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ON $\widehat{R}^*$ -τ – DISCONNECTEDNESS AND $\widehat{R}^*$-τ - CONNECTEDNESS IN TOPOLOGICAL SPACE

1. C. PANDIAMMAL, 2.K.VASUKI

1.Assistant Professor, Department Of Mathematics,

2.PG Department Of Mathematics,

NADAR SARASWATHI COLLEGE OF ARTS AND SCIENCE

VADAPUTHUPATTI, THENI – 625 531,Tamil Nadu.

ABSTRACT :

In this paper is to introduce new approach of separate sets, disconnected sets and connected sets called $\widehat{R}^*$-τ – separate sets, $\widehat{R}^*$ -τ – disconnected sets and $\widehat{R}^*$ -τ – connected sets of topological spaces with the help of $\widehat{R}^*$ - open and $\widehat{R}^*$- closed sets. On the basis of new introduce approach, some relationship of $\widehat{\boldsymbol{R}}^*$ -τ – disconnected and $\widehat{R}^*$-τ – connected set with $\widehat{R}^*$-τ – separate set have been investigated thoroughly.

KEYWORDS: $\widehat{R}^*$ -Open set , $\widehat{R}^*$-closed set, $\widehat{R}^*$ -closure, $\widehat{R}^*$- τ – separate sets, $\widehat{R}^*$-τ – disconnected sets and $\widehat{R}^*$-τ – connected sets.

1. INTRODUCTION

There are several natural approaches that can take to rigorously the concepts of connectedness for a topological space. Two most common approaches are connected and path connected and these concepts are applicable intermediate mean value theorem and use to help distinguish topological spaces. This concept play significant role in application in geographic information system studied by Egenhofer and Franzosa(1991) topological modeling studied by Clementine et al.(1994) and motion planning in robotic studied by Farber et al. (2003).

Generalization of open and closed set as like $\hat{R}*$ -open and $\hat{R}*$ - closed set which is nearly to open and closed set respectively. This notion are plays significant role in general topology. In this paper new approach of separate set, disconnected sets, connected sets are called τ – separate sets, $\hat{R}*$ -τ – disconnected sets and $\hat{R}*$ -τ – connected sets with the help of $\hat{R}*$ -open and $\hat{R}*$ - closed set

Throughout this paper (X, τ) and (X, $\tau_{\hat{R}*}$) will always be topological space. For a subset A of topological space X, Int(A), Cl(A) denotes the interior and closure of A respectively.

BASIC PRELIMINARIES

Definition 2.1 A subset A of a space (X, τ) is called $\widehat{R}^*$ - closed if $\hat{g}$ cl (A)⊆ U whenever

A ⊆U and U is R^* - open in (X, τ). The complement of $\widehat{R}^*$ - closed set is $\widehat{R}^*$ - open set.

A subset A of a topological space (X, τ) is called ***R****- closed set rcl (A) U whenever A ⊑ U and U is

regular semi open in (X, τ).

A subset A of a topological space (X, τ) is called regular semi open set U such that U ⊑ A ⊑ cl (U).

Theorem 2.2 : Every closed set is $\widehat{R}^*$- closed set in X.

Proof: Let A be a closed set in X and U be any $\widehat{R}^*$ open in X such that A⊆ U.

Since A is closed cl(A)=A. Hence there every closed set is $\hat{g}$ - closed set $\hat{g}$ cl(A) ⊆ cl(A) = A⊆U. Hence A is an$\widehat{R}^*$closed set in X.

Theorem 2.3: Every $\hat{g}$-closed set is $\widehat{R}^*$ closed set in X.

Proof: Let A be a $\hat{g}$-closed set in X and U be any R*-open in X such that A⊆ U.

Since A is $\hat{g}$ -closed set, $\hat{g}$ cl(A)= A⊆U. Hence A is a$\widehat{R}^*$-closed set in X.

Theorem 2.4: The union of any two $R*$-closed of X is$R*$-closed sets.

Proof: Let A and B are the R*-closed sets in topological space (X, τ). Let U be R*-open set in X such that A∪B ⊆ U, then A⊆U or B⊆U, Since A and B are the R*-closed sets, ^cl(A) ⊆ U or $\hat{g}$ cl(B) ⊆ U we know that $\hat{g}$ cl (A) ∪$\hat{g}$ cl (B) = $\hat{g}$ cl (A∪B) ⊆ U. Therefore A∪B is R*-closed set in X.

THEOREM 2.5: Intersection of any two$R*$-closed sets is also $R*$-closed.

Proof: Let A and B are the R*-closed sets in topological space (X, τ). Let U be R*-open set in X such that A ∩ B ⊆ U, then A⊆ U and B⊆U, since A and B are the R*-closed sets, $\hat{g}$ cl(A) ⊆ U, $\hat{g}$ cl(B) ⊆U, and we know that$\hat{g}$ cl(A) ∩ $\hat{g}$ cl(B) = $\hat{g}$ cl(A∩B) ⊆ U. Therefore A∩B is R*-closed set in X.

3.MAIN RESULTS

DEFINITION 3.1: Let (X ,τ) and (X ,$\tau_{\hat{R}^*}$) be topological spaces. Then the subsets A and B of

(X ,τ) are said to be $\hat{R}^*$ - τ- separate sets if and only if A and B are non – empty set. A ∩ $Cl_{\hat{R}^*}$ (B) and B ∩ $Cl_{\hat{R}^*}$ (A) are empty.

DEFINITION 3.2: Let (X ,τ) and (X ,$\tau_{\hat{R}^*}$) be topological spaces. Then the subsets A and B of (X ,τ) are said to be $\hat{R}^*$- τ- disconnected sets , if there exists if there exists $G_{\hat{R}^*}$ and $H_{\hat{R}^*}$ in $\tau_{\hat{R}^*}$ such that A ∩ $G_{\hat{R}^*}$and A ∩ $H_{\hat{R}^*}$ ≠ ∅.(A ∩ $G_{\hat{R}^*}$) ∩ (A ∩ $H_{\hat{R}^*}$) = ∅.(A ∩ $G_{\hat{R}^*}$) ∪(A ∩ $H_{\hat{R}^*}$) = A.(X ,$\tau_{\hat{R}^*}$) is said to be ∝ - τ- disconnected if there exists non - empty $G_{\hat{R}^*}$ and $H_{\hat{R}^*}$ in $\tau_{\hat{R}^*}$ such that $G_{\hat{R}^*}$∩ $H_{\hat{R}^*}$ ≠ ∅ , $G_{\hat{R}^*}$∪$H_{\hat{R}^*}$= X.

DEFINITION 3.3 : Let(X ,τ) be topological space and A be non – empty subset of X. Let $G_{\hat{R}^*}$be arbitrary in $\tau_{\hat{R}^*}$ then collection , $\tau^A_{\hat{R}^*}$ = {$G_{\hat{R}^*}$∩ A } : $G_{\hat{R}^*}$∈$\tau_\propto$ is a topology on A , called the relative topology of topology $\tau_{\hat{R}^*}$.

THEOREM 3.4: Let (X ,τ) and (X ,$\tau_{\hat{R}^*}$) be topological spaces , then (X ,τ) is $\hat{R}^*$ - τ- disconnected sets if and only if there exists non – empty proper subset of X which is both $\hat{R}^*$ - open and $\hat{R}^*$ - closed .

PROOF Necessity :Let (X ,$\tau_{\hat{R}^*}$) be $\hat{R}^*$ - τ- disconnected . Then by definition of $\hat{R}^*$- τ- disconnected , there exists non – empty set $G_{\hat{R}^*}$ and $H_{\hat{R}^*}$ in $\tau_{\hat{R}^*}$ such that $G_{\hat{R}^*}$∩ $H_{\hat{R}^*}$ = ∅ and $G_{\hat{R}^*}$U $H_{\hat{R}^*}$ = X. Since

$G_{\hat{R}^*}$∩ $H_{\hat{R}^*}$= ∅ and $H_{\hat{R}^*}$ is open in $\tau_{\hat{R}^*}$. Show that $G_{\hat{R}^*}$ = X - $H_{\hat{R}^*}$, but it is $\hat{R}^*$ - closed. Hence $G_{\hat{R}^*}$ is non – empty proper subset of X which is $\hat{R}^*$ - closedas well as$\hat{R}^*$ − open**.**

Sufficiency: Suppose A is non – empty proper subset of X such that it is $\hat{R}^*$ - open as well as

$\hat{R}^*$ - closed .Now A is non – empty $\hat{R}^*$ - closed show that X – A is non – empty and

$\hat{R}^*$ - open. Suppose B = X – A , then A U B = X and A ∩ B = ∅. Thus A and B are non –empty disjoint $\hat{R}^*$ - open as well as $\hat{R}^*$ - closed subset of X such that A U B = X .Consequently , X is $\hat{R}^*$ - τ- disconnected.

THEOREM 3.5: Every (X , $\tau_{\hat{R}^*}$) discrete space is $\hat{R}^*$ - τ- disconnected if the space contains more than one element.

Proof : Let (X , $\tau_{\hat{R}^*}$) discrete space such that X = { a , b } contains more than one element.

But τ is discrete topology so τ = {X , ∅ , {a},{b}} and family of all $\hat{R}^*$ - open sets is

$\tau_{\widehat{R}^*} = \{X, \emptyset, \{a\},\{b\}\}$. All $\widehat{R}^*$- closed sets are X , ∅ , {a},{b} . Since {a} is non – empty proper subset of X which is both $\widehat{R}^*$ - open and $\widehat{R}^*$- closed in X. Finally, we can say that $(X, \tau_{\widehat{R}^*})$ is $\widehat{R}^*$- τ- disconnected by using above theorem.

THEOREM 3.6: If (X, τ) is a disconnected space and $(X, \tau_{\widehat{R}^*})$ is a topological spaces , then and $(X, \tau_{\widehat{R}^*})$ is $\widehat{R}^*$ - τ- disconnected.

PROOF: As (X, τ) is a disconnected and $\tau_{\widehat{R}^*}$ is finer than τ , then by definition of $\widehat{R}^*$ - open and $\widehat{R}^*$ - closed set . $\tau_{\widehat{R}^*} \supseteq \tau$ .Since τ is a subspace of $\tau_{\widehat{R}^*}$ and τ is disconnected,$\tau_{\widehat{R}^*}$is disconnected .

THEOREM 3.7: Let A be a non – empty subset of topological space (X, τ) .Let $\tau^A_{\widehat{R}^*}$ be the relative topology on A , then A is $\widehat{R}^*$ - τ- disconnected if and only if A is $\widehat{R}^*$ - $\tau^A_{\widehat{R}^*}$– disconnected .

PROOF : Necessity : Let A be a $\widehat{R}^*$ - τ- disconnected and let $G_{\widehat{R}^*} \cup H_{\widehat{R}^*}$be a $\widehat{R}^*$- τ- disconnected on A . Then by definition of $\widehat{R}^*$- τ- disconnected , there exists non – empty $G_{\widehat{R}^*}$and $H_{\widehat{R}^*}$in $\tau_{\widehat{R}^*}$ such that $A \cap G_{\widehat{R}^*}$and $A \cap H_{\widehat{R}^*} \neq \emptyset$,$(A \cap G_{\widehat{R}^*}) \cap (A \cap H_{\widehat{R}^*}) = \emptyset$, $(A \cap G_{\widehat{R}^*}) \cup (A \cap H_{\widehat{R}^*}) = A$.Now by the definition of relative topology , if $G_{\widehat{R}^*}$and $H_{\widehat{R}^*}$in $\tau_{\widehat{R}^*}$, then there exists $G^1_{\widehat{R}^*}$ and $H^1_{\widehat{R}^*}$ in $\tau^A_{\widehat{R}^*}$ such that $G^1_{\widehat{R}^*} = A \cap G_{\widehat{R}^*}$and $H^1_{\widehat{R}^*} = A \cap H_{\widehat{R}^*}$. Now by (1) $G^1_{\widehat{R}^*}$ and $H^1_{\widehat{R}^*}$ are non – empty .Hence $A \cap G^1_{\widehat{R}^*}$ and $A \cap H^1_{\widehat{R}^*}$ non – empty .Similarly ,By (2) and (3) , we can say that $(A \cap G_{\widehat{R}^*}) \cap (A \cap H_{\widehat{R}^*}) = \emptyset$ and $(A \cap G_{\widehat{R}^*}) \cup (A \cap H_{\widehat{R}^*}) = A$. Consequently , A is $\widehat{R}^*$ - $\tau^A_{\widehat{R}^*}$– disconnected .

Sufficient : Suppose that A is $\widehat{R}^*$- $\tau^A_{\widehat{R}^*}$– disconnected and $M^A_{\widehat{R}^*} \cap N^A_{\widehat{R}^*}$ is a $\widehat{R}^*$ - $\tau^A_{\widehat{R}^*}$– disconnected on A . By definition , we can say that $M^A_{\widehat{R}^*}, N^A_{\widehat{R}^*} = \emptyset$, $M^A_{\widehat{R}^*}, N^A_{\widehat{R}^*} \in \tau^A_{\widehat{R}^*}$

$(A \cap M^A_{\widehat{R}^*}) \cap (A \cap N^A_{\widehat{R}^*}) = \emptyset$ and $(A \cap M^A_{\widehat{R}^*}) \cup (A \cap N^A_{\widehat{R}^*}) = A$.

Now (2) $\Rightarrow$ there exists $M^1_{\widehat{R}^*}$, $N^A_{\widehat{R}^*} \in \tau_\propto$ such that $M^A_{\widehat{R}^*} = A \cap M^1_{\widehat{R}^*}$, $N^A_{\widehat{R}^*} = A \cap N^1_{\widehat{R}^*}$. But by (1) We can say that $A \cap M^1_{\widehat{R}^*}$, $A \cap N^1_{\widehat{R}^*} \neq \emptyset$ .Now ,

$(A \cap M^A_{\widehat{R}^*}) = (A \cap M^1_{\widehat{R}^*}) = (A \cap A) \cap M^1_{\widehat{R}^*} = A \cap M^1_{\widehat{R}^*}$.

$(A \cap N^A_{\widehat{R}^*}) = (A \cap N^1_{\widehat{R}^*}) = (A \cap A) \cap N^1_{\widehat{R}^*} = A \cap N^1_{\widehat{R}^*}$

Now (3) $\Rightarrow (A \cap M^1_{\widehat{R}^*}) \cap (A \cap N^1_{\widehat{R}^*}) = \emptyset$.

Now (4) $\Rightarrow (A \cap M^1_{\widehat{R}^*}) \cup (A \cap N^1_{\widehat{R}^*}) = A$.

Hence A is $\widehat{R}^*$- τ- disconnected.

THEOREM 3.8: The union of two non – empty ∝ - τ- separate subsets of topological space

(X , $\tau_{\widehat{R}^*}$) is $\widehat{R}^*$ - τ- disconnected.

PROOF : Let A and B be $\widehat{R}^*$ - τ- separate subsets of (X , $\tau_{\widehat{R}^*}$) then by definition of

$\widehat{R}^*$- τ- separate sets , we can say that A and B are non – empty. A ∩ $Cl_{\widehat{R}^*}$ (B) and B ∩ $Cl_{\widehat{R}^*}$ (A) = ∅, A ∩ B =∅. Let X - $Cl_{\widehat{R}^*}$ (A) = $G_{\widehat{R}^*}$and X - $Cl_{\widehat{R}^*}$ (B) = $H_{\widehat{R}^*}$. Then $Cl_{\widehat{R}^*}$ (A) and $Cl_{\widehat{R}^*}$ (B) are non – empty $\widehat{R}^*$ - open subsets of X .Since ,

$G_{\widehat{R}^*}$ U $H_{\widehat{R}^*}$= (X - $Cl_{\widehat{R}^*}$ (A)) U (X - $Cl_{\widehat{R}^*}$ (B) = X - $Cl_{\widehat{R}^*}$ (A) ∩ $Cl_{\widehat{R}^*}$ (B)We have

(A U B) ∩ $G_{\widehat{R}^*}$= (A U B) ∩ (X - $Cl_{\widehat{R}^*}$ (A))= [A ∩ (X - $Cl_{\widehat{R}^*}$ (A)] ∪ [B ∩ (X - $Cl_{\widehat{R}^*}$ (A)] = ∅∪ B.(A U B) ∩ $G_{\widehat{R}^*}$ = B .Similarly ,(A∪ B) ∩ $H_{\widehat{R}^*}$= (A U B) ∩ (X - $Cl_{\widehat{R}^*}$ (B))= [A ∩ (X - $Cl_{\widehat{R}^*}$ (B)] ∪ [B ∩ (X - $Cl_{\widehat{R}^*}$ (B)] = ∅∪ A. (A U B) ∩ $H_{\widehat{R}^*}$ = A.Now (1) shows that (A ∪ B) ∩ $G_{\widehat{R}^*}$, (A ∪ B) ∩ $H_{\widehat{R}^*}$ ≠ ∅ . Additionally , (3) shows that [(A∪ B) ∩ $H_{\widehat{R}^*}$] ∩ [(A ∪ B) ∩ $G_{\widehat{R}^*}$] = ∅ and [(A∪ B) ∩ $H_{\widehat{R}^*}$]∪ [(A ∪ B) ∩ $G_{\widehat{R}^*}$] = A ∪ B .Finally , we can say that there exists $G_{\widehat{R}^*}$and $H_{\widehat{R}^*}$in $\tau_{\widehat{R}^*}$ such that

(A ∪B)∩ $G_{\widehat{R}^*}$, (A ∪ B) ∩ $H_{\widehat{R}^*}$ ≠ ∅ ,[(A∪ B) ∩ $H_{\widehat{R}^*}$] ∩ [(A ∪ B) ∩ $G_{\widehat{R}^*}$] = ∅ and

[(A∪ B) ∩ $H_{\widehat{R}^*}$]∪ [(A ∪ B) ∩ $G_{\widehat{R}^*}$] = A ∪ B .So ,$G_{\widehat{R}^*}$∪$H_{\widehat{R}^*}$ is $\widehat{R}^*$ - τ- disconnected of A ∪ B .

Hence , A ∪ B is $\widehat{R}^*$ - τ- disconnected.

THEOREM 3.10 : Let (X ,τ) and (X ,$\tau_{\widehat{R}^*}$) be topological spaces and A be a subset of X and let $G_{\widehat{R}^*}$∪$H_{\widehat{R}^*}$ be a $\widehat{R}^*$ −τ- disconnected of A . Then A ∩ $G_{\widehat{R}^*}$and A ∩ $H_{\widehat{R}^*}$ are $\widehat{R}^*$- τ- separate subsets of topological space (X , $\tau_{\widehat{R}^*}$).

Proof :Let $G_{\widehat{R}^*}$∪$H_{\widehat{R}^*}$ be a given $\widehat{R}^*$- τ- disconnected of subset A of (X ,$\tau_{\widehat{R}^*}$) .

To prove :A ∩ $G_{\widehat{R}^*}$and A ∩ $H_{\widehat{R}^*}$are $\widehat{R}^*$- τ- separate subsets , We must show that A ∩ $G_{\widehat{R}^*}$and A ∩ $H_{\widehat{R}^*}$ are non – empty ;[$Cl_{\widehat{R}^*}$ (A ∩ $G_{\widehat{R}^*}$)] and (A ∩ $H_{\widehat{R}^*}$) =∅ and [$Cl_{\widehat{R}^*}$ (A ∩ $H_{\widehat{R}^*}$)] and (A ∩ $G_{\widehat{R}^*}$) =∅.Now by our assumption and definition , we can say that there exist $G_{\widehat{R}^*}$and $H_{\widehat{R}^*}$in $\tau_{\widehat{R}^*}$ such that A ∩ $G_{\widehat{R}^*}$and A ∩ $H_{\widehat{R}^*}$ are non – empty ;(A ∩ $G_{\widehat{R}^*}$) ∩(A ∩ $H_{\widehat{R}^*}$) = ∅, (A ∩ $G_{\widehat{R}^*}$) ∪(A ∩ $H_{\widehat{R}^*}$) = A. Evidently , (4) ⇒ (1)To prove : (2) Suppose it is not possible (i.e)

[$Cl_{\widehat{R}^*}$ (A ∩ $G_{\widehat{R}^*}$)] and (A ∩ $H_{\widehat{R}^*}$) ≠ ∅.Then , there exists x ∈$Cl_{\widehat{R}^*}$ (A ∩ $G_{\widehat{R}^*}$) ∩ (A ∩ $H_{\widehat{R}^*}$) which implies that x∈$Cl_{\widehat{R}^*}$ (A ∩ $G_{\widehat{R}^*}$) and x ∈ A .x∈$H_{\widehat{R}^*}$, that is (A ∩ $G_{\widehat{R}^*}$) ∩ $H_{\widehat{R}^*}$ ≠ ∅.

Therefore $(A \cap G_{\widehat{R}^*}) \cap (A \cap H_{\widehat{R}^*}) \neq \emptyset$.But it is contrary to (5).Hence let (X,τ) and $(X,\tau_{\widehat{R}^*})$ be topological spaces and A be a subset of X and let $G_{\widehat{R}^*} \cup H_{\widehat{R}^*}$ be a $\widehat{R}^*$ −τ- disconnected of A . Then $A \cap G_{\widehat{R}^*}$ and $A \cap H_{\widehat{R}^*}$ are $\widehat{R}^*$- τ- separate subsets of topological space $(X, \tau_{\widehat{R}^*})$.

THEOREM 3.11: A subset Y of a topological space X is $\widehat{R}^*$- τ- disconnected iff it is union of two $\widehat{R}^*$ - τ- separate sets.

PROOF Necessity :Suppose Y = A ∪B , where A and B are $\widehat{R}^*$- τ- separate sets of X . By theorem (5) ,A ∪B is $\widehat{R}^*$ - τ- disconnected .Hence , Y is $\widehat{R}^*$ - τ- disconnected.

Sufficiency : Let Y is $\widehat{R}^*$ - τ- disconnected. To prove that there exist two $\widehat{R}^*$ - τ- separate subsets of A ,B in X such that Y = A ∪ B. By assumption ,Y is $\widehat{R}^*$ - τ- disconnected show that there exists a $\widehat{R}^*$ - τ- disconnected $G_{\widehat{R}^*} \cup H_{\widehat{R}^*}$ of Y.Y $\cap G_{\widehat{R}^*}$ and $Y \cap H_{\widehat{R}^*}$ are non – empty ;$(Y \cap G_{\widehat{R}^*}) \cap (Y \cap H_{\widehat{R}^*}) = \emptyset$. $(Y \cap G_{\widehat{R}^*}) \cup (Y \cap H_{\widehat{R}^*}) = Y$ .Since $Y \cap G_{\widehat{R}^*}$ and $Y \cap H_{\widehat{R}^*}$ are separated sets , if we write $A = (Y \cap G_{\widehat{R}^*})$ and $B = (Y \cap H_{\widehat{R}^*})$, then By (3) Y = A ∪B . Finally , we can say that there exists two $\widehat{R}^*$ - τ- separate subsets of A and B in X such that Y = A ∪ B .

THEOREM 3.12: If Y is an $\widehat{R}^*$ - τ- connected subset of topological space X such that Y ⊂ A ∪B , where A and B is $\widehat{R}^*$ - τ- connected , then Y ⊂A and Y ⊂B.

PROOF : Since the inclusion Y A ∪B , holds by the hypothesis we have (A ∪ B) ∩ Y = Y which yields that Y = (Y ∩ A) ∪ (Y ∩ B).Now we want to prove that (Y ∩ A) , (Y ∩ B) = ∅.Suppose , (Y ∩ A) ∪ (Y ∩ B) ≠∅.Now ,$(Y \cap A)\, Cl_{\widehat{R}^*}(Y \cap B) \subset (Y \cap A) \cap Cl_{\widehat{R}^*}(Y) \cap Cl_{\widehat{R}^*}(B)) = (Y \cap (Cl_{\widehat{R}^*}(Y)) \cap (A \cap (Cl_{\widehat{R}^*}(B)) = (Y \cap (A \cap (Cl_{\widehat{R}^*}(B))$

$(Y \cap A)\, Cl_{\widehat{R}^*}(Y \cap) = \emptyset$ .Similarly , we can prove that $Cl_{\widehat{R}^*}(Y \cap A) \cap (Y \cap B) = \emptyset$.

Hence , from the above result we can say that Y is a union of two $\widehat{R}^*$ - τ- separate sets

(Y ∩ A) *and* (Y ∩B) . Consequently, Y is $\widehat{R}^*$- τ- disconnected. But this contradicts the fact that Y is $\widehat{R}^*$ - τ- connected. Hence we can say that (Y ∩ A) ,$(Y \cap B) = \emptyset$.Now, if $(Y \cap A) = \emptyset$, then $Y = \emptyset \cup (Y \cap B) = (Y \cap B)$ which gives that Y ⊂ B. Similarly, Y ⊂ A if $(Y \cap A) = \emptyset$.

THEOREM 3.13: A topological space $(X,\tau_{\widehat{R}^*})$ is $\widehat{R}^*$- τ- connected if and only if there exists

non – empty proper subset which is both $\widehat{R}^*$- open and $\widehat{R}^*$ - closed in X is X itself .

PROOF : Necessity : Assume that $(X,\tau_{\widehat{R}^*})$ is $\widehat{R}^*$ - τ- connected . So , our assumption show that

(X ,$\tau_{\widehat{R}^*}$) is not $\widehat{R}^*$ - τ- connected . i.e .there does not exist a pair of non – empty disjoint $\widehat{R}^*$ - open and $\widehat{R}^*$ - closed A and B such that , A ∪ B = X . This shows that there exists non – empty subsets (other than X) which are both $\widehat{R}^*$ - open and $\widehat{R}^*$ - closed in X .

Sufficiency : Suppose that (X ,$\tau_{\widehat{R}^*}$) is topological space such that the only non – empty subsets of X which is $\widehat{R}^*$ - open and $\widehat{R}^*$ - closed in X is X itself . By hypothesis , there does not exist a partition of the space X . Hence X is not $\widehat{R}^*$ - τ- disconnected i.e $\widehat{R}^*$ - τ- connected.

4.CONCLUSION

We have introduce new approach of separate sets, disconnected sets and connected sets called $\hat{R}*$ -τ – separate sets, $\hat{R}*$ -τ – disconnected sets and $\hat{R}*$ -τ – connected sets of topological spaces with the help of $\hat{R}*$ - open and $\hat{R}*$ - closed sets and investigated their properties.

Finally , we construct a new topological space on a connectedness and disconnectedness

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