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Bianchi Type-III Cosmological Model With Special Form of Deceleration Parameter in $f(R,T)$ Gravity

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Abstract **:** The present study deals with the Bianchi type-III cosmological model with special form of deceleration parameter in presence of perfect fluid in $f(R,T)$ theory of gravity proposed by Harko *et al.*(2011). The solutions of the field equations have been obtained by applying the law of special form deceleration parameter proposed by Singha and Debnath (2009). The physical and geometrical parameters of the model are obtained and discussed in details.

Keywords **:** Bianchi type-III Space-time, $f(R,T)$ Gravity, Perfect fluid, Special form of Deceleration Parameter.

I. INTRODUCTION

Recent cosmological observations such as Type-Ia Supernova (Reiss *et al.* 1998, Perlmutter *et al.*1999, Bernardis *et al.* 2000, Hanany *et al.* 2000, Peeble & Ratra 2003, Padmanabhan 2003) challenge to gravitational theories on the late time acceleration of the universe and the existence of the dark matter. In the last two decades, high red-shift supernovae type Ia (SNIa) (Riess *et al.* 1998, Perlmutter *et al.* 1999, Bennett *et al.* 2003) have carried out cosmological experiments to understand the evolution of present Universe. The observational data obtained from these experiments confirmed the cosmic accelerated expansion of the present universe. Further, recent cosmic observations like Cosmic Microwave Background (CMB) (Spergel *et al.* 2003, Oli 2012), the large-scale structure Tegmark *et al.* (2004) and the CMB radiation (CMBR) (Caldwell & Doran 2004, Carroll *et al.* 2004) discovered the late time cosmic acceleration of the Universe. Hence modified theories of gravitation become popular in astrophysics and modern cosmology. These modified theories of gravitation assumed that General Relativity (GR) imparts at large scales and hence for describing the gravitational field, extension of an Einstein–Hilbert action is required without using dark energy components in the field equations. All these Quantitative observations suggests that there is a hitherto unknown component, dubbed dark energy which is responsible for the cosmic acceleration. In view of this it is now believed that energy composition of universe has 4% ordinary matter, and 20% dark matter and 76% dark energy.

During last decade, there are several modified theories of gravitation have been proposed to understand the mechanism behind late-time acceleration and the presence of dark energy, dark matter in the Universe. Among the various modifications of modified theories, the $f(R)$ theory of gravity has been extensively investigated by several authors (Capozziello *et al.* 2005, Nojiri *et al.* 2006, Nojiri & Odintsov 2007). This $f(R)$ theory gravity can be established through the Einstein–Hilbert action principle in which the matter Lagrangian is replaced by an arbitrary function. Initially, Buchdahl (1970) has proposed the class of the modified theories of gravity. The $f(R)$ theory became more popular after the developments done by Starobinsky (1980). Various aspects of $f(R)$ gravity have been studied by Akbar & Cai (2006), Chiba *et al.* (2007) and Multamäki & Vilja (2006, 2007). Copeland *et al.* (2006) widely discussed reviews of several models of modified $f(R)$ theory of gravity.

After some modification in $f(R)$ theory of gravity, Harko *et al.* (2011) introduced $f(R,T)$ theory of gravitation representing the realistic alternative to Einstein's theory. This modified theory has been attracting more and more attention of researchers in recent years to explain late time acceleration and dark energy. In this modified theory of gravitation, the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace T of the stress-energy tensor. They have obtained the gravitational field equations in the metric formalism, as well as, the equations of motion for test particles, which follow from the covariant divergence of the stress-energy tensor. Generally, the gravitational field equations depend on the nature of the matter source. They have presented several particular models, corresponding to some explicit forms of the function $f(R,T)$. Harko *et al.* (2011) have investigated FRW cosmological model in this theory by choosing appropriate function $f(T)$. They have also discussed the case of scalar fields since scalar fields which plays a vital role in cosmology. The equations of motion of test particles and a Brans-Dicke type formulation of the model are also presented.

Motivated by the work of Harko *et al*. (2011), several researchers have extended this work in $f(R,T)$ theory of gravity for the different cosmological models. Reddy *et al*. (2012) have extended this work for Kaluza-Klein Universe and Adhav (2012) for LRS Bianchi type-I cosmological model in the presence of perfect fluid source in the framework of $f(R,T)$ gravity. Reddy *et al*. (2012) have investigated Bianchi type-III cosmological model filled with perfect fluid in $f(R,T)$ theory. Naidu *et al*. (2013) have studied bulk viscous Bianchi type-V string cosmological model in the framework of $f(R,T)$ theory. Chandel & Ram (2013) have studied spatially homogeneous Bianchi type-III model in the generalized $f(R,T)$ theory with perfect fluid representing empty space for large time. Chaubey & Shukla. (2013) have investigated a new class of Bianchi cosmological models and discussed the well-known astrophysical phenomena, namely the Hubble parameter $H(z)$, luminosity distance (d_L) and distance modulus $\mu(z)$ with redshift. In this literature, various aspects of $f(R,T)$ gravity have been studied by the several researchers (Samanta & Dhal 2013, Ram *et al*. 2013, Sahoo & Mishra 2014, Pawar and Solanke 2015, Katore *et al*. 2016, Sahoo *et al*. 2016a, 2016b, 2017a, 2017b). Also, Samanta & Myrzakulov (2017) have investigated the FRW model having a big-rip singularity, with imperfect fluid in modified $f(R,T)$ gravity. Samanta & Bishi (2017) discussed astrophysical parameters such as the look-back time, luminosity distance, jerk parameter of Bianchi type-III model in the framework of $f(R,T)$ gravity with wet dark fluid using the equation of state. Magnetized SQM with $f(R,T)$ gravity have been studied in detail (Aktas & Aygün 2017, Agrawal & Pawar 2017, Pradhan & Jaiswal 2018). Pawar *et al*.(2019) studied the modified holographic Ricci dark energy model in modified $f(R,T)$ gravity.

Inspired by the above investigations and discussion, in the present paper, Bianchi type-III cosmological model with special form of deceleration parameter in presence of perfect fluid in $f(R,T)$ theory of gravity has been studied. The solutions of the field equations have been obtained by applying the law of special form deceleration parameter proposed by Singha and Debnath (2009). The physical and geometrical parameters of the model are obtained and discussed in details.

II. METRIC AND FIELD EQUATIONS

Consider a spatially homogeneous and anisotropic Bianchi type-III metric in the form as

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{-2mx}B^2(t)dy^2 - C^2(t)dz^2 \quad , \tag{1}$$

where A , B and C are the scale factors (metric tensors) and functions of cosmic time t only and m is a positive constant.

In the present study, assume that the energy momentum tensor for perfect fluid is

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - p g_{\mu\nu} \ , \tag{2}$$

where ρ is the energy density, p is the pressure of the fluid and $u_\mu = (1,0,0,0)$ is the four-velocity vector in the comoving coordinates system which satisfies the condition $u_\mu u^\mu = 1$.

Gravitational theories are depending on the nature of matter source, it is possible to study the different cosmological models of $f(R,T)$ theory of gravity. Harko *et al*. (2011) introduced the new modified gravitational $f(R,T)$ theory with some particular classes of $f(R,T)$ gravity by specifying functional form of f as

$$\left.\begin{array}{l}(i) \quad f(R,T) = R + 2f(T) \\ (ii) \quad f(R,T) = f_1(R) + f_2(T) \\ (iii) \quad f(R,T) = f_1(R) + f_2(R)f_3(T)\end{array}\right\}. \tag{3}$$

Here considers the cosmological consequence for the arbitrary function given by Harko *et al*. (2011) of the form

$$f(R,T) = R + 2\, f(T), \tag{4}$$

where R is the Ricci scalar and T is the stress-energy tensor of the matter $T_{\mu\nu}$.

The gravitational field equations in $f(R,T)$ theory of gravity are given by

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu} + 2\, f'(T)\, T_{\mu\nu} + [2p\, f'(T) + f(T)]g_{\mu\nu} \ , \tag{5}$$

where the overhead prime denotes differentiation with respect to the argument.

Now, in the present study, choose the function $f(T)$ of the trace of the stress-energy tensor of the matter as

$$f(T) = \lambda T \quad , \qquad \text{where } \lambda \text{ is a constant.} \tag{6}$$

The corresponding $f(R,T)$ gravity field equations (5) for Bianchi type-III metric (1) with the help of equations (2) & (6) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = (8\pi + 3\lambda)\, p - \lambda\, \rho \quad , \tag{7}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=(8\pi+3\lambda)\,p-\lambda\,\rho \quad , \tag{8}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{m^2}{A^2}=(8\pi+3\lambda)\,p-\lambda\,\rho \quad , \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}+-\frac{m^2}{A^2}=\lambda\,\rho-(8\pi+3\lambda)\,p \quad , \tag{10}$$

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=0 \quad , \tag{11}$$

where the overhead dot (.) denote derivative with respect to the cosmic time t.

The spatial volume (V) and average scale factor (a) are defined as

$$V=ABC \ , \qquad a=V^{\frac{1}{3}}=(ABC)^{\frac{1}{3}} \ . \tag{12}$$

The directional Hubble parameters in the directions of x, y and z axes respectively are defined as

$$H_x=\frac{\dot{A}}{A}\ , \quad H_y=\frac{\dot{B}}{B}\ , \quad H_z=\frac{\dot{C}}{C}\ . \tag{13}$$

The mean Hubble parameter (H) is given by

$$H=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) . \tag{14}$$

The expansion scalar (θ) is defined as

$$\theta=3H. \tag{15}$$

The deceleration parameter (q) is given by

$$q=-\frac{a\ddot{a}}{\dot{a}^2} . \tag{16}$$

The Shear scalar (σ^2) is defined as

$$\sigma^2=\frac{1}{2}\left[\sum_{i=1}^{3}H_i^2-\frac{1}{3}\theta^2\right] . \tag{17}$$

The mean anisotropic parameter (Δ) of the expansion is defined as

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 , \tag{18}$$

where $H_i(i=1,2,3)$ represents the directional Hubble parameters in the direction of x, y and z axes respectively.

III. SOLUTIONS OF THE FIELD EQUATIONS

Integrating equation (11), one can obtain

$$B=c_1.A \ , \tag{19}$$

where c_1 is an integration constant.

Using equation (19), the field equations (7) – (10) will reduce to

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=(8\pi+3\lambda)\,p-\lambda\,\rho \quad , \tag{20}$$

$$2\frac{\ddot{A}}{A}+\left(\frac{\dot{A}}{A}\right)^2-\left(\frac{m}{A}\right)^2=(8\pi+3\lambda)\,p-\lambda\,\rho \quad , \tag{21}$$

$$\left(\frac{\dot{A}}{A}\right)^2+2\frac{\dot{A}\dot{C}}{AC}-\left(\frac{m}{A}\right)^2=\lambda\,p-(8\pi+3\lambda)\,\rho \ . \tag{22}$$

The field equations (20) – (22) are a system of three independent differential equations with four unknowns A, C, p and ρ. In order to solve the system completely, we use a special form of deceleration parameter which is defined by Singha and Debnath (2009) for FRW metric as

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=-1+\frac{\alpha}{1+a^{\alpha}} . \tag{23}$$

where $\alpha>0$ is a constant and a is mean scale factor of the universe.

Here the model of universe which begins with a decelerating expansion and evolves into a late time accelerating universe which is in agreement with SNe Ia astronomical observations (Riess, *et al.* 2004); Adhav *et al.*(2013a, 2013b) has extended this law for

Bianchi type-I,III,V,VI0 and Kantowski-Sachs cosmological models with dynamical equation of state (EoS) parameter respectively. Recently, several researchers have extended this law for the different cosmological models in presence of different matters source (Adhav, *et al*. 2014 , Katore & Shaikh 2015 , Khade *et al*. 2018).

After solving equation (23) one can obtain the mean Hubble parameter (H) as

$$H = \frac{\dot{a}}{a} = k(1 + a^{-\alpha}), \tag{24}$$

where k is a constant of integration.

On integrating equation (24), we obtain the mean scale factor as

$$a = (e^{k\alpha t} - 1)^{1/\alpha}. \tag{25}$$

Using equations (25) in equation (12), the volume (V) of the universe is given by

$$V = a^3 = (e^{k\alpha t} - 1)^{\frac{3}{\alpha}}. \tag{26}$$

Using equations (12) and (19) in equation (26), we can obtain

$$(e^{k\alpha t} - 1)^{\frac{3}{\alpha}} = c_1\, A^2 C\ . \tag{27}$$

Now, to solve the above equation (27), we use a physical condition that the expansion scalar is proportional to shear scalar. We assume that the expansion scalar (θ) in the model is proportional to the shear's scalar σ which leads to

$$A = C^n\ , \tag{28}$$

where $n \neq 1$ is arbitrary constant.

Using equations (19), (26) and (27), the metric tensors A , B and C are found to be as follows

$$A = (c_1)^{-\frac{n}{2n+1}} .\ (e^{k\alpha t} - 1)^{\frac{3n}{\alpha(2n+1)}}\ , \tag{29}$$

$$B = (c_1)^{\frac{n+1}{2n+1}} .\ (e^{k\alpha t} - 1)^{\frac{3n}{\alpha(2n+1)}}\ , \tag{30}$$

$$C = (c_1)^{-\frac{1}{2n+1}}(e^{k\alpha t} - 1)^{\frac{3}{\alpha(2n+1)}}\ , \tag{31}$$

Using Equations (29)-(31) in equation (1), the Bianchi type-III space-time can be written in the form as

$$ds^2 = dt^2 - \left[(c_1)^{\frac{-2n}{2n+1}} .(e^{k\alpha t} - 1)^{\frac{6n}{\alpha(2n+1)}}\right]dx^2 - \left[(c_1)^{\frac{2(n+1)}{2n+1}} .e^{-2mx}.(e^{k\alpha t} - 1)^{\frac{6n}{\alpha(2n+1)}}\right]dy^2 - \left[(c_1)^{\frac{-2}{2n+1}}.(e^{k\alpha t} - 1)^{\frac{6}{\alpha(2n+1)}}\right]dz^2\ . \tag{32}$$

IV. PHYSICAL PROPERTIES OF THE MODEL

To discuss the physical behaviors of the model given by equation (32), we find the following some physical and kinematical parameters of the model which are very important to discuss the physics of the cosmological model are

Using equations (29) – (31) in equation (12), the volume V and average scale factor a of the model are found to be

$$V = (e^{k\alpha t} - 1)^{\frac{3}{\alpha}}\ . \tag{33}$$

$$a = (e^{k\alpha t} - 1)^{1/\alpha}. \tag{34}$$

Using equations (29) – (31) in equation (13), the directional Hubble parameters in the directions of x, y and z axes respectively are found to be

$$H_x = H_y = \left(\frac{3nk}{2n+1}\right)\frac{e^{k\alpha t}}{(e^{k\alpha t}-1)}\ ,\quad H_z = \left(\frac{3k}{2n+1}\right)\frac{e^{k\alpha t}}{(e^{k\alpha t}-1)}\ . \tag{35}$$

Using equations (29) – (31) in equation (14), the mean Hubble parameter H is given by

$$H = \frac{k\ .\ e^{k\alpha t}}{(e^{k\alpha t}-1)}\ . \tag{36}$$

Using equation (36) in equation (15), the expansion scalar (θ) is found to be

$$\theta = \frac{3k\ .\ e^{k\alpha t}}{(e^{k\alpha t}-1)}\ . \tag{37}$$

Using equation (34) in equation (16), the deceleration parameter (q) is found to be

$$q == -1 + \frac{\alpha}{e^{k\alpha t}} . \tag{38}$$

Using equations (36) and (37) in equation (17), the Shear scalar (σ^2) is found to be

$$\sigma^2 = 3k^2 \left(\frac{n-1}{2n+1}\right)^2 \left(\frac{e^{k\alpha t}}{e^{k\alpha t}-1}\right)^2 . \tag{39}$$

Using equations (35) and (36) in equation (18), the mean anisotropic parameter of the expansion (Δ) is found to be

$$\Delta = 2\left(\frac{n-1}{2n+1}\right)^2. \tag{40}$$

Since the mean anisotropy parameter $\Delta \neq 0$, then the model does not approach isotropy for $n \neq 1$.

Using equations (29) and (31) in equations (21) and (22) and solving one can obtain the energy density and pressure for the model as

$$\rho = \frac{m^2(4\pi+\lambda)\ c_1^{\frac{2n}{2n+1}}}{4(8\pi^2+6\pi\lambda+\lambda^2)}\left[(e^{k\alpha t}-1)^{\frac{-6n}{2n+1}}\right] - \frac{3k^2 n\{2\lambda(n\alpha+1)+e^{k\alpha t}[9\lambda+12\pi(n+2)]\}}{4(8\pi^2+6\pi\lambda+\lambda^2)(2n+1)^2}\left[\frac{e^{k\alpha t}}{(e^{k\alpha t}-1)^2}\right]. \tag{41}$$

$$p = \frac{m^2(8\pi+3\lambda)(4\pi+\lambda)\ c_1^{\frac{2n}{2n+1}}}{4\lambda(8\pi^2+6\pi\lambda+\lambda^2)}\left[(e^{k\alpha t}-1)^{\frac{-6n}{2n+1}}\right] + \frac{3k^2 n}{\lambda(2n+1)^2}\left\{3(n+2)e^{k\alpha t}\ \frac{(8\pi+3\lambda)\{2\lambda(n\alpha+1)+e^{k\alpha t}[9\lambda+12\pi(n+2)]\}}{4(8\pi^2+6\pi\lambda+\lambda^2)}\right\}\left[\frac{e^{k\alpha t}}{(e^{k\alpha t}-1)^2}\right]. \tag{42}$$

V. DISCUSSION

From equation (33), it is observed that the spatial volume V is vanishes at $t = 0$, then it expands exponentially as t increase and become infinitely large as $t \to \infty$ as shown in Fig-1. The directional Hubble parameters are infinite at $t = 0$ and finite at $t = \infty$. It is observed that the expansion of universe is infinite at time $t = 0$, but as cosmic time t increases, it decreases and halts at a finite value after some finite value of t. It is also observed that the shear scalar $\sigma^2 \to \infty$ as time $t \to 0$ and it decreases to null as time increases as shown in Fig-2.

From equation (40), it is also observed that the mean anisotropic parameter of the model is independent of cosmic time and it is a constant throughout the evolution of the Universe. It is observed that the mean anisotropic parameter (Δ) is non-zero for $n \neq 1$ and hence the present model does not approach isotropy. Hence the Universe expands anisotropically.
From equations (41) and (42), observed that the model starts with big bang having infinite energy density and pressure and as time increases then density and pressure both are monotonically decreases with respect to the cosmic time. In future, both will be zero.

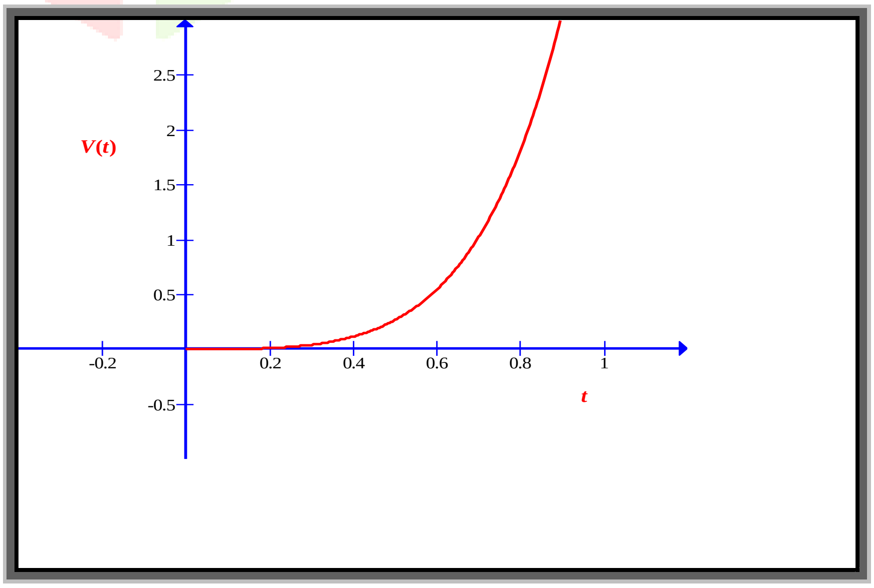

Figure-1 : Spatial volume $V(t)$ against cosmic time t for $\alpha = k = 1$.

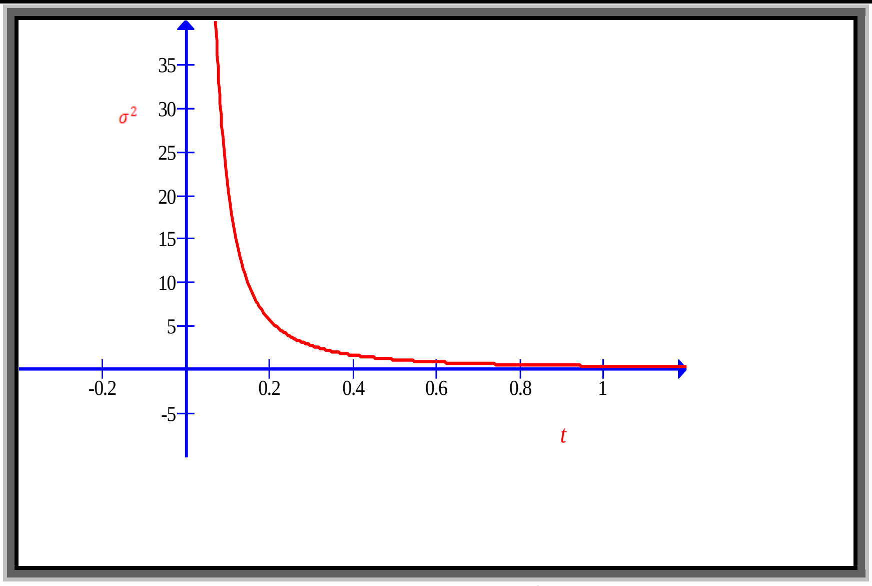

Figure-2 : The variation of Shear scalar σ^2 against cosmic time *t*.

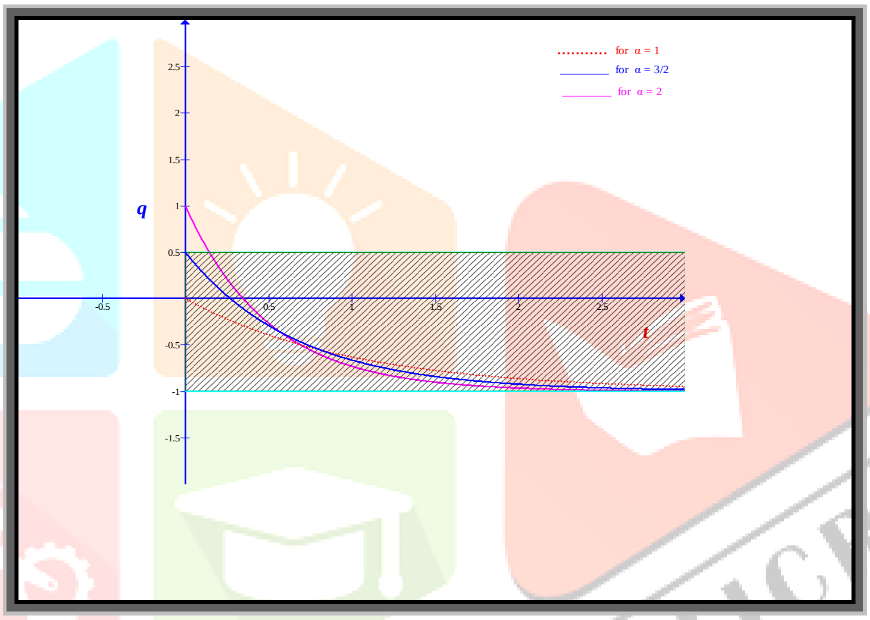

Figure-3 : The deceleration parameter q against cosmic time t .

It is observed that the deceleration parameter q varies from $+1$ to -1 for $\alpha = 2$ as shown in Fig-3. It shows that the universe accelerates after an epoch of deceleration. Also, observations of SN Ia (Reiss, *et al*. 1998, 2004; Perlmutter, *et al*. 1997,1999) reveal that the present universe is accelerating and the value of deceleration parameter lies somewhere is in the range $-1 < q < 0$. For $\alpha = 3/2$, the deceleration parameter q is in the range $-1 \leq q \leq 0.5$ (shaded region in the Figure-3) which matches with the observations made by Riess *et al*. (1998) and Perlmutter *et al*. (1999) and the present-day universe is undergoing accelerated expansion. It is follows that in the derived model, one can choose the values of deceleration parameter consistent with the observations.

VI. CONCLUSION

In this paper, we have investigated the Bianchi type-III cosmological model with special form of deceleration parameter in presence of perfect fluid in $f(R,T)$ theory of gravity proposed by Harko *et al.* (2011). The solutions of the field equations have been obtained by using special form of deceleration parameter.

It is observed that the model has no initial singularity and shows the late time accelerated expansion of the universe for large t . The directional Hubble parameters, the shear scalar, energy density and pressure are infinite at initial epoch while they approach to null at large time. It is also observed that the anisotropy of the model is a constant throughout the evolution of the Universe. It is observed that the mean anisotropic parameter (Δ) is non-zero for $n \neq 1$ and hence the present model does not approach isotropy.

Also, the observations of SN Ia (Reiss, *et al.*1998, 2004; Perlmutter, *et al.*1997, 1999) reveal that the present universe is accelerating. The deceleration parameter for different values of α $(\alpha = 1\,,\ 3/2\,,\ 2)$ lies somewhere is in the range $-1 \leq q \leq 0$ which results matches with the observations made by Riess *et al.* (1998) and Perlmutter *et al.* (1999) and the present-day universe is undergoing accelerated expansion. It follows that in the present model, one can choose the values of deceleration parameter consistent with the observations. The model obtained and presented here which represents an accelerating and expanding cosmological model of the universe. Also, this model is consistent with the recent observations of type-Ia supernovae. It is observed that the results of the present model are different from the work of Reddy, *et al*. (2012, 2013), Adhav, *et al*. (2013), Katore, *et al*. (2015).

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