY IDEAL

THEOREM: 2.1

Let f be a homomorphism from a ring R into a ring S and Let B be a multi-fuzzy left ideal of S. Then the pre-image, $f^{-1}(B)$ is a multi-fuzzy left ideal on R.

PROOF:

Consider a ring homomorphism $f: R \to S$.

Let B be a multi-fuzzy left ideal of S. For all $x, y \in R$

(i) $f^{-1}(B)(x - y) = B(f(x - y))$
$= B(f(x) - f(y))$
$\geq \min\{Bf(x), Bf(y)\}$
$= \min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$
$f^{-1}(B)(x - y) \geq \min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

(ii) $f^{-1}(B)(xy) = B(f(xy))$
$\geq \max\{Bf(x), Bf(y)\}$
$=\max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$
$f^{-1}(B)(xy) \geq \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

(iii) $f^{-1}(B)(xy) = Bf(xy)$
$= B(f(x), f(y))$
$\geq B(f(y))$
$= f^{-1}(B)(y)$
$f^{-1}(B)(xy) \geq f^{-1}(B)(y)$

Therefore,

$f^{-1}(B)$ is a multi-fuzzy left ideal of R.

THEOREM: 2.2

Let f be a homomorphism from a ring R into a ring S and Let B be a multi-fuzzy right ideal of S. Then the pre-image, $f^{-1}(B)$ is a multi-fuzzy right ideal on R.

PROOF:

Consider a ring homomorphism f: R→ S.

Let B be a multi-fuzzy right ideal of s. For all x, y∈ R.

(i) $f^{-1}(B)(x-y) = Bf(x-y)$

$= B\big(f(x)-f(y)\big)$

$\geq \min\{Bf(x), Bf(y)\}$

$= \min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

$f^{-1}(B)(x-y \geq \min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

(ii) $f^{-1}(B)(xy) = B\big(f(xy)\big)$

$\geq \max\{Bf(x), Bf(y)\}$

$= \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

$f^{-1}(B)(xy) \geq \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

(iii) $f^{-1}(B)(xy) = Bf(xy)$

$= B\big(f(x)f(y)\big)$

$\geq B(f(x))$

$= f^{-1}(B)(x)$

$f^{-1}(B)(xy) \geq f^{-1}(B)(x)$

Therefore,

$f^{-1}(B)$ is a multi-fuzzy right ideal of R.

THEOREM: 2.3

Let f be a homomorphism from a ring R into a ring S and let A be a multi-fuzzy left ideal of a ring R with sub property. Then the image f (A) is a multi-fuzzy left ideal of a ring S.

PROOF:

Consider a ring homomorphism f: R→ S.

Let A be a multi-fuzzy left ideal of R. For all x, y∈ R.

(i) $f(A)\big(f(x)-f(y)\big) = f(A)f(x-y)$

$= A(x-y)$

$\geq \min\{A(x), A(y)\}$

$= \min\{f(A)(x), f(A)(y)\}$

$f(A)(f(x)-f(y)) \geq \min\{f(A)(x), f(A)(y)\}$

(ii) $f(A)\big(f(x)f(y)\big) = f(A)f(xy)$

$= A(xy)$

$\geq \max\{A(x), A(y)\}$

$= \max\{f(A)(x), f(A)(y)\}$

$f(A)(f(x)f(y)) \geq \max\{f(A)(x), f(A)(y)\}$

(iii) $f(A)\big(f(x)f(y)\big) = f(A)f(xy)$

$= A(xy)$

$\geq A(y)$

= □(□)(□(□))

□(□)(□(□)□(□)) ≥ □(□)(□(□))

Therefore,

f (A) is a multi-fuzzy left ideal of S.

THEOREM: 2.4

Let f be a homomorphism from a ring R into a ring S and let A be a multi-fuzzy right ideal of a ring R with sub property. Then the image f (A) is a multi-fuzzy right ideal of a ring S.

PROOF:

Consider a ring homomorphism f:□ → □.

Let A be a multi-fuzzy ideal of R. For all x, y∈ □

(i) $$\square(\square)\big(\square(\square)-\square(\square)\big)=\square(\square)\square(\square-\square)$$
$$=\square(\square-\square)$$
$$\geq \min\{\square(\square),\square(\square)\}$$
$$=\min\{\square(\square)(\square),\square(\square)(\square)\}$$
$$\square(\square)(\square(\square)-\square(\square))\geq \min\{\square(\square)(\square),\square(\square)(\square)\}$$

(ii) $$\square(\square)\big(\square(\square)\square(\square)\big)=\square(\square)\square(\square\square)$$
$$=\square(\square\square)$$
$$\geq \max\{\square(\square),\square(\square)\}$$
$$=\max\{\square(\square)(\square),\square(\square)(\square)\}$$
$$\square(\square)(\square(\square)\square(\square))\geq \max\{\square(\square)(\square),\square(\square)(\square)\}$$

(iii) $$\square(\square)\big(\square(\square)\square(\square)\big)=\square(\square)\square(\square\square)$$
$$=\square(\square\square)$$
$$\geq \square(\square)$$
$$=\square(\square)(\square(\square))$$
$$\square(\square)(\square(\square)\square(\square))\geq \square(\square)(\square(\square))$$

Therefore,

f(A) is a multi-fuzzy right ideal of S.

THEOREM: 2.5

Let f be an anti-homomorphism from a ring R into a ring S and let B be a multi-fuzzy left ideal of S.Then the pre-image $\square^{-l}(\square)$ is a multi-fuzzy right ideal of R.

PROOF:

Consider a ring anti-homomorphism f:□ → □.

Let B be a multi-fuzzy left ideal of S. For all x,y∈ □

(i) $$\square^{-l}(\square)(\square-\square)=\square\square(\square-\square)$$
$$=\square(\square(\square)-\square(\square))$$
$$\geq \min\{\square\square(\square),\square\square(\square)\}$$
$$=\min\{\square^{-l}(\square)(\square),\square^{-l}(\square)(\square)\}$$
$$=\min\{\square^{-l}(\square)(\square),\square^{-l}(\square)(\square)\}$$
$$\square^{-l}(\square)(\square-\square)\geq \min\{\square^{-l}(\square)(\square),\square^{-l}(\square)(\square)\}$$

(ii) □$^{-l}$(□)(□□) = □(□(□□))
= □(□(□)□(□))
≥ max{□□(□), □□(□)}
= max{□$^{-l}$(□)(□), □$^{-l}$(□)(□)}
= max{□$^{-l}$(□)(□), □$^{-l}$(□)(□)}
□$^{-l}$(□)(□□) ≥ max{□$^{-l}$(□)(□), □$^{-l}$(□)(□)}

(iii) □$^{-l}$(□)(□□) = □□(□□)
= □(□(□)□(□))
≥ □(□(□))
= □$^{-l}$(□)(□)
□$^{-l}$(□)(□□) ≥ □$^{-l}$(□)(□)

Therefore, □$^{-l}$(□) is a multi-fuzzy right ideal of R.

THEOREM: 2.6

Let f be an anti-homomorphism from a ring R into a ring S and let B be a multi-fuzzy right ideal of S. Then the pre-image, □$^{-l}$(□) is a multi-fuzzy left ideal of R.

PROOF:

Consider a ring anti-homomorphism □: □ → □.

Let B be a multi-fuzzy right ideal of S. For all x,y∈ □

(i) □$^{-l}$(□)(□ − □) = □□(□ − □)
= □(□(□) − □(x))
≥ min{□□(□), □□(□)}
= min{□$^{-l}$(□)(□), □$^{-l}$(□)(□)}
= min{□$^{-l}$(□)(□), □$^{-l}$(□)(□)}
□$^{-l}$(□)(□ − □) ≥ min{□$^{-l}$(□)(□), □$^{-l}$(□)(□)}

(ii) □$^{-l}$(□)(□□) = □(□(□□))
= □(□(□)□(□))
≥ max{□□(□), □□(□)}
= max{□$^{-l}$(□)(□), □$^{-l}$(□)(□)}
= max{□$^{-l}$(□)(□), □$^{-l}$(□)(□)}
□$^{-l}$(□)(□□) ≥ max{□$^{-l}$(□)(□), □$^{-l}$(□)(□)}

(iii) □$^{-l}$(□)(□□) = □□(□□)
= □(□(□)□(□))
≥ □(□(□))
= □$^{-l}$(□)(□)
□$^{-l}$(□)(□□) ≥ □$^{-l}$(□)(□)

Therefore, □$^{-l}$(□) is a multi-fuzzy left ideal of R.

THEOREM: 2.7

Let f be an anti-homomorphism from a ring R into a ring S and let A be a multi-fuzzy left ideal of a ring R with sub property. Then the image f (A) is a multi-fuzzy right ideal of a ring S.

PROOF:

Consider a ring anti-homomorphism □: □ → □.

Let A be a multi-fuzzy left ideal of R. For all x,y∈ □

(i) □(□)(□(□) − □(□)) = □(□)□(□ − □)
= □(□ − □)
≥ min{□(□)□(□)}
= min{□(□)(□), □(□)(□)}
= min{□(□)(□), □(□)(□)}
□(□)(□(□) − □(□)) ≥ min{□(□)(□), □(□)(□)}

(ii) □(□)(□(□)□(□)) = □(□)□(□□)
= □(□□)
≥ max{□(□), □(□)}
= max{□(□)(□), □(□)(□)}
= max{□(□)(□), □(□)(□)}
□(□)(□(□)□(□)) ≥ max{□(□)(□), □(□)(□)}

(iii) □(□)(□(□)□(□)) = □(□)□(□□)
= □(□□)
≥ □(□)
= □(□)(□(□))
(□(□)□(□)) ≥ □(□)(□(□))

Therefore,

f (A) is a multi-fuzzy right ideal of S.

THEOREM: 2.8

Let f be an anti-homomorphism from a ring R into a ring S and let A be a multi-fuzzy right ideal of a ring R with sup property. Then the image, f (A) is a multi-fuzzy left ideal of a ring S.

PROOF:

Consider a ring anti-homomorphism □: □ → □.

Let A be a multi-fuzzy right ideal of R. For all x,y∈ □.

(i) □(□)(□(□) − □(□)) = □(□)□(□ − □)
= □(□ − □)
≥ min{□(□), □(□)}
= min{□(□)(□), □(□)(□)}
□(□)(□(□) − □(□)) ≥ min{□(□)(□), □(□)(□)}

(ii) □(□)(□(□)□(□)) = □(□)□(□□)
= □(□□)
≥ max{□(□), □(□)}
= max{□(□)(□), □(□)(□)}
□(□)(□(□)□(□)) ≥ max{□(□)(□), □(□)(□)}

(iii) □(□)(□(□)□(□)) = □(□)□(□ − □)
= □(□□)
≥ □(□)

□(□)(□(□)□(□)) ≥ □(□)(□(□))

Therefore,

f(A) is a multi-fuzzy left ideal of S.

THEOREM: 2.9

Let f be a homomorphism from a ring R into a ring S and let B be a multi-fuzzy left ideal of S.Then the anti pre-image, □^−l(□) is a multi- anti fuzzy left ideal of R.

PROOF:

Consider a ring homomorphism □: □ → □.

Let B be a multi anti-fuzzy left ideal of S. For all x,y∈ □.

(i) □^−l(□)(□ − □) = □□(□ − □)
= □(□(□) − □(□))
≤ max{□□(□), □□(□)}
□^−l(□)(□ − □) ≤ max{□^−l(□)(□), □^−l(□)(□)}

(ii) □^−l(□)(□□) = □(□(□□))
= □(□(□□))
≤ max{□□(□), □□(□)}
□^−l(□)(□□) ≤ max{□^−l(□)(□), □^−l(□)(□)}

(iii) □^−l(□)(□□) = □□(□□)
= □(□(□)□(□))
≤ □(□(□))
□^−l(□)(□□) ≤ □^−l(□)(□)

Therefore,

□^−l(□) is a multi-anti fuzzy left ideal of R.

THEOREM: 2.10

Let f be a homomorphism from a ring R into a ring S and let B be a multi-anti fuzzy right ideal of S.Then the anti pre-image,□^−l(□) is a multi-anti fuzzy right ideal of R.

PROOF:

Consider a ring homomorphism $\square : \square \rightarrow \square$.

Let B be a multi anti-fuzzy right ideal of S. For all x,y$\in \square$.

(i) $\square^{-1}(\square)(\square - \square) = \square\square(\square - \square)$

$= \square(\square(\square) - \square(\square))$

$\leq \max\{\square\square(\square), \square\square(\square)\}$

$\square^{-1}(\square)(\square - \square) \leq \max\{\square^{-1}(\square)(\square), \square^{-1}(\square)(\square)\}$

(ii) $\square^{-1}(\square)(\square\square) = \square(\square(\square\square))$

$= \square(\square(\square\square))$

$\leq \max\{\square\square(\square), \square\square(\square)\}$

$\square^{-1}(\square)(\square\square) \leq \max\{\square^{-1}(\square)(\square), \square^{-1}(\square)(\square)\}$

(iii) $\square^{-1}(\square)(\square\square) = \square\square(\square\square)$

$= \square(\square(\square)\square(\square))$

$\leq \square(\square(\square))$

$= \square^{-1}(\square)(\square)$

$\square^{-1}(\square)(\square\square) \leq \square^{-1}(\square)(\square)$

Therefore,

$\square^{-1}(\square)$ is a multi-anti fuzzy right ideal of R.

CONCLUSION

In this paper we discussed the properties of image of a multi-fuzzy ideal of a ring under homomorphism and anti-homomorphism and the properties of image of multi-anti fuzzy ideal of a ring under homomorphism and anti-homomorphism.

Hence,we also extended the concept of multi-anti fuzzy subgroup to multi-anti fuzzy ideal of a Ring.we verified that every function of homomorphism from a ring R into a ring S. If the multi-fuzzy set be a multi-fuzzy left & Right ideal of a ring with sub property.

Then,

The image of multi-fuzzy set is a multi-fuzzy left & Right ideal of a Ring S.

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