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CHROMATIC AND INDEPENDENCE NUMBER OF FUZZY GRAPHS

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Abstract:

The minimum number of colors required to color all the vertices such that adjacent vertices do not receive the same color is the **chromatic number** $\boldsymbol{\chi(G)}$. The maximum cardinality among all maximal fuzzy independent set is called **fuzzy independence number** and it is denoted by $\beta^{f}(G)$.

Keywords: **Fuzzy graph, independence set, Chromatic number, independence number, relation between independence and chromatic number , fuzzy tree**

I. Introduction:

Zadeh introduced the fuzzy set as a class of object with a continuum of grades of membership. Fuzzy relation on a set was first defined by Zadeh in 1965, the first definition of a Fuzzy graph was introduced by Kaufmann in1973and the structure of Fuzzy graphs developed by Azriel Rosenfeld in 1975.

Colouring of graphs is a most important concept in which we partition the vertex or edge set of any associated graph so that adjacent vertices or edges belongs to different sets of the partition. The colouring problem consists of determining the chromatic number of a graph and an associated colouring function.

Let G be a simple graph with n vertices. A colouring of the vertices of G is a mapping f: V (G)→N, such that adjacent vertices are assigned different colours. In 1965, Behzad and Vizing have posed independently a new concept of a graph colouring called total colouring. A k-colouring of graph G is an assignment of k colours to the vertices and edges in such a way that adjacent vertices and incident edges are received different colours.

In this chapter we determine the relation between the chromatic number and independence number of fuzzy graphs. The proper coloring of a graph is the coloring of the vertices with minimal number of colors such that no two adjacent vertices have the same color. The minimum number of colors required to color all the vertices such that adjacent vertices do not receive the same color is the **chromatic number** $\boldsymbol{\chi(G)}$. The maximum cardinality among all maximal fuzzy independent set is called **fuzzy independence number** and it is denoted by $\beta^f(G)$.

II. Preliminary Definitions

Definition 1:

A fuzzy set A defined on a non empty X is the family A = {(x,(x))/$x\epsilon X$} where μ_A: X →[0, 1]

Definition 2:

Let V be a finite non empty set. The triple $\tilde{G}$= (V,σ, μ) is called a fuzzy graph on V where σ and μ are fuzzy sets on V and E respectively, such that $\mu(uv) \leq (u) \wedge (v)$ for all u,v ϵ V and uvϵ E. For fuzzy graph $\tilde{G}$= (v,σ, μ), the elements V and E are called set of vertices and set of edges of G respectively.

Definition 3:

Two vertices u and v in $\tilde{G}$ are called adjacent if (1/2) $[\sigma(u) \wedge \sigma(v)] \leq \mu(uv)$.

Definition 4:

The degree of vertex v in $\tilde{G}$, denoted by deg(v) is the number of adjacent vertices to v and the maximum degree of $\tilde{G}$ is defined by $\Delta(\tilde{G})$ =max$\{deg\tilde{G}$(v)/vϵ $V\}$.

Definition 5:

Two edges $v_i v_j$ and $v_i v_k$ are said to be incident if $2\{\mu(v_i,v_j) \wedge \mu(v_i v_k)\} \leq \sigma(v_i)$ for i = 1,2,...$|v|$ and $1 \leq j, k \leq |v|$.

Definition 6:

A family $\Gamma = \{ \gamma_1, \gamma_2, \dots \gamma_k\}$ of fuzzy sets on V is called a k- fuzzy coloring if $\tilde{G}$ = (v,σ,μ) if

(a) $V\Gamma = \sigma$

(b) $\gamma_i \wedge \gamma_j = 0$

(c) For every strong edge uv of $\tilde{G}$, γ_i(u) $\wedge$ γ_i(v) = 0 for $1 \leq i \leq k$.

The above definition of k- fuzzy coloring was defined by the authors Eslahchi and Onagh on fuzzy set of vertices. This has been extended to both fuzzy set of vertices and fuzzy set of edges by Lavanya.S and Sattanathan .R as k- fuzzy total coloring as follows.

Definition 8:

A family $\Gamma = \{\gamma_1, \gamma_2, \dots \gamma_k\}$ of fuzzy sets on V∪E is called a k- fuzzy total colouring of $\tilde{G}$ = (v,σ, μ) if

a) $\max_i$ {γ_i(v)} = $\sigma(v)$ for all vϵV and maxi { γ_i(uv)} = $\mu(uv)$ for all edges uv ϵ E.

b) $\gamma_i \wedge \gamma_j$ j = 0

c) for every adjacent vertices u,v of min $\{\gamma_i(u), \gamma_i(v)\} = 0$ and for every incident edges min $\{\gamma_i(v_jv_k) / v_jv_k$ are set of incident edges from the vertex $v_j\} = 0$, j = 1,2,...$|v|$.

Definition 9:

The chromatic number of fuzzy graph G: (V, σ, μ) is defined as $\chi(G) = \max\{(\chi_\alpha / \alpha \epsilon L)\}$ *where* $\chi_\alpha = \chi(G_\alpha)$.

Definition 8:

If H is an induced subgraph of G, two points are adjacent in H iff they are adjacent in G.

Definition 9:

An induced subgraph of a graph which is complete is called a clique, the number of vertices in clique is called clique number and it is denoted by w (G).

Theorem 1.1:

For a fuzzy tree the chromatic number is less than or equal to independence number

(i.e)$\chi^f(G) \leq \beta^f(G)$.

Proof:

Let us assume that the given graph satisfies the condition $\mu(x,y) < CONN_f(x,y)$

Therefore the given graph is a fuzzy tree

We know that the chromatic number of a fuzzy tree is two

Now we have to find the independence number

Case (i)

If n = 3

Let $\{u_1, u_2, u_3\}$ be the vertices of the given fuzzy tree

Let us assume that there is no arc between u_1 and u_3

Let S be the independent set of G

Therefore the vertices u_1 and u_3 are independent

The maximal independent set is two

Therefore the maximum of maximal independent set is two which is the independence number

For this case, $\chi^f(G) = \beta^f(G)$.

Case(ii)

If n > 3

Let $\{u_1,u_2,\ldots u_n\}$ be the vertices of fuzzy tree

Let us assume that $u_1,u_2,\ldots u_k$ be the independent set S of G, k> 2

We know that the chromatic number of a fuzzy tree is two

The maximal independent set is k

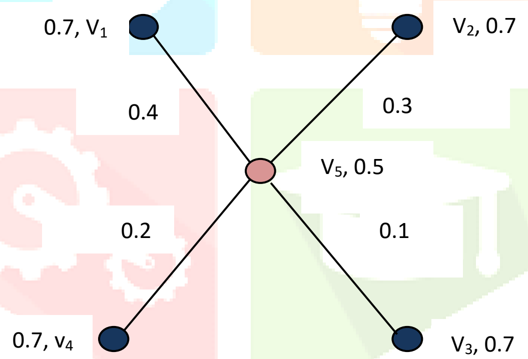
Therefore the independence number is k which is greater than two

Therefore $\chi^f(G) < \beta^f(G)$.

From this two cases we conclude that $\chi^f(G) \le \beta^f(G)$.

Example 1.2:

For a fuzzy tree T_5



Let $\Gamma = \{\gamma_1, \gamma_2\}$ be a family of fuzzy sets defined on V∪E as follows

For set of vertices

V	v_1	v_2	v_3	v_4	v_5	Max
γ_1	0.7	0.7	0.7	0.7	0	0.7
γ_2	0	0	0	0	0.5	0.5

The fuzzy coloring is

$$\Gamma = \{\gamma_1, \gamma_2\}$$

$$\gamma_1(v_i) = \begin{cases} 0.7 & i = 1,2,3,4 \\ 0 & otherwise \end{cases}$$

$$\gamma_2(v_i) = \begin{cases} 0.5 & i = 5 \\ 0 & otherwise \end{cases}$$

$\therefore \chi^f(G) = \max\{(\chi_\alpha / \alpha \epsilon L)\}\ where\ \chi_\alpha = \chi(G_\alpha).$

$\Rightarrow \chi^f(T_5) = 2.$

Independence Number:

In the above example there is no arc between $\{v_1,v_2\}$ $\{v_2,v_3\}$ $\{v_3,v_4\}$ $\{v_1,v_3\}$

Therefore the maximal independent set is $\{v_1, v_2, v_3, v_4\}$

Therefore the independence number $\beta^f(T_5) = 4$

$\Rightarrow \chi^f(T_5) < \beta^f(T_5)$

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