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A STUDY ON TOPOLOGICAL CHARACTERIZATIONS OF LINEARITY

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ABSTRACT

The space having uniformities with a totally ordered base are characterized in bigger classes of non-archimedean space and suboderable space, consequntey serveral new matrization results are obtained by exampie,we show that the condition used in our main theorem cannot be weakened essentially.our example may be interesting else where. In this paper we shall give the most important topological porperties of semi-linear uniform space( )

INTRODUCTION

All the spaces under consideration are supposed to be T_1- spaces. If the x is a space, then X$'$ is the set of all cluster points of X,φX,Xx, cX, dX, wX, denote the pseudocharacter, character, cellularity, density weight of X, respectively. Define ad X $\geq$x if any intersection of lessthan x open sets in X is open.

A topological space X is called linearly uniformizable if there is uniformity generating the topology of X and having a linearly ordered base (with respect to inclusion if we regard uniformities as entourages of diagonals or with respect to, refinement if we regard uniformities as covers) obviously, any metrizable topological spaces is linearly uniformizable.

As it is well- known , a metric d on a space X is called a non-archimedean metric if itsatisfies the strong triangular inequality $d(x, z) \leq$ max*$d(x, y)$, $d(x, y)$+ for all x, y, z∈X. It iseasy to see that then any two balls (x), $B(y)$ are either disjoint or one contains the other.

Hence a topological spaces X is called a non-archimedean topological space if it has a base Ɓ such that any two members of Ɓ are either disjoint or comparable by inclusion.

By result of a spaces X is non-archimedeanly metrizable iff it is a metrizable non- archimedean topological space. Proved the metrizabe space is non-archimedeanly metrizable iffIndX=0.

One can show that non-archimedean topological spaces X are always strongly zero dimensional but, conversely, The sorgenfrey –line S satisfies IndS=0 and it is not a non- archimaedean topological space. The Michael line is an example of a non-archimedean topological space with is not metrizable .every non-archimedean topological spaces is paracompact hence here ditarily para compact.

Let E be a Banach space and X a T_1-space. we shall consider a function $F(x)$ fromX into the space 2^E of all subsets of E and assume it to be lower semi-continuous in the following sense: the set $*x \in$ X/$F(x) \cap U$ G Ø+ is open X for every open subset U of E.

In this paper we shall give the most important topological properties of uniformspace which still valid in semi-linear uniform spaces.

CHAPTER-II

TOPOLOGICAL CHARACTERIZATION OF UNIFORMABLE SPACE

THE MAIN RESULT

A natural problem is to characterize those topological spaces which are linearlyuniformizable.one of the first solution was given by who generalized the metrizability theorem of Nagta – Smirnov to higher cardinals. Proved a lot of further $x-$ matrization theorems yield

„classical" metrization theorems if specialized to $x = \omega$, even though the proofs , in mostcases,have to use different methods for uncountable x. But , in some sense, this situation is unsatisfying : As we shall indicate by examples, and as it seen by the result below, the theory of non-metrizable linearly uniformizable spaces is not just a direct generalization of metric spaces, it has many autonomous and independent aspects which do not have analogues for metrizable spaces.it is this aspects which we want to emphasize in this paper; and in this sense, our theorems characterization linear uniformizibility seem to be more adept to the whole concept. If one specializes our theorems to the countable case, one does not get general metrizabilitytheorems but one obtains necessary and sufficient conditions for a space to be strongly zerodimensional and metrizable.so, from this point of view, one could consider the theory of linearly uniformizable spaces to be a generalization of strongly zero dimensional metric spaces. but again, this view is not justified completely, too. Again there are characteristic differences between the countable and the non-countable case: in the countable case, these spaces areorderable, whereas this is not true generally in the uncountable case; Baire"s theorem holds for complete metric spaces but not for every complete ω_1-metrizable space.

As we could see above, there are interesting relations between the investigated classes and ordered spaces. We will try to find more about those relations. But at first we want to assemble several properties needed here after:

i) If X is a (strongly) sub orderable space, then ind X=0 iff Ind X=0. It is easy to prove this assertion by modifying a proof that every (sub)-orderable space is normal, or to deriveit from results.

ii) If every point of a space X has a monotone base of its neighborhoods and

$(x) = (\Delta x)$ for every non isolated $x \in$ X, then adX $\geq (\Delta$X) .

iii) If X is sub orderable and adX = $(\Delta$X), then X is paracompact.

Perhaps, the last statement needs a proof for non-discrete X: If X is not paracompact then, by ,X contains a closed homeomorphic copy of a stationary set of an uncountable regular $x$and since such a set cannot be discrete we have, $x > ad$ X.But the diagonal of no stationary set in X is an intersection of less than x is open set (every such an open set must contain a square of a „tail‟).for further information on sub orderable spaces .

Trying to characterize linearly uniformizable spaces X, one usually gets problemswith the set of the isolated points $x \in$ X.therefore, we need some special condition for „handing‟ the isolated points. In facts we shall need condition guaranteeing that there are not too many isolated points too close by the derived setX$'$ of X. It turns out that a property, which we call

„property (B)‟, is an appropriate one. the background of property(B) is in the theory of generalized metric spaces. In Hodel introduced the following concept: X is said to be a β-spaceif for every point $x \in$ X,there is a family $*\text{U}_n (x)/n \in \omega+$ of neighborhood of x suchthat ß $\text{U}_n (x) = *x^{+}$and if ß$*\text{U}_n (x)/n \in \omega^{+}$ G ϕ,then $*x_n{}^{+}$ has an accumulation point. we shall use here the following generalization:

DEFINITION: 2.1

A spaces X is said to have property B , λ a cardinal, if for every $x \in$ X, there is a family $*\text{U}_\alpha (x)/\alpha \in \lambda+$ of neighborhoods of x such that ß$*\text{U}_\alpha (x)/\alpha \in \lambda$={x}, and if ß$*\text{U}_\alpha (x_\alpha)/\alpha \in \lambda$ contains an isolated point, then $*x_\alpha{}^{+}$ has a complete accumulation point.

Clearly, anyβ-space and hence every metrizable space has B_ω; every x- uniformizable space has

B_x;every dense –in –itself space X has $B_{\varphi \text{K}}$;any λ −stratifiable space has$B$;every space X has

$B_{|\text{K}|}$, and every discrete space has$B$ for any λG0.

If X has B and & is an open base for X then we many suppose that the above

$\text{U}_\alpha (x)$ belong to &.The Michel line X is a first-countable space which does not have property

$B_\omega$as follows from corollary.

THEOREM: 2.2

For any topological space, the following condition are equivalent:

a) X is either linearly uniformizable and non-uniformizable, or X is metrizable, or X is metrizable and strongly zero dimensional.

b) X is a non-arichmedean topology space ;$\varphi(x) = \varphi(\Delta x)$for every non isolated

$x \in$ X; and X has property $_{(\Delta K)}$.

c) X is a non-arichimedean topological space;$(x) = \varphi(\Delta x)$ for every non isolatedpoint $x \in$ X; and X′is a $G_{\varphi(\Delta K)}$-set in X.

d) X is strongly sub orderable , zero dimensional; ad(X)=$\varphi(\Delta$X);and X′ is a

$_{(\Delta K)}$ −set in X .

(Remember that X′ denotes the set of non isolated points of X.)Proof:

The implication (a)⇒(b) follows from the above observations.(b)⇒ (c).

Suppose that X satisfies (b),ΔX = ß*$V_\alpha/\alpha \in \lambda$+, λ= $\varphi(\Delta$X), V_α open in X×X and that & is an non-archimedean base for X. We may also assume that the sets$U_\alpha(x)$ from property

B belongs to& and $U_\alpha(x) \times U_\alpha(x) < V_\alpha$ and$U_\alpha(x)$ = *x+ if x is isolated.

Now, ß*$\cup$*$U_\alpha(x)/x \in$ X′+/$\alpha \in \lambda$+; indeed, if the last set contains an isolated point x,say

$x \in$ ß*$U_\alpha(x_\alpha)/\alpha \in \lambda$)+, $x_\alpha \in$ X′, the {x_α+ has a complete accumulation pointy, Now take $\alpha \in$

$\beta \in \lambda$ such that x Ø$U_\alpha(y)$, $(U_\alpha(y) \times U_\alpha(y)) - V_\beta$ G Ø, (y) 3 x_β. Then $(x_\beta) \cap U_\alpha(y)$ G Ø,

hence$U_\beta(x_\beta) < U_\alpha(y)$ or $U_\beta(x_\beta)$ ➻$U_\alpha(y)$. But $U_\beta(x_\beta) \times U_\beta(x_\beta) < V_\beta$, hence x Ø$U_\beta(x_\beta)$ –

a contradiction. Thus ,(b)⇒(c).

(c) ⇒(d). Assume that X satisfies (c):i.e; let & be a non-archimedean base of X,$\varphi(\Delta X) = \lambda$ and ΔX = ß*U$_\alpha/\alpha \in \lambda^+$,U$_\alpha$ open in X×X as well as X$'$ = ß*$G_\alpha/\alpha \in \lambda^+, G_\alpha$ open in X by our assumptions. Then, by(ii),adX ≥ $\varphi(\Delta X)$, and it remains to prove that X is strongly sub or+1derable.

Since X is paracompact and Ind X=0 ,for every α∈ λ, there is an open decompostion &$_\alpha$ of X, &$_\alpha$ < &, refining the cover

({ ,- ∩ $G_\alpha/x \in G_{\alpha^+}$ ∪ *x/x Ø G_{α^+});

We may suppose that, for $\alpha \in \beta \in \lambda$, &$_\beta$ refines &$_\alpha$because ad X=λ it is almost clear that ∪{&$_\alpha/\alpha \in \lambda$+ is a base for X(if x is isolated, then {x}∈ &$_\alpha$for some α; if$x \in$ X$'$, U∈ &, x ∈U and W#U for every W∈ ∪*&$_\alpha/\alpha \in \lambda$+ with $x \in$ W, then W ➸U, henceß{U$_\alpha$,x-/$\alpha \in \lambda$+ ➸U).

If λ≤ ω,then X is metrizable, hence orderable by the result compare also where herrlich shows that a totally disconnected metric space X is orderable iff Ind X=0.

If $\lambda > \omega$, we may suppose that |*$V \in$ &$_{\alpha+1}/V$ < W+| ≥ ω for any W∈ &$_\alpha$ with |W| ≥ ω. We can order every &$_\alpha$in such a way that for $\alpha \in \beta \in \lambda$ the canonical „refinement" map &$_\beta$ → &$_\alpha$ is an order homomorphism and for W ∈ &$_\alpha$,|W| ≥ ω,the set {V∈ &$_{\alpha+1}/V$ < W+ is discretely ordered without a first and a last element. If , for x,y∈ X, we define $x < y$ whenever V<W for some V,W∈ &$_\alpha$, $\alpha \in \lambda$, with $x \in V$, $y \in$ W, then the order < induces the topology at every non isolated point of X. indeed, let$x$ ∈ X‘, W ∈ &$_\alpha$, x ∈ W, and take T,U,V∈ &$_{\alpha+1}$ in such a way that TU U ∪ V < W, $x \in U$, T is the predecessor of U in &$_{\alpha+1}$ then for $t \in T$, $r \in V$, we have x ∈-t, r,< W. Clearly, every open interval is open set in X.

To finish the proof, we must show (d)⇒(a). Let X is satisfy (d), and $(\Delta X) = \lambda$. Hence X′=ß*$G_\alpha/\alpha \in \lambda$+ and ΔX = ß*U$_\alpha/\alpha \in \lambda$+ where each $G_\alpha$and U_α are open. for each $x \in$ X′. There is open interval $V_{x,} < G_\alpha$ containing x and such that $V_{x,} \times V_{x,} < U_\alpha$; for $x \in$ (X − X$'$)we put $V_{x,\alpha}$ = *x+. If we put V_α = ∪{$V_{x,\alpha} \times V_{x,\alpha}/x \in$ X$'$} ∪ ΔX, then $V_\alpha < U_\alpha$,V_α is open in X× X and each V_α,x- is a convex set. Since ß{V_α,x-/$\alpha \in \lambda$+ = *x+ for every $x \in$ X, we see that

*V_α,-/$\alpha \in \lambda$+ is local base at$x$ (for $x \in$ X′ because all V_α,- are convex;for x − X − X′ because

V_α,- = *x+ when ever x Ø G_α.

Since X is paracompact by(iii),ß{$V_\alpha/\alpha \in x$+, $x \in \lambda$,generates a uniformity on X with a linearly ordered base.(in paracompact space, every collection of neighbourhood of ΔX is apart of a uniformity on X;we take such a smallest uniformity.)

It remains to remark that X is metrizable iff (ΔX) = ω and then Ind X=dimX=0 by(i)on p.

REMARK: 2.3

Notice that we have not used zero dimensional in the main part of (d)⇒ (a).Thus any strongly sub orderable space X with ad X=φ(ΔX) is linearly uniformizable iff X′ is a $G_{\varphi(\Delta K)}$-set.

To get different view of the situation, we indicate another proof for (d)=(a), too that for any orderable space X , we have m(X) = φ(ΔX) where m(X) is the metrizability-degree of the space X. Mutatis mutandis, the same proof works for strongly sub orderable space X where X′ is a $G_{\varphi(\Delta K)}$- set. Now the implication (d)⇒(a) follows from the fact space X is x-metizable iff

m(X) = ad X = x.

A further , equivalent, description of space which are either linearly uniformizable and non-metrizable or metrizable and strongly zero dimensional (i.e. condition(a)of our theorem above)will be stated in proposition.

COROLLARIE: 2.4

1) Every non archimedean topological space X with (x) = (ΔX) for everypoint $x \in$ X is linearly uniformizable .

2) Every non-archimedean β-space with a $G_ð$ –diagonal is metrizable.

3) Every strongly sub orderable space with $G_ð$-diagonal and such that the set of all isolated points isF_σ, is metrizable.

4) Every orderable space X with ad X=(ΔX) is linearly unifomizable .

5) Every perfect normal non-archimedean space with a $G_ð$ diagonal ismetrizable (van Douwen).

6) Every non-archimedean space X with $\varphi(x) = d$X for any non isolated

$x \in$ Xis linearly uniformizable (thus every separable non-archimedean space is metrizable.

7) The Michel-line is first countable but is not a $\beta -$ space.

PROOF:

Corollaries and follow directly from our theorem ,3 from the preceding remark, and 7follows from 2. To prove corollary 4, it suffices to show that in any orderable space X with

$(\Delta x) = ad$ X $= x$ the set X′ is a G_x-set , and then to use the preceding remark. Indeed , if

ΔX = ß*U$_\alpha/\alpha \in x^+$ for an open decreasing family *U$_{\alpha^+}$ in X × X , then iX′ = ß{∪{$V_{x,\alpha}$/∈ X′}/α ∈ x}, where $V_{x,\alpha}$ are open intervals such that $x \in V_{x,\alpha}$, $V_{x,\alpha} \times V_{x,\alpha}$ <U$_\alpha$. If x is isolated and condition and contained in the last intersection, then $x \in$ ß*$V_{x\alpha}$, $\alpha/\alpha \in x^+$ for some sequence *$x_\alpha/\alpha \in$ x+ in X′. Then we may suppose that $x_\alpha < x$ for some cofinal set in x. Now let y be the predecessor of x, then $(y, x) \in$ ß*U$_\alpha/\alpha \in x$+, which is contradiction.

Corollary , follows from the following reasoning. Let& be a non-archimedean base of X, and denote d(X) = λ . one can easily show that |&| ≤ λ (for every $x \in$ X the set&$_x$ = *$B \in$ &/$x \in B$ } is a linearly ordered set of clopen sets ,hence |&$_x$| ≤ λ and ∪*&$_x$/$x \in S$+ where S is a dense set in X).Now, for every disjoint pair B,C∈ & we denote V(B,C)=(X×X)-(B×C).there are at most λ many such open sets V(B,C)and their intersection is

ΔX.consequently ,(ΔX) < λ (and, since φX = λ) if X is not discrete by our assumption one has φ(ΔX) = λ . By d(X) = λ it follows that |X − X′| ≤ λ. Therefore ,X-X′ is an $F_{\varphi(\Delta K)}$ set inX and it follows from assertion (c) in the above theorem that X is linearly uniformizable .

REMARK: 2.4

In different ways, corollaries 3 and 4 generalized Lutzer‟s result from ,that every orderable space with $G_ð$-diagonal ,the set of isolated points is an F_σ-set .see e.g the proof of corollary4).

Corollary which can also be proved differently, answers partly the question posed in, whether every perfectly normal non-archimedean space is metrizable. This question is a Souslineline. Compare also the space described after our corollary

Corollary generalizes one of the oldest result on non-archimedean spaces proved and also by other authors, e.g., which states that every separable non archimedean space is metrizable.

since $d(\mathrm{X}) = w\mathrm{X}$ for archimedean spaces (see the proof above) and since every regular space X with wX$\leq ad$ X is linearly uniformizable ,as follows froma result in ,our corollary gives also the following.

COROLLARY:2.5

The following conditions for a space X are equivalent: a)X is regular ,IndX=0,wX$\leq ad$ X.

b) X is wX –unifromizable, and Ind X = 0 if X is metrizable.

c) X is a non-archimedean space with $(x) = d\mathrm{X}$ for every non-isolated$x \in \mathrm{X}$.

REMARK: 2.6

Condition (b)says in fact that either X is a sperable zero dimensional metrizable space (if w(X)=w)or X is w(X)-metrizable.

EXAMPLE:

It is easy to find examples of spaces which are not linearly uniformizable, but satisfy all the condition of our problem except $(x) = (\Delta\mathrm{X})$ or ad X = $(\Delta\mathrm{X})$, respectively; take for instance a disjoint sum of a x −uniformizable space and a λ-uniformizable space with $x > \lambda >$

ω. Then the resulting space is non-archimedean, and orderable provided the original spaces are dense-in -itself.

EXAMPLE:

A modified Michael-line .We shall show now that in our theorem ,the conditions concerning the isolated points are essential :to prove that a space X is linearly uniformizable we must assure that in a certain sense-there are not too many isolated points which are "too close to the subspace X′".As a first example , take the Michel-line X which is non-archimedean , strongly sub orderable and which satisfies $\varphi(x) = \varphi(\Delta X) = ad$ X for all non-isolated points x. But the Michel – line is not metrizable. Therefore ,it may serve as a counter example in the countables case. By Barie‟s theorem, X′ is not G_δ nor has X property B_ω.

The following example concepts concerns the uncountable case, too .Let Z be the ordered set of integers, x a regular infinite cardinal, Y the lexicographically power Z*and X the strongly sub orderable space with the same ordered set as Y in which all points $x \in Y$ for which

$*\alpha pr_\alpha x$ G 0+is cofinal in x are isolated .Then the space X is non-archimedean, strongly sub orderable,$(x) = \varphi(\Delta X) = ad$ X $= x$ for all non-isolated points x and X is not linearly uniformizable.

The facts that X is strongly sub orderable and $(x) = ad$ X $= x$ for non-isolated ∈ X , are clear. Since Y is x –uniformizable (the partion‟s $\{pr_\sigma{}^{-1}*x^{+}/x \in z *_{+}, \alpha \in x,$ from a base for the uniformity), one gets that Y- and hence X-is non-archimedean; moreover, we have

$(\Delta X) = (\Delta Y) = x$ .To prove that X is not linearly uniformizable, it suffices to show to show that X′ is not a G_x-set in Y.

Suppose that $X' = ß*U_\alpha/\alpha \in x+$, U_α open in Y. Since X-X′ in dense in Y, there isanx_0 ∈U_0− X′ and a neighbourhood V_0 of x_0 in Y such thatV_0 <U_0,

$V_0 = *x/pr_\alpha x = pr_\alpha x_0 for\ \alpha \in \beta_0\}$ for some β_0 ,and $pr_\alpha x_0$ G 0 for some $\alpha \in \beta_0$. Since X′, and hence U_1,is dense in Y, there is an$x_1 \in U_1 \cap V_0 -$ X′ with a neighborhood $V_1$of x_1 in Y such that

$V_1 < U_1 \cap V_{0,1} = *x/pr_\alpha x = pr_\alpha x_1\ for\ \alpha \in \beta_{1^+}$ for some β_1 , and $pr_\alpha x_1$ G 0 for some

$\alpha \in \beta_1, \geq 1$. By transfinite induction we get points $x_\alpha \in U_\alpha -$ X′ and their neighbourhoods

$V_\alpha < U_\alpha$ such that $x_\alpha \in ß\{V_\gamma/\gamma \in \alpha\}, V_\alpha = *x/pr_\gamma x = pr_\gamma x_\alpha for\ \gamma \in \beta_{\alpha^+}$ for some β_α and

$pr_\gamma x_\alpha$ G 0 for some $\gamma \in \beta_\alpha$, $\gamma \geq \alpha$.Thus there is an isolated point

$x \in ß*V_\alpha/\alpha \in x+ < ß*U_\alpha/\alpha \in x =$ X′, which is contradiction. (Here for each $\alpha \in x, pr_\gamma x = pr_\gamma x_\alpha$ for all$\gamma \in \beta_\alpha$.)

EXAMPLE:

A modified sorgenfrey-line .The next examples shows that one cannot replace

„strongly suborderable‟ by „suborderable‟ in condition.

Let Z be a the ordered set of integers,x a regular infinite cardinal, Y the lexicographically power Z* and X be the set Y with the half open intervals [a, b[as a base for the topology.one can easily show that ad=X=ad Y=x, $\varphi(\Delta X) = \varphi(\Delta Y) = x$ (hence X is represented paracompact by, Ind X=0 and X is dense -in-itself) .If $2^x < 2^x$, then X×Xis not collectionwise normal: the subspace $\{(x, -x)_+/x \in X_+$ is closed discrete in X×X of cardinally 2*,but$c(X \times X) \leq d(X \times X) = dX = x^2 2^x$. If $2^{2x} < 2^{2x}$,then X×X is not normal (use jone‟ classical method for counting continuous functions). Thus, for $x = \omega$, the space X is not metrizable. Suppose now that $x > \omega$. If X would be linearly uniformizable, then X was orderable by a dense orderd (e.g. the order obtain un the proof (c) $\Rightarrow (d)$ above is dense provided X is dense-in-it-self). On other hand , X cannot be orderable by a dense order,, because the finest order-completion of X would be connected then, which contradicts corollary.

Therefore ,we have: for every regular cardinal x there is a paracompact sub orderable 0- dimensional space X without isolated points with ad X = $\varphi(\Delta X) = x$, (= $\varphi(x)$, $for\ any\ x$), which is not linearly uniformizable (even not non-archimedean

EXAMPLE:

Our next step is to show that one cannot weaken the condition "ad X=(ΔX) " in (d) to "ad X=$\varphi(x)$, for all non-isolated $x \in$ X". Let be the reals and I be the subspace of the irrational and let X < $\times I$ be the following space ;X = $*(r, s)/either\ r \in I, or\ s = \sqrt{2}+$. Take the lexicographically order on X and the order topology r,then Michael-line M is embeddable into X. Let &1 be a non-archimedean base of $\overline{M}$ and &2 be a non-archimedean base of the irrational I. Then

$$\& = **x+ \times B\,|\,x \in I, B \in \&_2+ \cup *(B \times I) \cap X\,|\,B \in \&_1+$$

Is an non-archimedean base of (X,).Therefore , X is a non-archimedean space without isolated points; it satisfies $\varphi(x) = ad$ X $= \omega$ for every $x \in$ X,but (X,r) is not metrizable since it contains the contains the Michael-line as a base subspace. Therefore , $\varphi(\Delta X) > \omega$.

REMARK: 2.7

The construction described above can be generalized to arbitrary regualar x if one replaces the Michael-line by the space of our example and the irrational by Z* (compare example).

Therefore, for every regular cardinal x there is a paracompact zero dimensional orderable space X without isolated points, with adX=$\varphi(x) = x\ for\ every\ x \in$ X , which is not linearly uniformizable .

REMARK: 2.8

The space constructed above was in fact non-archimedean. Here is an example of a paracompact zero dimensional space which serves the same purpose as above, but is not non- archimedean: For some infinite regular cardinal x, let D be an ordered discrete space ofcardinality and cofinality x. Define X = $Z^* \times ((D \times Z^*) \cup *\infty+)$, where all the products and powers are lexicographical, and ∞ is a „last point" added to D× Z^*.then ad X = $\varphi(x) = x$ for every $x \in$ X, but X is not linearly uniformizable, because its subspace $Z^* \times *\infty+$ is homeomorphic with the space from our example.

EXAMPLE:

We will show now that we cannot weaken the condition "ad X = $\varphi(\Delta X)$" in(d) to "$\varphi(x) = \varphi(\Delta X) for\ all\ x$", and at the same time ,that we cannot weaken the non-archimedean property in(b),(c) to „hereditarily paracompact 0-dimensional", even if the space in question is dense –in-itself.

The example is a modification of the last of the example: X=$((D \times Z^*) \cup *\infty+) + Æ^*$, where again the product, the powers and the sum are lexicographical, and ∞ is a „last point" added to $D \times Æ^*$. Then X is hereditarily paracompact,$(x) = x$ for all x, X cannot be linearly uniformizable for$x > \omega$ because ad X = ω (coinitiality of Æ* is ω). It remains to show that (ΔX) = x: we know that both spaces (D × Æ*) *∞+, Æ* are x-uniformizable , hence, their diagonals are intersection of monotone open families *U$_\alpha/\alpha \in x+$ and*$V_\alpha/\alpha \in x+$,respectively. If *$y_\alpha/\alpha \in x+$ is cofinal in D ×

Z^*, *$x_n/n \in \omega+$ coinitial in Æ*,and the map $\varphi: x \to \omega$ given by the formula α = £. ω + $f\alpha$, then

ß{W$_\alpha/\alpha$ ∈ x+ = ΔX, where

$$W_a = U_a U\ V_a\ U\ (\ -y_\alpha, x_{\varphi\alpha}, x\text{-}y_\alpha, x_{\varphi\alpha},).$$

Our result is: For every uncountable regular x there is a hereditarily paracompact 0- dimensinal orderable space without isolated points, with(x) = φ(ΔX) = x for all x ∈ X, which is not linearly unifomizable (even not non-archimedean).

For $x = \omega$ such a space cannot exist because of Lutzer‟s result that orderable spaces with aG_δ –diagonal are metrizable. If in this case, we replace „orderability‟ by „suborderability‟, then the sorgenfrey-line as a analogue of the example above.

REMARK: 2.9

The space X constructed above has metrizability degree (x) = x, but X is not

x −metrizable.

EXAMPLE:

If we look at the last three examples, we can see that the constructed spaces aredense-in-itself. For such space, our theorem has the following form:

COROLLARY: 2.10

If X is dense-in-itself then the following assertionare equivalent:

1) X is linearly uniformizable and non-metrizable or metizable and dim X = 0.

2) X is non-archimedean and (x) = (ΔX)$for\ every\ x \in$ X.

3) X is orderable, 0-dimensional and ad X = (ΔX).

REMARKS: 2.11

In the preceding examples we have shown that the conditions in (2) and (3) cannot be weakened; e.g, the non-archimedean property in (2) to orderability, Here, orderability of X would be weaker than the no-archimedean property, since if $\varphi(\Delta X) = \omega$, X is orderable sinceit is metrizable and Ind X=0, and if $\varphi(\Delta X) > \omega$, orderability of X follows from(theorem).

Further, we showed that orderability in(3) cannot be weakened to suborderability. we also showed that the condition "ad X=$\varphi(x)$" in(3) cannot be replaced by "ad X = $\varphi(x)$ for every non-isolated $x \in$ X " and moreover (in example) that the condition "$\varphi(x) = \varphi(\Delta X)$ " in(2) cannot be weakened to"$\varphi(x) = ad$ X ".

Let S be a souslin-tree (which exists if a souslin-line exists; e.g. Moreover, we can assume that every element $s \in S$ has infinitely many immediate successors.

Now let X be the space described as follows: points $x \in$ X are maximal chains of S; a basic neighborhood of x will exists of those maximal chains of $y \in$ X which coincide at a fixed element α of x (and hence at all elements $\beta \leq \alpha$). Then it follows from the construction that X is a non-archimedean topological space, X is first countable and dense-in-itself. Moreover, because of our assumption concerning the souslin-tree S, X is linearly orderable (which follows analogously as in the proof (c) ⇒(d) of our theorem above. More or less, this is a standard construction. On the other side, X cannot be metrizable, since $c(X) = \omega$ but obviously

$d(X) > \omega$. Hence (ΔX) > ω.Further, X is perfectly normal because X is orderable, dense-in- itself and$(x) = \omega$.

Using x^+ souslin-trees, the situation can be generalized to higher cardinals.

APPLICATION, PROBLEMS AND FURTHER RESULTS:

At the end of our paper we want to show several, results where the preceeding theorems are used as tools.

Recall that a Hausdorff-space (X, r)is linearly stratifiable sapces if there is a cardinal λ anda map S: $(\lambda \times r) \to r$ having the following properties:

i) $\in \beta \in \lambda, \quad \in r \Rightarrow (\alpha, U) < S(\beta, U) < \overline{\beta U}$ <U.

ii) $U \in r \Rightarrow U$ =∪ $*(\alpha, U)/\alpha \in \lambda+$.

iii)$, V \in r, U < V, \alpha \in \lambda \Rightarrow (\alpha, U) < (\alpha, V)$.

A linearly stratifiable space is called λ-straitifable, if λ is the smallest cardinal which canbe used in the above definition. Every linearly stratifiable space is hereditarily paracompact and every linearly uniformizable space is linearly stratifiable. Indeed, if * $_{\alpha}/\alpha \in \lambda+$ is linearly

ordered base of a (covering) uniformity on X, and we put $S(\alpha, U) = U\backslash \overline{st(X)}$ we obtain a

λ-stratification of X. Every λ-stratifiable space has property β .Therefore, the following proposition show how can weaken the other condition in our theorem if we replace property ( )by λ-stratifiability. That a sub orderable space X is linearly uniformizable if X is linearly stratifiable.

Therefore, by combining this with our results, we obtain:

PROPOSITION: 2.12

If X is linearly stratifiable and non-metrizable, then the following are equivalent:

1) X is strongly sub orderable
2) X is sub orderable.

And consequently, we get.

PROPOSITION:2.13

For any space X, the following are equivalent:

1) X is either linearly uniformizable and non-metrizable, or X is metrizableand $dim X = 0$.
2) X is zerodimensional, suborderable and linearly stratifiable.

Another application concerns metrizability of orderable spaces. We state the next threeproposition for orderable spaces, which may be more convenient for the reader; but it follows from our theorem that all these proposition are valid for strongly suborderable space X whose set of isolated points is anF_x –set where $x = ad$ X.

PROPOSITION: 2.14

An orderable space X is metrizable iff it is a k-space and (ΔX) ≤ adX.

PROOF:

Suppose that an orderable space X is a nondiscrete k-space and $\omega_1 \leq$ (ΔX) ≤ adX. Then every compact subspace of X is finite (since it is ω_1-additive),thus X is discrete a contradiction. Hence (ΔX) = ω and everything follows from Lutzer‟s result that any orderabele space with a G_δ-diagonal is metrizable.

PROPOSITION: 2.15

If X is an orderable space with cX = ω, (ΔX) ≤ ad X, then X is metrizable.

PROOF:

If X is nonmetrizable, then (ΔX) > ω, and X has a strongly decreasing chain of clopen sets length ω_1,so that cX ≥ ω_1 ;a contradiction.

REMARK: 2.16

In proposition , the condition "cX = ω" is essential, as the following example shows.

If X is a one- point Lindel$\ddot{o}$fication of a discrete space of cardinality ω_1,then X is orderable. Nonmetrizable, = ω_1, (ΔX) = ad X = ω_1.

REMARK: 2.17

Examples of such a space must be scattered, at least under $\neg CH$, as follows from the following proposition:

PROPOSITION: 2.18

If X is orderable, not scattered ,X $< 2^{\omega}$, and $\varphi(\Delta X) \leq ad$ X, then X is metrizable.

PROOF:

Take a subspace Y of X without isolated pints. Then since X is orderable

$cY < 2^{\omega}$, $(\Delta X) \leq ad\ Y$, and we can construct a cantor tree of length $x = ad\ Y$ composed of open subsets of Y such that each α-level of this tree would consist of 2^{ω}open sets. Hence

$(\Delta X) \leq ad$ X $\leq ad\ Y = \omega$, and X must be metrizable.

REMARK:2.19

Since the same argument holds for every α-level of the tree, we in fact have shown that for dense-in-itself spaces X, one has cX $\geq 2^{2d\ K}$.

On the other hand X=$Z^{2\omega}$,orderable lexicographically, satisfies all the conditions of proposition 4 except "cX $< 2^{\omega}$",and X is not metrizable.

The Michael-line X as, a typical example of a non-archimedean toplogical space, originates from the space of the result turning some points (the irrational)into isolated points and leaving the neighborhoods of all other points as they are. "X is a discretization of the space

by the subset \ ".

It was shown that every non-archimedean topological space is a discretization of a certain space Y which can be gotten from X as an inverse limit of some quotient space of X[6].By the examples above, we know that a non-archimedean topological space X with $\varphi(x) = \varphi(\Delta X)$ for every non-isolated point $x \in$ X need not be linearly uniformizable. In fact, to obtain linear uniformizability we need some further condition assuring that, there are not „not many " isolated points which are „too close" to the subspace X′;(property $B_{\varphi(\Delta K)}$of condition (b)in our theorem

above can serve as a typical example).

Therefore, it may not be surprising that we can prove the following theorem:

PROPOSITION:2.20

A non-archimedean topological space X is homeomorphic with a discretization of a linearly uniformizable space Y if and only if $(x) = (\Delta X)$ for every non-isolated point $\in$ X .

COROLLARY: 2.21

Any non-archimedean space X with a G_δ-diagonal is homeomorphic with a discretization of a metric space Y.

PROOF:

We have to show that only sufficient of the condition. To show that, we adapt thenotation of the proof of (b)⇒ (c) of our main theorem. For every point $x \in$ X and for every $\alpha \in$

λ, choose a basis set $B_{x,} \in$ & such that $(B_{x,} \times B_{x,}) < V_\alpha$, and consider the covering

= *$B_{x,}/x \in$ X+.Then, for every $x \in$ X, ß*$st(x, \quad _\alpha)/\alpha \in \lambda$ = *x+,since ßV_α = ΔX.

Since every non-archimedean space is paracompact and strongly zero-dimensional, we can choose partition &$_\alpha$ < &of X refining u_α for each $\alpha \in \lambda$. Moreover, we may suppose thatthe covering &$_\alpha$, $\alpha \in \lambda$,are linearly ordered by refinement, becaused ad X $\geq \varphi(\Delta X)$; see (ii) on p.

Therefore ,*&$_\alpha/\alpha \in \lambda$+ defines a uniformity G on the underlying set of X with a linearly ordered base such that for every non-isolated point $x \in$ X, *$st(x$, &$_\alpha)/\alpha \in \lambda$+ is local base of the topology of X.

CHAPTER-VAPPLICATION

Topology being somehow very recent in nature but has got tremendous applications over almost all other fields. Theoretical or fundamental topology is a bit dry but the application part is what drives crazy once we get used. In this paper we discuss some application in various fields of science and Technology, like application to Biology, robotics, GIS, Engineering

,computer science. Topology though being a part of mathematics but it has influenced the whole with so strong effects and incredible applications.

CONCLUSION

The topological space linearly uniformizable if there if there is uniformity generating The topology of having a linearly orderd base. Obviously any metrizable topological. Space is linearly uniformizable

By a result it is a metrizable non-archimedean topological spaces every non- archimedean topological space is para compact hence here ditarly para compact the most important topological properties of uniform space which still valid in semi-linear uniform space.

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