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On GPR-Compactness in topological spaces

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Abstract: ***The purpose of this paper to introduce a new class of continuous functions like GPR-compactness functions and study some of their properties.***

Key words: GPR-continuity, gpr-continuity, gpr-continuous, ect.

I. INTRODUCTION

Maki et al [5,6] defined α-generalized closed sets and the notion of generalized preclosed set (briefly gp closed) sets in topological which are weaker forms of closed it is observed that every αg-closed set is gp-[2] first defined a new closed set using regular-open set known as generalized pre regular closed. Palaniappan and Rao [8] defined the notion of regular generalized (briefly rg-closed) Gnanambal closed set (briefly gpr-closed). In this chapter we study the properties of gpr-closed sets and gpr-continuous functions

II PRELIMINARIES :

Throughout this dissertation work (X, τ), (Y, σ) and (Z,η) represent non-empty topological spaces on which no separation axioms are assumed unless explicitly stated, and they are simply written X, Y and Z respectively. For a subset A of (X, τ), the closure of A, the interior of A with respect to τ are denoted by cl(A) and int(A) respectively. The complement of A is denoted by A^c. The α-closure of A is the smallest α-closed set containing A and this is denoted by αcl(A).

Definition : A subset A of a topological space (X, τ) is called a

1. semi-open set [3] if A ⊆ cl(int(A)) and semi-closed set if int (cl(A)) ⊆ A.
2. pre-open set [4] if A ⊆ int(cl(A)) and pre-closed set if cl(int(A)) ⊆ A.
3. α-open set [7] if A ⊆ int(cl (int(A))) and α-closed set if cl (int (cl (A))) ⊆ A
4. semi pre open set [1] if A ⊆ cl(int(cl(A))) and semi pre closed [1] if int(cl(int (A))) ⊆ A .
5. regular open set [9] if A=int(cl(A)) and a regular closed set if A=cl(int(A)).

III GPR-Compactness in topological spaces.

In this section we study the concept of GPR-compactness and GPR-connectedness using gpr-open sets and studied some of their characterizations.

Definition 3.1: A collection $\{A_i : i\in I\}$ of gpr-open sets in a topological space (X,τ) is called a gpr - open cover of a subset A in (X, τ) if $A \subseteq \bigcup_{i\in I} A_i$ holds.

Definition 3.2 : A topological space (X, τ) is GPR-compact if every gpr-open cover of (X, τ) has a finite subcover.

Definition 3.3 : A subset A of a topological space (X,τ) is said to be GPR -compact relative to (X,τ), if for every collection $\{A_i : i\in I\}$ of gpr-open subsets of gpr-open subsets of (X, τ) such that $A \subseteq \bigcup_{i\in I} A_i$ there exists a finite subset I_0 of I such that $A \subseteq \bigcup_{i\in I_0} A_i$.

Definition 3.4 : A subset A of a topological space (X,τ) is called GPR-compact if A is GPR-compact of the subspace of (X,τ).

Theorem 3.5 : A gpr -closed subset of a GPR-compact space (X,τ) is GPR -compact relative to (X,τ).

Proof: Let A be gpr-closed subset of a GPR-compact space X. Then $X – A$ is gpr-open. Let Ω be a gpr-open cover for A. Then $\{\Omega, X – A\}$ is a gpr – open cover for X. Since X is GPR-compact, it has a finite subcover, say, $\{p_1, p_2,\ldots, p_n\} = \Omega_1$. If $X – A \notin \Omega$, then Ω_1 is a finite subcover of A. If $X – A \in \Omega_1$, then Ω_1- (X – A) is a sub cover of A. Thus A is a GPR -compact relative to (X,τ).

Theorem 3.6: The image of a GPR-compact space under GPR-continuous map is compact.

Proof: Let $f: X \to Y$ be gpr-continuous map from a GPR-compact space (X, τ) onto a topological space (Y, μ). Let $\{A_i : i\in I\}$ be an open cover of (Y, σ). Since f is gpr -continuous, $\{f^{-1}(A_i) : i\in I\}$ is an gpr-open cover of (X, τ). Since (X, τ) is GPR-compact, the $\omega\alpha$-open cover of (X, τ), $\{f^{-1}(A_i) : i\in I\}$ has a finite subcover say$\{f^{-1}(A_i) : i = 1,\ldots,n\}$. Therefore $X = \bigcup_{i=1}^{n} f^{-1}(A_i)$ which implies $f(X) = \bigcup_{i=1}^{n} A_i$, this implies $Y = \bigcup_{i=1}^{n} A_i$ that is,$\{A_1, A_2 \ldots A_n\}$ is a finite subcover of $\{A_i : i\in I\}$ for (Y, μ). Hence (Y, μ) is compact.

Theorem 3.7 : If a map $f: X \to Y$ is gpr-irresolute and a subset S of X is GPR-compact relative to (X, τ), then the image f(S) is GPR-compact relative to (Y, σ).

Proof: Let $\{A_i : i\in I\}$ be a collection of gpr-open sets in (Y, σ) such that $f(S) \subseteq \bigcup_{i\in I} A_i$ .Then $S \subseteq \bigcup_{i=1}^{n} f^{-1}(A_i)$, where $\{f^{-1}(A_i) : i\in I\}$ is gpr-open set in (X, τ). Since S is GPR-compact relative to (X, τ), there exist finite subcollections $\{A_1, A_2, \ldots ,A_n\}$ such that $S \subseteq \bigcup_{i=1}^{n} f^{-1}(A_i)$..That is $f(S) \subseteq \bigcup_{i=1}^{n} A_i$. Hence f(S) is GPR-compact relative to (Y, σ).

Definition 3.8 : A topological space (X,τ) is said to be GPR-connected if (X,τ) cannot be written as a disjoint union of two non-empty gpr-open sets.

A subset of (X,τ) is GPR-connected if it is GPR-connected as a subspace.

Theorem 3.9 : For a topological space (X,τ) the following are equivalent

(i) (X,τ) is GPR-connected.

(ii) The only subsets of (X, τ) which are both gpr-open and gpr-closed are the empty set ϕ and X.

(iii) Each gpr-continuous map of (X, τ) into a discrete space (Y, μ) with at least two points is a constant map.

Proof: (i) $\Rightarrow$ (ii) Let G be an gpr-open and a gpr-closed subset of (X,τ). Then $X - G$ is also both gpr-open and gpr-closed. Then $X = G \cup (X - G)$, a disjoint union of two non-empty gpr-open sets which contradicts to the fact that (X, τ) is GPR-connected. Hence $G = \phi$ or X.

(ii) $\Rightarrow$(i) Suppose that $X = A \cup B$ where A and B are disjoint non-empty gpr-open subsets of (X,τ). Since $A = X - B$ then A is both gpr-closed and gpr-open. By assumption $A = \phi$ or X, which is a contradiction. Hence (X, τ) is GPR-connected

(ii) $\Rightarrow$(iii) Let $f: (X, \tau) \to (Y, \mu)$ be a gpr-continuous map, where (Y, μ) is discrete space with at least two points. Then $f^{-1}(\{y\})$ is gpr-closed and gpr-open for each $y \in Y$. That is (X, τ) is covered by gpr-closed and gpr-open covering $\{f^{-1}(\{y\}) : y \in Y\}$. By assumption, $f^{-1}(\{y\}) = \phi$ or X for each $y \in Y$. If $f^{-1}(\{y\}) = \phi$ for each $y \in Y$, then f fails to be a map. Therefore there exist at least one point say $f^{-1}(\{y_1\}) \neq \phi$, $y_1 \in Y$ such that $f^{-1}(\{y_1\}) = X$. This shows that f is a constant map.

(iii) $\Rightarrow$ (ii) Let G be both gpr-closed and gpr-open in (X, τ). Suppose $G \neq \phi$. Let $f: (X, \tau) \to (Y, \mu)$ is gpr-continuous map defined by $f(G) = \{a\}$ and $f(X - G) = \{b\}$ where $a \neq b$ and $a, b \in Y$. By assumption, f is constant and so $G = X$.

THEOREM 3.10 : EVERY GPR-CONNECTED SPACE IS CONNECTED.

Proof: Let (X, τ) be an GPR-connected space. Suppose that (X, τ) is not connected. Then $X = A \cup B$ where A and B are disjoint non-empty open subsets of (X, τ). Then A and B are gpr-open disjoint sets and $X = A \cup B$ where A and B are disjoint non-empty gpr-open subsets of (X, τ). This contradicts to the fact that (X, τ) is GPR-connected and so (X, τ) is connected.

The converse of the above theorem need not be true as seen from the following example.

Example 3.11 : Let $X= \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{a, b\}, X\}$. Then (X, τ) is not GPR-connected space but it is connected space because every subset of X is gpr-open. The only clopen sets of X are ϕ, X. Therefore X is connected.

Theorem 3.12 : If $f: (X, \tau) \to (Y, \sigma)$ is gpr-continuous surjection and (X, τ) is GPR-connected, then (Y, σ) is connected.

Proof: Suppose that (Y, σ) is not connected. Let Y = A ∪ B where A and B are disjoint non-empty open subsets in (Y, σ). Since f is gpr-continuous, X = $f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty gpr-open subsets in (X, τ). This contradicts to the fact that (X, τ) is GPR-connected. Hence (Y, σ) is connected.

Theorem 3.13 : If f: (X, τ) → (Y, σ) is gpr-irresolute surjection and (X, τ) is GPR-connected, then (Y, σ) is GPR-connected.

Proof: Suppose that (Y, σ) is not GPR-connected. Let Y = A ∪ B where A and B are disjoint non-empty gpr-open subsets in (Y, σ). Since f is gpr-continuous and surjection, X = $f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty gpr-open subsets in (X, τ). This contradicts to the fact that (X, τ) is GPR-connected. Hence (Y, σ) is GPR-connected.

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