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LINEAR ALGEBRA IN VARIOUS DISCIPLINES

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Abstract:

Linear Algebra is one of the streams of Mathematics which targets on solving the system of linear equations. Solving system of linear equations involve plenty of applications in various fields. Matrix notations and the way of finding solutions to linear equations made complicated findings easy in daily life as well as in other fields like Physics, Chemistry, Computers. In the present paper, we explained the relevant applications and calculations with appropriate examples which illustrate the power of the linear algebra tools.

Key Words:

Linear Algebra, Linear equations, Applications, Daily life, Physics, Chemistry, Computers, Cryptology

Linear Algebra in Daily life

It is always easy and convenient to explain some mathematical terms with the appropriate examples which are having applications in our day to day life. So, let us discuss such an example here.

Example:

Three men represented by M_1, M_2 & M_3 wish to buy some goods like Bread, Buns, Cakes and Rolls. They need these items in different quantities and to purchase these items, they have two places of choice say shops D_1 & D_2. Now the question is that Which shop is the best for M_1, M_2 & M_3 to pay as small amount as possible?

The Demand quantity of Food stuff and the cost of each item in shops D_1 and D_2 are given in the following table.

Demand Quantity of Food stuff

	Bread	Bun	cakes	Roll
M_1	6	5	3	1
M_2	3	6	2	2
M_3	3	4	3	1

Price in Shops D_1 and D_2

	D_1	D_2
Bread	1.50	1.00
Bun	2.00	2.50
Cakes	5.00	4.50
Roll	16.00	17.00

To Answer the given question, we have to follow the following procedure where for every person we have to calculate the amounts individually.

The amount that has to be paid by the person M_1 in the shop D_1 is:

$$(6\,X\,1.5)+(5\,X\,2)+(3\,X\,5)+(1X\,16)=50$$

and in the shop D_2 :

$$(6\,X\,1)+(5\,X\,2.5)+(3\,X\,4.5)+(1X\,17)=49$$

The amount that has to be paid by the person M_2 in the shop D_1 is:

$$(3X\,1.5)+(6X\,2)+(2\,X\,5)+(2\,X\,16)=58.50$$

and in the shop D_2 :

$$(3\,X\,1)+(6\,X\,2.5)+(2\,X\,4.5)+(2X\,17)=61$$

The amount that has to be paid by the person M_3 in the shop D_1 is:

$$(6\,X\,1.5)+(5\,X\,2)+(3\,X\,5)+(1X\,16)=43.50$$

and in the shop D_2 :

$$(6\,X\,1)+(5\,X\,2.5)+(3\,X\,4.5)+(1X\,17)=43.50$$

We can represent these calculations as a product of two matrices and the we can get the required result in short span of time directly with in one step.

$$\mathbf{P} = \begin{bmatrix} 6 & 5 & 3 & 1 \\ 3 & 6 & 2 & 2 \\ 3 & 4 & 3 & 1 \end{bmatrix} \text{ (Demand Matrix) and}$$

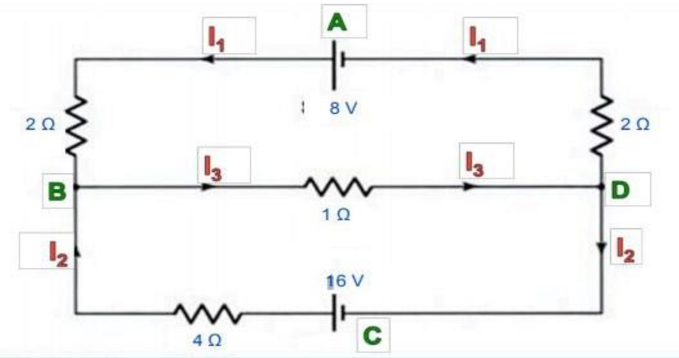
$$\mathbf{Q} = \begin{bmatrix} 1.50 & 1 \\ 2 & 2.50 \\ 5 & 4.50 \\ 16 & 17 \end{bmatrix} \text{ (Price Matrix).}$$

$$\mathbf{R} = \mathbf{PQ} = \begin{bmatrix} 50 & 49 \\ 58.50 & 61 \\ 43.50 & 43.50 \end{bmatrix}$$

Here the Resultant matrix 'R' Expresses that person M_1 must pay Rs.50/- in shop D_1 and Rs. 49/- in the shop D_2. Person M_2 has to pay Rs.58.50/- in the shop D_1 and Rs. 61/- in shop D_2. Person M_3 has to pay Rs. 43.50/- in both shops D_1 and D_2. So, it is better for the person M_1 to purchase items in the shop D_2, for the person M_2 in shop D_1 and the person M_3 will pay the same price in D_1 and D_2.

Linear Algebra in Physics

Systems of linear equations play essential role in determining the currents that flow through various branches of electrical networks. To explain this application, now we will consider one electrical circuit as follows.



Now our task is to find out the amounts of currents I_1, I_2 and I_3 that flow through the electrical circuit. To fulfill the task, we utilize Ohm's Law and Kirchhoff's Law which are very familiar in Physics. By applying these Ohm's and Kirchhoff's Law, we can be able to construct a system of linear equations through which we can evaluate the amounts currents I_1, I_2 and I_3 in the above circuit. By the observation of Kirchhoff's Law, in the above circuit, we have two junctions present in the circuit i.e., the points B and D. Also we have two complete closed paths ABDA and CBDC. Applying Kirchhoff's Law to the junctions and paths results in:

Junctions

$$B: I_1 + I_2 = I_3 \quad \& \quad D: I_3 = I_1 + I_2$$

These two equations result in a single linear equation

$$I_1 + I_2 - I_3 = 0$$

Paths

$$ABDA: 2I_1 + 1I_3 + 2I_1 = 8 \quad \& \quad CBDC: 4I_2 + 1I_3 = 16$$

We can write this system of three linear equations with three unknowns as follows

$$I_1 + I_2 - I_3 = 0$$
$$4I_1 + 1I_3 = 8$$
$$4I_2 + 1I_3 = 16$$

Now we solve this system of equations by using "Gauss – Jordan Method" to find the required current components. For that first we have to write the Augmented matrix and then we will apply the row reduction algorithms.

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 0 & 1 & 8 \\ 0 & 4 & 1 & 16 \end{bmatrix}_{R_2 \to R_2 - 4R_1}$$

$$\approx \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & -4 & 5 & 8 \\ 0 & 4 & 1 & 16 \end{bmatrix}_{R_2 \to \frac{R_2}{-4}}$$

$$\approx \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & \frac{-5}{4} & -2 \\ 0 & 4 & 1 & 16 \end{bmatrix}_{\substack{R_1 \to R_1 - R_2 \\ R_3 \to R_3 - 4R_2}}$$

$$\approx \begin{bmatrix} 1 & 0 & \frac{1}{4} & 2 \\ 0 & 1 & \frac{-5}{4} & -2 \\ 0 & 0 & 6 & 24 \end{bmatrix}_{R_3 \to \frac{R_3}{6}}$$

$$\approx \begin{bmatrix} 1 & 0 & \frac{1}{4} & 2 \\ 0 & 1 & \frac{-5}{4} & -2 \\ 0 & 0 & 1 & 4 \end{bmatrix}_{\substack{R_1 \to R_1 - \frac{1}{4}R_3 \\ R_2 \to R_2 + \frac{5}{4}R_3}}$$

$$\approx \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

From the above equivalent row reduced augmented matrix, the currents I_1, I_2, I_3 are as follows:

I_1 = 1 Amp

I_2 = 3 Amps

I_3 = 4 Amps

Linear Algebra in Chemistry

In chemistry, balancing chemical equation is quite complex. To carry out the chemical reaction, balanced chemical equation i.e., correct ratios of reagents and products are very essential. By using one of the linear algebras technique 'reduced row echelon form' the task of balancing the chemical equation became easier. Now let us discuss this application with a small example.

Let us consider one simple chemical reaction as follows

$$CH_4 + O_2 \longrightarrow CO_2 + H_2O$$

Now our aim is to evaluate the values of the coefficients in front of each compound. So, let us assign a letter value of N_1, N_2, N_3, N_4 in place of coefficient as follows.

$\mathbf{N_1}$ CH4 + $\mathbf{N_2}$ O2 → $\mathbf{N_3}$ CO2 + $\mathbf{N_4}$ H2O

From the above equation, we can notice that there are only three elements; Carbon, Hydrogen and Oxygen, that make up the compounds which will carry the chemical reaction. Thus, the system of equations formed for three elements are as follows:

Carbon: $N_1 = N_3$

Hydrogen: $4N_1 = 2N_4$

Oxygen: $2N_2 = 2N_3 + N_4$

This system of equations can be rewritten in standard form.

$N_1 - N_3 = 0$

$4N_1 - 2N_4 = 0$

$2N_2 - 2N_3 - N_4 = 0$

Now we will solve this system of equations by using Gauss – Jordan Method as follows.

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 4 & 0 & 0 & -2 \\ 0 & 2 & -2 & -1 \end{bmatrix}_{R_2 \leftrightarrow R_3}$$

$$\approx \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 \\ 4 & 0 & 0 & -2 \end{bmatrix}_{R_3 \to R_3 - 4R_1}$$

$$\approx \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 \\ 0 & 0 & 4 & -2 \end{bmatrix}_{R_2 \to \frac{R_2}{2}}$$

$$\approx \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & \frac{-1}{2} \\ 0 & 0 & 4 & -2 \end{bmatrix}_{R_3 \to \frac{R_3}{4}}$$

$$\approx \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & -1/2 \\ 0 & 0 & 1 & -1/2 \end{bmatrix} \begin{matrix} R_1 \to R_1 + R_3 \\ R_2 \to R_2 + R_3 \end{matrix}$$

$$\approx \begin{bmatrix} 1 & 0 & 0 & -1/2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1/2 \end{bmatrix}_{C_4 \to c_4 (-1)}$$

$$\approx \begin{bmatrix} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1/2 \end{bmatrix}$$

Here, we got the matrix of reduced row echelon form. Here, by observation, we can say that N_4 is a free variable and thus we can write the solutions as follows:

$$N_1 = \frac{1}{2} N_4$$

$$N_2 = N_4$$

$$N_3 = \frac{1}{2} N_4$$

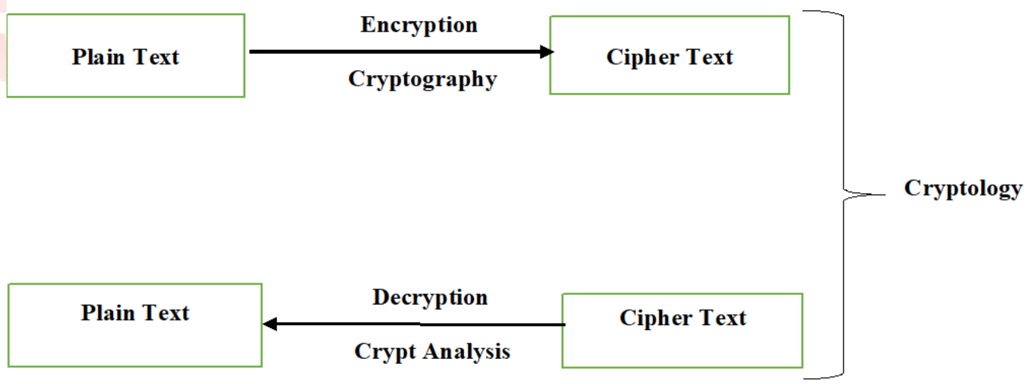
Let N_4 = 2. This is because generally we prefer the full integers as coefficients. So, here in order to get the required coefficients, we have to choose the least common denominator between the equations i.e., 2. Then we get the values of N_1, N_2, N_3 and N_4 values as follows.

$$N_1 = 1; \ N_2 = 2; \ N_3 = 1; \ N_4 = 2$$

Thus the required balanced chemical equation is **$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$**

Linear Algebra in Computers

The applicability of linear algebra in computer science is known as "Numerical linear algebra" which we also termed as "Applied linear algebra". Here we will discuss the applicability of linear algebra in Cryptology which is the science of disguising messages so that only the intended recipient can decipher the received message. We can understand the process of Cryptology with the following diagram.



In the above process we use a matrix called **"Key Matrix"** through which a plaintext message is encrypted or cipher text message will be decrypted.

Linear Algebra - cryptographic Techniques

To explain the application of Linear Algebra, first we need to assign numerical values 0 to 25 to all 26 alphabets from A to Z as follows.

Table: I

A	B	C	D	E	F	G	H	I	J	K	L	M
0	1	2	3	4	5	6	7	8	9	10	11	12
N	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**	**W**	**X**	**Y**	**Z**
13	14	15	16	17	18	19	20	21	22	23	24	25

The Encryption process:

The encryption algorithm of this method is: C =AP mod N.

Where 'C' is the column vector of the numerical values of cipher text

'P' is the column vector of the numerical values of plaintext

'A' is an (n × n) matrix, which is the key of the algorithm for which modular inverse should exist

'N' is the number of letters of the alphabet used in the cryptography.

The Decryption process:

The decryption algorithm of this method is: P = A^{-1} C mod N.

Here A^{-1} is the modular inverse of the matrix A for mod 26.

Process of choosing key Matrix and finding modular inverse of Key matrix:

Here for encryption, use the Key Matrix $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

Here, we use this matrix because, modular inverse of matrix A for mod 26 exists.

<u>To determine the modular inverse of matrix A for mod 26</u>

- $\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} = 8 - 3 = 5$
- $Adj\ A = Adj \begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ -3 & 2 \end{vmatrix}$
- 5^{-1} mod 26 = 21
- $21Adj\ A = 21 \begin{vmatrix} 4 & -1 \\ -3 & 2 \end{vmatrix} = \begin{vmatrix} 84 & -21 \\ -63 & 42 \end{vmatrix}$
- $\begin{bmatrix} 84 & -21 \\ -63 & 42 \end{bmatrix} mod\ 26 = \begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}$

Therefore, the required matrix for decryption is $\begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}$

Now we explain the process of Cryptography and Crypt analysis with the following example.

Cryptography (Process of Encryption):

The Message to be encrypted is: FAITH AND PRAYER BOTH ARE INVISIBLE

Step I:

Divide the phrase into 2X1 vectors and assign numerical values to each alphabet from table I

$$\begin{bmatrix} F \\ A \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad \begin{bmatrix} I \\ T \end{bmatrix} = \begin{bmatrix} 8 \\ 19 \end{bmatrix} \quad \begin{bmatrix} H \\ A \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix} \quad \begin{bmatrix} N \\ D \end{bmatrix} = \begin{bmatrix} 13 \\ 3 \end{bmatrix} \quad \begin{bmatrix} P \\ R \end{bmatrix} = \begin{bmatrix} 15 \\ 17 \end{bmatrix}$$

$$\begin{bmatrix} A \\ Y \end{bmatrix} = \begin{bmatrix} 0 \\ 24 \end{bmatrix} \quad \begin{bmatrix} E \\ R \end{bmatrix} = \begin{bmatrix} 4 \\ 17 \end{bmatrix} \quad \begin{bmatrix} B \\ O \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \end{bmatrix} \quad \begin{bmatrix} T \\ H \end{bmatrix} = \begin{bmatrix} 19 \\ 7 \end{bmatrix} \quad \begin{bmatrix} A \\ R \end{bmatrix} = \begin{bmatrix} 0 \\ 17 \end{bmatrix}$$

$$\begin{bmatrix} E \\ I \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix} \quad \begin{bmatrix} N \\ V \end{bmatrix} = \begin{bmatrix} 13 \\ 21 \end{bmatrix} \quad \begin{bmatrix} I \\ S \end{bmatrix} = \begin{bmatrix} 8 \\ 18 \end{bmatrix} \quad \begin{bmatrix} I \\ B \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix} \quad \begin{bmatrix} L \\ E \end{bmatrix} = \begin{bmatrix} 11 \\ 4 \end{bmatrix}$$

Step II:

Multiply the above resultant 2 X 1 matrices with the encryption matrix $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 8 \\ 19 \end{bmatrix} = \begin{bmatrix} 35 \\ 100 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 7 \\ 0 \end{bmatrix} = \begin{bmatrix} 14 \\ 21 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 13 \\ 3 \end{bmatrix} = \begin{bmatrix} 29 \\ 51 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 15 \\ 17 \end{bmatrix} = \begin{bmatrix} 47 \\ 113 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 24 \end{bmatrix} = \begin{bmatrix} 24 \\ 96 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 4 \\ 17 \end{bmatrix} = \begin{bmatrix} 25 \\ 80 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 14 \end{bmatrix} = \begin{bmatrix} 16 \\ 59 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 19 \\ 7 \end{bmatrix} = \begin{bmatrix} 45 \\ 85 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 17 \end{bmatrix} = \begin{bmatrix} 17 \\ 68 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 16 \\ 44 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 13 \\ 21 \end{bmatrix} = \begin{bmatrix} 47 \\ 123 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 8 \\ 18 \end{bmatrix} = \begin{bmatrix} 34 \\ 96 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \end{bmatrix} = \begin{bmatrix} 17 \\ 28 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 11 \\ 4 \end{bmatrix} = \begin{bmatrix} 26 \\ 49 \end{bmatrix}$$

Step III:

Transform all the above resultant matrices to Mod 26

$$\begin{bmatrix}10\\15\end{bmatrix}\bmod 26=\begin{bmatrix}10\\15\end{bmatrix}\quad \begin{bmatrix}35\\100\end{bmatrix}\bmod 26=\begin{bmatrix}9\\22\end{bmatrix}\quad \begin{bmatrix}14\\21\end{bmatrix}\bmod 26=\begin{bmatrix}14\\21\end{bmatrix}$$

$$\begin{bmatrix}29\\51\end{bmatrix}\bmod 26=\begin{bmatrix}3\\25\end{bmatrix}\quad \begin{bmatrix}47\\113\end{bmatrix}\bmod 26=\begin{bmatrix}21\\9\end{bmatrix}\quad \begin{bmatrix}24\\96\end{bmatrix}\bmod 26=\begin{bmatrix}24\\18\end{bmatrix}$$

$$\begin{bmatrix}25\\80\end{bmatrix}\bmod 26=\begin{bmatrix}25\\2\end{bmatrix}\quad \begin{bmatrix}16\\59\end{bmatrix}\bmod 26=\begin{bmatrix}16\\7\end{bmatrix}\quad \begin{bmatrix}45\\85\end{bmatrix}\bmod 26=\begin{bmatrix}19\\7\end{bmatrix}$$

$$\begin{bmatrix}17\\68\end{bmatrix}\bmod 26=\begin{bmatrix}17\\16\end{bmatrix}\quad \begin{bmatrix}16\\44\end{bmatrix}\bmod 26=\begin{bmatrix}16\\18\end{bmatrix}\quad \begin{bmatrix}47\\123\end{bmatrix}\bmod 26=\begin{bmatrix}21\\19\end{bmatrix}$$

$$\begin{bmatrix}34\\96\end{bmatrix}\bmod 26=\begin{bmatrix}8\\18\end{bmatrix}\quad \begin{bmatrix}17\\28\end{bmatrix}\bmod 26=\begin{bmatrix}17\\2\end{bmatrix}\quad \begin{bmatrix}26\\49\end{bmatrix}\bmod 26=\begin{bmatrix}0\\23\end{bmatrix}$$

Step IV:

Use Table I to transform all the above resultant matrices to Alphabets.

$$\begin{bmatrix}10\\15\end{bmatrix}=\begin{bmatrix}K\\P\end{bmatrix}\quad \begin{bmatrix}9\\22\end{bmatrix}=\begin{bmatrix}J\\W\end{bmatrix}\quad \begin{bmatrix}14\\21\end{bmatrix}=\begin{bmatrix}O\\V\end{bmatrix}\quad \begin{bmatrix}3\\25\end{bmatrix}=\begin{bmatrix}D\\Z\end{bmatrix}\quad \begin{bmatrix}21\\9\end{bmatrix}=\begin{bmatrix}V\\J\end{bmatrix}$$

$$\begin{bmatrix}24\\18\end{bmatrix}=\begin{bmatrix}Y\\S\end{bmatrix}\quad \begin{bmatrix}25\\2\end{bmatrix}=\begin{bmatrix}Z\\C\end{bmatrix}\quad \begin{bmatrix}16\\7\end{bmatrix}=\begin{bmatrix}Q\\H\end{bmatrix}\quad \begin{bmatrix}19\\7\end{bmatrix}=\begin{bmatrix}T\\H\end{bmatrix}\quad \begin{bmatrix}17\\16\end{bmatrix}=\begin{bmatrix}R\\Q\end{bmatrix}$$

$$\begin{bmatrix}16\\18\end{bmatrix}=\begin{bmatrix}Q\\S\end{bmatrix}\quad \begin{bmatrix}21\\19\end{bmatrix}=\begin{bmatrix}V\\T\end{bmatrix}\quad \begin{bmatrix}8\\18\end{bmatrix}=\begin{bmatrix}I\\S\end{bmatrix}\quad \begin{bmatrix}17\\2\end{bmatrix}=\begin{bmatrix}R\\C\end{bmatrix}\quad \begin{bmatrix}0\\23\end{bmatrix}=\begin{bmatrix}A\\X\end{bmatrix}$$

Step V:

Write the encrypted message from step 4

The encrypted message is **KPJWOVDZVJYSZCQHTHRQQSVTISRCAX**

Crypt analysis (Process of Decryption):

The process to decrypt the phrase **KPJWOVDZVJYSZCQHTHRQQSVTISRCAX** is similar to encryption, except we use the modular inverse of the original encryption matrix, which is $\begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}$

Step I:

Divide the phrase into 2X1 vectors and assign numerical values to each alphabet from table I to decrypt the message

$$\begin{bmatrix} K \\ P \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \end{bmatrix} \quad \begin{bmatrix} J \\ W \end{bmatrix} = \begin{bmatrix} 9 \\ 22 \end{bmatrix} \quad \begin{bmatrix} O \\ V \end{bmatrix} = \begin{bmatrix} 14 \\ 21 \end{bmatrix} \quad \begin{bmatrix} D \\ Z \end{bmatrix} = \begin{bmatrix} 3 \\ 25 \end{bmatrix} \quad \begin{bmatrix} V \\ J \end{bmatrix} = \begin{bmatrix} 21 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} Y \\ S \end{bmatrix} = \begin{bmatrix} 24 \\ 18 \end{bmatrix} \quad \begin{bmatrix} Z \\ C \end{bmatrix} = \begin{bmatrix} 25 \\ 2 \end{bmatrix} \quad \begin{bmatrix} Q \\ H \end{bmatrix} = \begin{bmatrix} 16 \\ 7 \end{bmatrix} \quad \begin{bmatrix} T \\ H \end{bmatrix} = \begin{bmatrix} 19 \\ 7 \end{bmatrix} \quad \begin{bmatrix} R \\ Q \end{bmatrix} = \begin{bmatrix} 17 \\ 16 \end{bmatrix}$$

$$\begin{bmatrix} Q \\ S \end{bmatrix} = \begin{bmatrix} 16 \\ 18 \end{bmatrix} \quad \begin{bmatrix} V \\ T \end{bmatrix} = \begin{bmatrix} 21 \\ 19 \end{bmatrix} \quad \begin{bmatrix} I \\ S \end{bmatrix} = \begin{bmatrix} 8 \\ 18 \end{bmatrix} \quad \begin{bmatrix} R \\ C \end{bmatrix} = \begin{bmatrix} 17 \\ 2 \end{bmatrix} \quad \begin{bmatrix} A \\ X \end{bmatrix} = \begin{bmatrix} 0 \\ 23 \end{bmatrix}$$

Step II:

Multiply the above resultant 2 X 1 matrices with the Decryption matrix $\begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}$

$$\begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}\begin{bmatrix} 10 \\ 15 \end{bmatrix} = \begin{bmatrix} 135 \\ 390 \end{bmatrix} \quad \begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}\begin{bmatrix} 9 \\ 22 \end{bmatrix} = \begin{bmatrix} 164 \\ 487 \end{bmatrix} \quad \begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}\begin{bmatrix} 14 \\ 21 \end{bmatrix} = \begin{bmatrix} 189 \\ 546 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}\begin{bmatrix} 3 \\ 25 \end{bmatrix} = \begin{bmatrix} 143 \\ 445 \end{bmatrix} \quad \begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}\begin{bmatrix} 21 \\ 9 \end{bmatrix} = \begin{bmatrix} 171 \\ 459 \end{bmatrix} \quad \begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}\begin{bmatrix} 24 \\ 18 \end{bmatrix} = \begin{bmatrix} 234 \\ 648 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}\begin{bmatrix} 25 \\ 2 \end{bmatrix} = \begin{bmatrix} 160 \\ 407 \end{bmatrix} \quad \begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}\begin{bmatrix} 16 \\ 7 \end{bmatrix} = \begin{bmatrix} 131 \\ 352 \end{bmatrix} \quad \begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}\begin{bmatrix} 19 \\ 7 \end{bmatrix} = \begin{bmatrix} 149 \\ 397 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}\begin{bmatrix} 17 \\ 16 \end{bmatrix} = \begin{bmatrix} 182 \\ 511 \end{bmatrix} \quad \begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}\begin{bmatrix} 16 \\ 18 \end{bmatrix} = \begin{bmatrix} 186 \\ 528 \end{bmatrix} \quad \begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}\begin{bmatrix} 21 \\ 19 \end{bmatrix} = \begin{bmatrix} 221 \\ 619 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}\begin{bmatrix} 8 \\ 18 \end{bmatrix} = \begin{bmatrix} 138 \\ 408 \end{bmatrix} \quad \begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}\begin{bmatrix} 17 \\ 2 \end{bmatrix} = \begin{bmatrix} 112 \\ 287 \end{bmatrix} \quad \begin{bmatrix} 6 & 5 \\ 15 & 16 \end{bmatrix}\begin{bmatrix} 0 \\ 23 \end{bmatrix} = \begin{bmatrix} 115 \\ 368 \end{bmatrix}$$

Step III:

Transform all the above resultant matrices to Mod 26

$$\begin{bmatrix}135\\390\end{bmatrix}\text{mod }26=\begin{bmatrix}5\\0\end{bmatrix}\quad\begin{bmatrix}164\\487\end{bmatrix}\text{mod }26=\begin{bmatrix}8\\19\end{bmatrix}\quad\begin{bmatrix}189\\546\end{bmatrix}\text{mod }26=\begin{bmatrix}7\\0\end{bmatrix}$$

$$\begin{bmatrix}143\\445\end{bmatrix}\text{mod }26=\begin{bmatrix}13\\3\end{bmatrix}\quad\begin{bmatrix}171\\459\end{bmatrix}\text{mod }26=\begin{bmatrix}15\\17\end{bmatrix}\quad\begin{bmatrix}234\\648\end{bmatrix}\text{mod }26=\begin{bmatrix}0\\24\end{bmatrix}$$

$$\begin{bmatrix}160\\407\end{bmatrix}\text{mod }26=\begin{bmatrix}4\\17\end{bmatrix}\quad\begin{bmatrix}131\\352\end{bmatrix}\text{mod }26=\begin{bmatrix}1\\14\end{bmatrix}\quad\begin{bmatrix}149\\397\end{bmatrix}\text{mod }26=\begin{bmatrix}19\\7\end{bmatrix}$$

$$\begin{bmatrix}182\\511\end{bmatrix}\text{mod }26=\begin{bmatrix}0\\17\end{bmatrix}\quad\begin{bmatrix}186\\528\end{bmatrix}\text{mod }26=\begin{bmatrix}4\\8\end{bmatrix}\quad\begin{bmatrix}221\\619\end{bmatrix}\text{mod }26=\begin{bmatrix}13\\21\end{bmatrix}$$

$$\begin{bmatrix}138\\408\end{bmatrix}\text{mod }26=\begin{bmatrix}8\\18\end{bmatrix}\quad\begin{bmatrix}112\\287\end{bmatrix}\text{mod }26=\begin{bmatrix}8\\1\end{bmatrix}\quad\begin{bmatrix}115\\368\end{bmatrix}\text{mod }26=\begin{bmatrix}11\\14\end{bmatrix}$$

Step IV:

Use Table I to transform all the above resultant matrices to Alphabets.

$$\begin{bmatrix}5\\0\end{bmatrix}=\begin{bmatrix}F\\A\end{bmatrix}\quad\begin{bmatrix}8\\19\end{bmatrix}=\begin{bmatrix}\text{I}\\\text{T}\end{bmatrix}\quad\begin{bmatrix}7\\0\end{bmatrix}=\begin{bmatrix}\text{H}\\\text{A}\end{bmatrix}\quad\begin{bmatrix}13\\3\end{bmatrix}=\begin{bmatrix}\text{N}\\\text{D}\end{bmatrix}\quad\begin{bmatrix}15\\17\end{bmatrix}=\begin{bmatrix}\text{P}\\\text{R}\end{bmatrix}$$

$$\begin{bmatrix}0\\24\end{bmatrix}=\begin{bmatrix}\text{A}\\\text{Y}\end{bmatrix}\quad\begin{bmatrix}4\\17\end{bmatrix}=\begin{bmatrix}\text{E}\\\text{R}\end{bmatrix}\quad\begin{bmatrix}1\\14\end{bmatrix}=\begin{bmatrix}\text{B}\\\text{O}\end{bmatrix}\quad\begin{bmatrix}19\\7\end{bmatrix}=\begin{bmatrix}\text{T}\\\text{H}\end{bmatrix}\quad\begin{bmatrix}0\\17\end{bmatrix}=\begin{bmatrix}\text{A}\\\text{R}\end{bmatrix}$$

$$\begin{bmatrix}4\\8\end{bmatrix}=\begin{bmatrix}\text{E}\\\text{I}\end{bmatrix}\quad\begin{bmatrix}13\\21\end{bmatrix}=\begin{bmatrix}\text{N}\\\text{V}\end{bmatrix}\quad\begin{bmatrix}8\\18\end{bmatrix}=\begin{bmatrix}\text{I}\\\text{S}\end{bmatrix}\quad\begin{bmatrix}8\\1\end{bmatrix}=\begin{bmatrix}\text{I}\\\text{B}\end{bmatrix}\quad\begin{bmatrix}11\\4\end{bmatrix}=\begin{bmatrix}\text{L}\\\text{E}\end{bmatrix}$$

Step V:

Write the encrypted message from step 4

The decrypted message is **FAITH AND PRAYER BOTH ARE INVISIBLE**

Conclusion

Linear Algebra is the one of the most important branches of Mathematics which has wide range of applications in almost all branches of Sciences. Combination of Linear Algebra and modular arithmetic both played a vital role in Cryptology which explains the lively nature of Mathematics. The matrices that we discussed are useful and powerful in the mathematical analysis and collecting data. Strongly we can say that Linear Algebra, Concept of Matrices all these are all just not a part of academics but these areas of Mathematics are playing a very prominent role in our day to day life. Not only these two areas but also all the fields of Mathematics are cradles of all creations.

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